Ceng 375 Numerical Computing
Final
Jan 11, 2008 15.00–17.00
Good Luck!

Each question is 25 pts. Solve only 4 of them.

1. The following table and figure are given as

years	Population (million)
1965	31,4
1975	40,4
1985	50,7
1990	56,5
2000	67,8

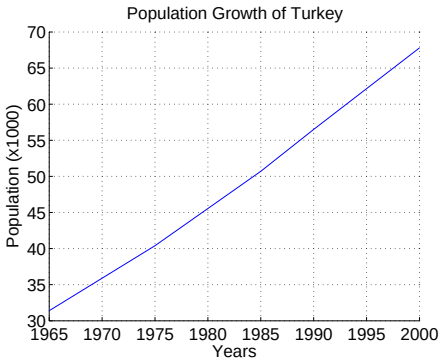

Figure 1: The population growth of Turkey between years of 1965 and 2000

i What is the relationship that the graph suggests? Use least squares method to find out the necessary parameters of this suggested formula.

ii Estimate the population at the years of 1995, 2007,2010 with least squares method.

iii Fit a cubic (P_3) polynomial to the given data.

iv Estimate the population at the years of 1995, 2007,2010 with fitted polynomial.

v Compare your results for both least squares and interpolated polynomial methods.

2. Consider the difference approximation

$$f'_n = \frac{-f_{n+2} + 4f_{n+1} - 3f_n}{2h}$$

where $\underline{\underline{f_n \text{ means } f(x)}}$ and $\underline{\underline{f_{n+1} \text{ means } f(x+h)}}$

i Use this formula to approximate the derivative of $f(x) = cos(x)$ at $x = 0$ using step sizes of $h = 0.10$ and 0.20.

ii Make an error analysis. Estimate the order of error ($O(h^?)$).
Hints: The ratio of errors and the difference with the exact value.

3. Find the power fit $y = Ax^2$ for the following data,

x_k	y_k
2.0	5.1
2.3	7.5
2.6	10.6
2.9	14.4
3.2	10.0

Hint: Use the least-squares method and find only the value of "A".

4. Write the expression to economize the the Maclaurin series for e^{3x} with the precision 4.0 by using Chebyshev polynomials.
   Hint: The two-term recursion formula

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$$
$$T_0(x) = 1$$
$$T_1(x) = x$$

5. Consider the following table of data

x_i	f_i
0.0000	0.0000
0.2000	0.5879
0.4000	1.0637
0.6000	1.3927
0.8000	1.5573
1.0000	1.5575
1.2000	1.4091

i Approximate $\int_0^{1.2} f(x)\mathrm{dx}$ using the *Trapezoidal Rule* and a step size of $h = 0.4$.

ii Approximate $\int_0^{1.2} f(x)\mathrm{dx}$ using the *Trapezoidal Rule* and a step size of $h = 0.2$.

iii Estimate the *error* in your answer to previous item. **Hint:** Use the procedure to estimate the proportionality factor, C.