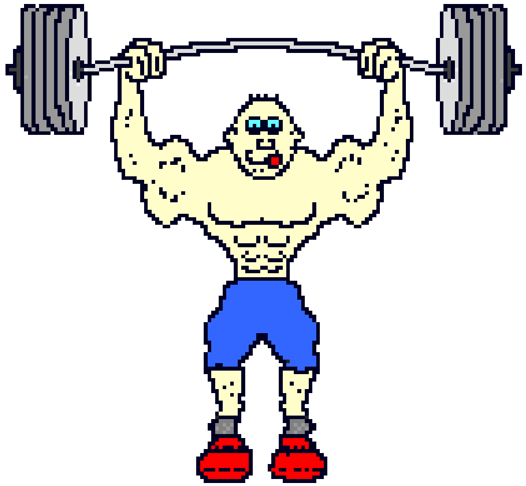
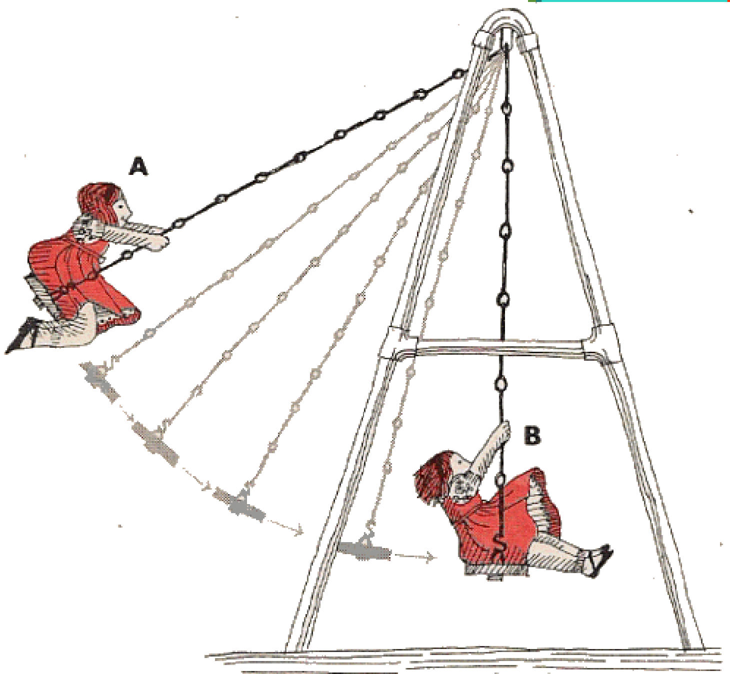


Chapter 7

Kinetic Energy and Work



İZMİR KÂTİP ÇELEBİ ÜNİVERSİTESİ

7 KINETIC ENERGY AND WORK

7.2 What is Energy?

- **Question:** What is energy?
- **Answer:** Energy is a scalar quantity associated with the *state* (or condition) of one or more objects.

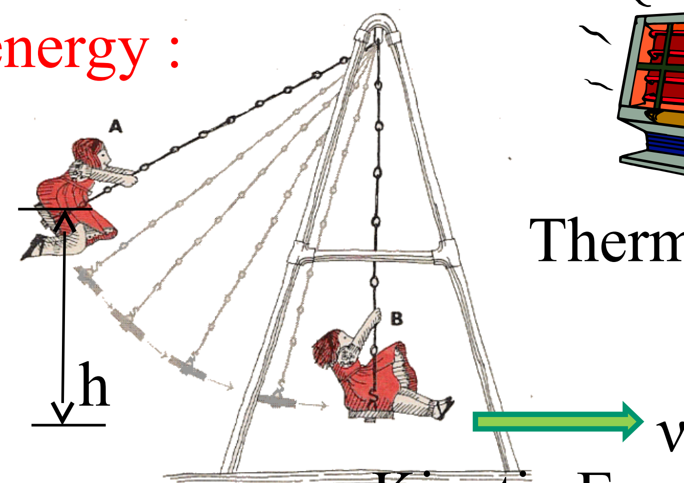
position, velocity

- Energy is required for any sort of motion. Some characteristics:

  1)Energy can be transformed from one type to another and **transferred** from one object to another.

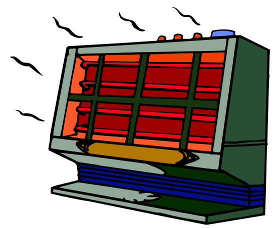
  2)The total amount of energy is always the **same** (energy is ***conserved***).

