Ceng 375 Numerical Computing
Final
Aug 8, 2005 09.00–11.00
Good Luck!

1 (20 Pts)

i A three digit, decimal machine which rounds all intermediate calculations, calculates the value of

$$f(x) = x^2 - 6x + 8 \; for \; x = 1.99 \; as \; \overline{f}(1.99) = 0.0600$$

What are the forward error associated with this calculation?

2 (20 Pts) In Newton's method the approximation x_{n+1} to a root of $f(x) = 0$ is computed from the approximation x_n using the equation

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

i Derive the above formula, using a Taylor series of $f(x)$.

ii For $f(x) = 2x - 5^{-2x}$, refine the approximation $x_0 = 0.23$ to the unique root of $f(x)$ by carrying out one iteration of Newton's method.

iii What are the assumptions under which the above formula for Newton's method work?

3 (25 Pts) Consider the matrix

$$A = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 1 & 3 \\ -3 & 0 & 5 \end{bmatrix}$$

i Use the Gaussian elimination method to triangularize this matrix and from that gets its determinant.

ii Get the inverse of the matrix through Gaussian elimination.

iii Get the inverse of the matrix through Gauss-Jordan method.

4 (25 Pts)

i Find the Fourier coefficients for $f(x) = x^3$ if it is periodic and one period extends from $x = -1$ to $x = 2$. Do not evaluate the integrals.

ii Write the Fourier series expansion for this function up to 3^{rd} term.

5 (20 Pts) Consider the difference approximation

$$f'_n = \frac{-f_{n+2} + 4f_{n+1} - 3f_n}{2h}$$

where f_n means $f(x)$ and f_{n+1} means $f(x+h)$

i Use this formula to approximate the derivative of $f(x) = cos(x)$ at $x = 0$ using step sizes of $h = 0.10$ and 0.20.

ii Make an error analysis. Estimate the order of error $(O(h^?))$.
Hints: The ratio of errors and the difference with the exact value.