Ceng 375 Numerical Computing
Midterm
July 18, 2005 09.00–11.00
Good Luck!

1 (20 Pts) Consider the function:

$$f(x) = 5x - e^{-x}$$

  i Show that this function has a simple root in the interval $0 < x < 1$

  ii Estimate this root using two iterations of the Secant Method.

  iii Estimate the error in your answer to part ii.

2 (25 Pts) Consider the function:

$$f(x) = cos(x) - 2x = 0$$

i Use two iterations of Newton s method to estimate the root of this function between $x = 0.0$ and $x = 1.0$

ii Estimate the error in your answer to part i.

iii Approximately how many iterations of the bisection method would have been required to achieve the same error?

3 (30 Pts) Solve this system by Gaussian elimination with pivoting

$$\begin{bmatrix} 1 & -2 & 4 & 6 \\ 8 & -3 & 2 & 2 \\ -1 & 10 & 2 & 4 \end{bmatrix}$$

i How many row interchanges are needed?

ii Repeat without any row interchanges. Do you get the same results?

iii You could have saved the row multipliers and obtained a LU equivalent of the coefficient matrix. Use this LU to solve but with right-hand sides of $[1, -3, 5]^T$

4 (25 Pts) Consider solving the following linear system by the Jacobi method.

$$4x_1 + x_2 = 5$$
$$x_1 + 5x_2 = 6$$

i Write down the Jacobi iteration formula for this problem given initial guess $x^{(0)} = 0$.

ii Assume that the error (vector) at iteration k is denoted by $e^{(k)}$ and that $||e^{(0)}|| = 1$. How many iterations do we need before $||e^{(k)}|| \leq 10^{-4}$?