1 Approximation of Functions

1. Write a MATLAB program to generate Chebyshev polynomials. (**Hint:** Use the M-file given in the book.)
   Solution:

```
function T=Tch(n)
if n==0
disp('1')
elseif n==1
disp('x')
else
t0='1';
t1='x';
for i=2:n
T=symop('2*x','*',t1,'-',t0);
t0=t1;
t1=T;
end
end
```

   save with the name *Tch.m*. Then;

```
>>Tch(5)
ans =2*x*(2*x*(2*x*(2*x^2-1)-x)-2*x^2+1)-2*x*(2*x^2-1)+x
>>collect(ans)
ans= 16*x^5-20*x^3+5*x
```

   For Matlab7 users,

```
function s = symop(varargin);
%SYMOP Obsolete Symbolic Toolbox function.
% SYMOP takes any number of arguments, including  %'+','-','*','/','^',
% '(' and ')', concatenates them and symbolically evaluates the %result.
% Copyright (c) 1993-98 by The MathWorks, Inc.
% $Revision: 1.2 $ $Date: 1997/11/29 01:06:41 $
s = [];
for k = 1:nargin
   v = varargin{k};
   if ~ischar(v), v = char(sym(v)); end
   switch v
```

```
   case {'+','-','*','/','^','(',')'}
      s = [s v];
   otherwise
      s = [s 'sym(''' v ''')'];
   end
end
s = eval(s);
```

2. Write a MATLAB program using Chebyshev polynomials to economize a Maclaurin series for e^x in the interval [0,1] with a precision of 0.0001. Tabulate the error values. (**Hint:** Utilize the M-file given in the book.) Solution:

```
>> syms x
>> ts=taylor(exp(x),8)
ts =1+x+1/2*x^2+1/6*x^3+1/24*x^4+1/120*x^5+1/720*x^6+1/5040*x^7
>> cs=collect(Tch(7))
cs = 64*x^7-112*x^5+56*x^3-7*x
>> es=ts-cs/factorial(7)/2^6
es =
1+46081/46080*x+1/2*x^2+959/5760*x^3+1/24*x^4+5/576*x^5+1/720*x^6
>> vpa(es,7)
ans = 1.+1.000022*x+.5000000*x^2+.1664931*x^3+.4166667e-1*x^4
+.8680556e-2*x^5+.1388889e-2*x^6
```

3. The Chebyshev series and Maclaurin series for e^x are given as the following;

$$e^x = 0.9946 + 0.9973x + 0.5430x^2 + 0.1772x^3$$
$$e^x = 1 + x + 0.5x^2 + 0.1667x^3$$

- Tabulate the error values for the interval [-1,1].
- Plot the error values for the interval.

Solution:

```
function week8lsgitem3(ll,ul,s)
format short;
%format long;
disp('     x         e^x    Chebyshev   Error   Maclaurin   Error')
x =(ll:s:ul)';
taylor=exp(x);
max=(ul-ll)/s+1;
for i=1:max
chebyshev=(0.9946 + 0.9973*x(i) + 0.5430*x(i)^2 + 0.1772*x(i)^3);
errorchebyshev(i)=taylor(i)-chebyshev;
maclaurin=(1+x(i)+0.5*x(i)^2+0.1667*x(i)^3);
errormaclaurin(i)=taylor(i)- maclaurin;
D=[x(i),taylor(i),chebyshev,errorchebyshev(i),maclaurin,errormaclaurin(i)];
disp(D);
end
plot(x,errorchebyshev,'o',x,errormaclaurin,'-')
```

save with the name *week8lsgitem3.m*. Then;

```
>> week8lsgitem3(-1,1,0.1)
     x         e^x    Chebyshev   Error   Maclaurin   Error
   -1.0000    0.3679    0.3631    0.0048    0.3333    0.0346

   -0.9000    0.4066    0.4077   -0.0011    0.3835    0.0231

   -0.8000    0.4493    0.4536   -0.0042    0.4346    0.0147

   -0.7000    0.4966    0.5018   -0.0052    0.4878    0.0088

   -0.6000    0.5488    0.5534   -0.0046    0.5440    0.0048

   -0.5000    0.6065    0.6096   -0.0030    0.6042    0.0024

   -0.4000    0.6703    0.6712   -0.0009    0.6693    0.0010

   -0.3000    0.7408    0.7395    0.0013    0.7405    0.0003

   -0.2000    0.8187    0.8154    0.0033    0.8187    0.0001
```

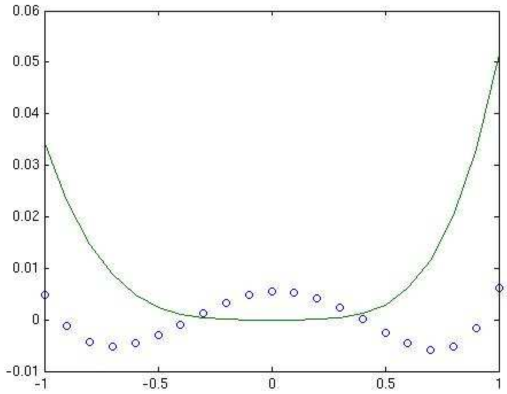

Figure 1: plot(x,errorchebyshev,'o',x,errormaclaurin,'-').

-0.1000	0.9048	0.9001	0.0047	0.9048	0.0000
0	1.0000	0.9946	0.0054	1.0000	0
0.1000	1.1052	1.0999	0.0052	1.1052	0.0000
0.2000	1.2214	1.2172	0.0042	1.2213	0.0001
0.3000	1.3499	1.3474	0.0024	1.3495	0.0004
0.4000	1.4918	1.4917	0.0001	1.4907	0.0012
0.5000	1.6487	1.6511	-0.0024	1.6458	0.0029
0.6000	1.8221	1.8267	-0.0046	1.8160	0.0061
0.7000	2.0138	2.0196	-0.0058	2.0022	0.0116
0.8000	2.2255	2.2307	-0.0051	2.2054	0.0202
0.9000	2.4596	2.4612	-0.0016	2.4265	0.0331
1.0000	2.7183	2.7121	0.0062	2.6667	0.0516