Forms of energy :

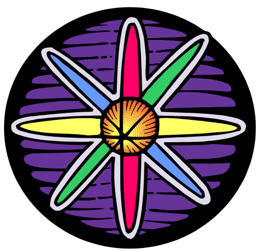


Potential Energy

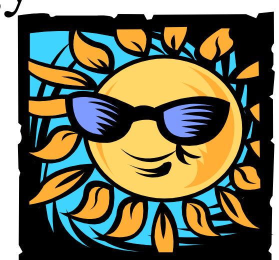
Kinetic Energy



Thermal Energy



Nuclear Energy



Radiant Energy

In this chapter we will focus on only one type of energy (**kinetic energy**) and on only one way in which energy can be transfered (**work**).

7.3 Kinetic Energy

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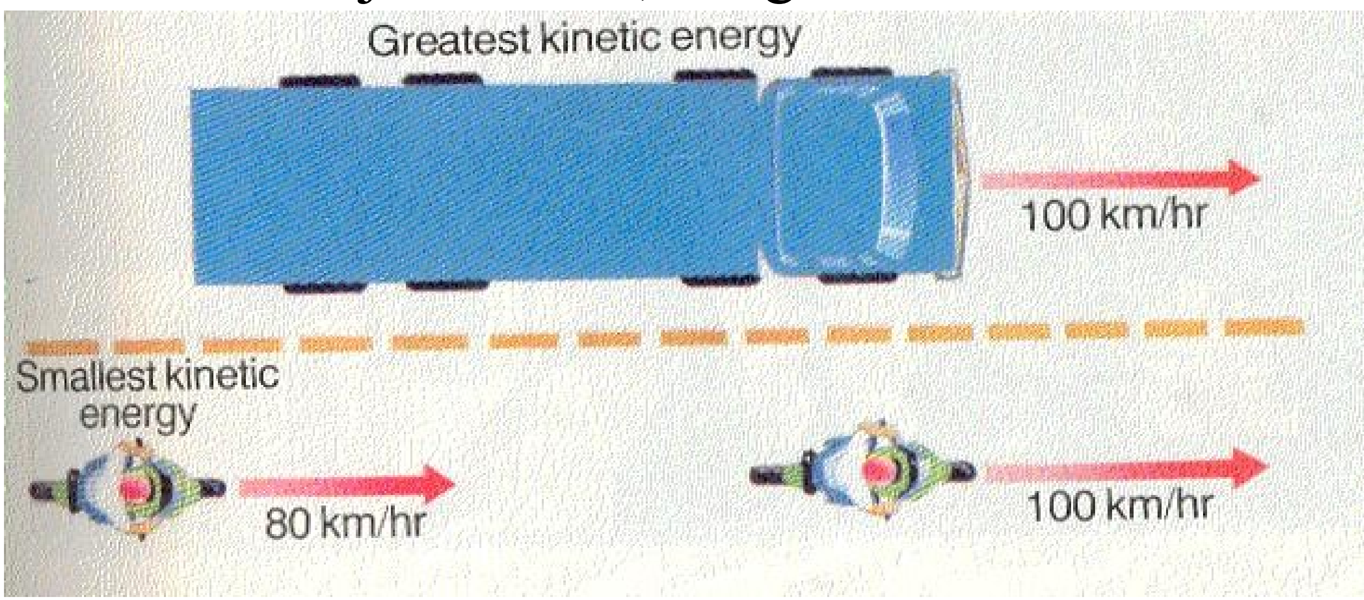
- **Kinetic energy *K*** is energy associated with the **state of motion** of an object.
- For an object of mass *m* whose speed *v* is well below the speed of light,

$$K = \tfrac{1}{2}mv^2 \qquad \text{(kinetic energy).}$$

Newtonian mechanics

- The faster the object moves, the greater is its kinetic energy.



