1 Approximation of Functions, Fourier Series

To construct the trigonometric polynomial of order M of the form

$$f(x) = \frac{A_0}{2} \sum_{j=1}^{M} [A_j cos(jx) + B_j sin(jx)]$$

based on the N equally spaced values $x_k = -\pi + 2\pi k/N$,for $k = 1, 2, \ldots, N$.
The construction is possible provided that $2M + 1 \leq N$.
The following program constructs vectors A and B that contain the coefficients A_j and B_j, respectively, of the equation above of order M.

```
function [A,B]=tpcoeff(X,Y,M)
%Input    - X is a vector of equally spaced abscisssas in [-pi, pi]
%         - Y is a vector of ordinates
%         - M is the degree of the trigomometric polynomial
%Output   - A is a vector containing the coefficients of cos(jx)
%         - B is a vector containing the coefficients of sin(jx)

N=length(X)-1;
max1=fix((N-1)/2);
if M>max1
   M=max1;
end
A=zeros(1,M+1);
B=zeros(1,M+1);
Yends=(Y(1)+Y(N+1))/2;
Y(1)=Yends;
Y(N+1)=Yends;
A(1)=sum(Y);
for j=1:M
   A(j+1)=cos(j*X)*Y';
   B(j+1)=sin(j*X)*Y';
end
A=2*A/N;
B=2*B/N;
A(1)=A(1)/2;
```

You are given the function $Y(X) = X$ for the interval $[-\pi, \pi]$.

1. Use the MATLAB program given above to calculate A_js and B_js. (**Hint:** You should first calculate all the Y values for a given M, say 100.)

2. The following program will evaluate the $f(x)$ of order M at a particular value of x. A, B and M values are taken from the previous item.

```
function z=tp(A,B,x,M)
z=A(1);
for j=1:M
   z=z+A(j+1)*cos(j*x)+B(j+1)*sin(j*x);
end
```

Study the following commands:

```
>>x=-pi:.01:pi
>>y=tp(A,B,x,M)
>>plot(x,y,X,Y,'o')
```

3. Repeat the procedure for the M-values, 10, 20, 50. Compare the results