$m \uparrow$, $v \uparrow$ } $KE \uparrow$

- The SI unit of kinetic energy (and every other type of energy) is the **Joule (J),**

$$1 \text{ joule} = 1 \text{ J} = 1 \text{Nm} = 1 \text{ kgm}^2/\text{s}^2$$

Sample Problem: Kinetic Energy, train crash

Energy released by 2 colliding trains with given weight and acceleration from rest. Find the final velocity of each locomotive:

$$v^2 = v_0^2 + 2a(x - x_0).$$

$$v^2 = 0 + 2(0.26\ \mathrm{m/s^2})(3.2 \times 10^3\ \mathrm{m}),$$
$$v = 40.8\ \mathrm{m/s} = 147\ \mathrm{km/h}.$$

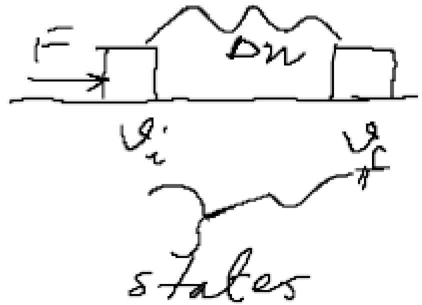
Fig. 7-1 The aftermath of an 1896 crash of two locomotives. *(Courtesy Library of Congress)*

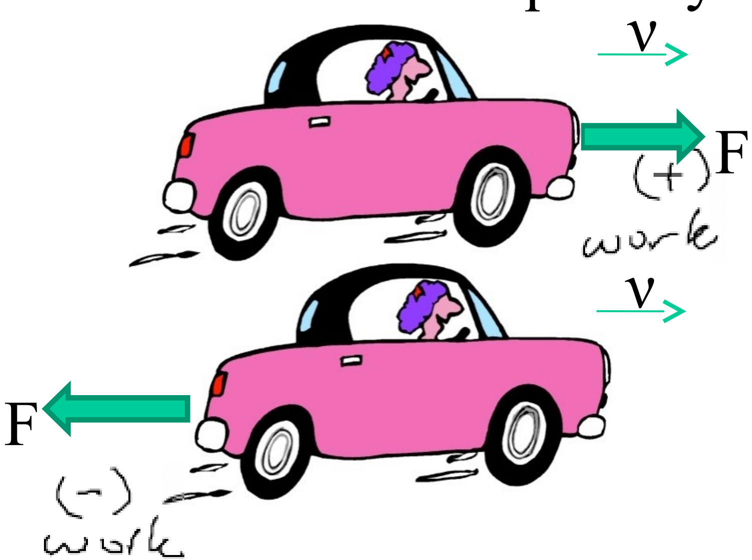
- Convert weight to mass: $$m = \frac{1.2 \times 10^6\ \mathrm{N}}{9.8\ \mathrm{m/s^2}} = 1.22 \times 10^5\ \mathrm{kg}.$$
- Find the kinetic energy:

$$K = 2(\tfrac{1}{2}mv^2) = (1.22 \times 10^5\ \mathrm{kg})(40.8\ \mathrm{m/s})^2$$
$$= 2.0 \times 10^8\ \mathrm{J}. \qquad \text{(Answer)}$$

- Work is <u>energy transferred</u> *to or from* an object by means of a <u>force acting</u> on the object.
- *Energy transferred*
    - <u>to</u> the object is <u>positive work</u>,
    - <u>from</u> the object is <u>negative work</u>.
- " Work", then, is **transferred energy**; $F \Delta x \sim W$
- "doing work" is the **act of transferring** the energy.
- Work has the same units as energy (1 joule = 1 J = 1Nm = 1 kgm^2/s^2.)
    - Work is a scalar quantity and represented by "**W**". NOT a vector

$v_f > v_i$ something into the system





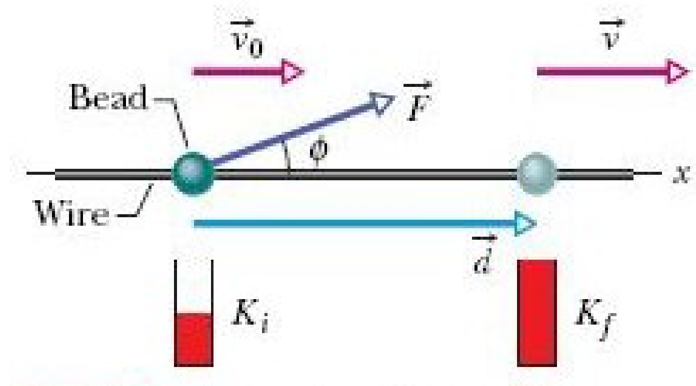
The car accelerates, Kinetic energy increases ($v_f > v_i$)

Positive Work

The car decelerates, Kinetic energy decreases ($v_f < v_i$)

Negative Work

- Start from force equation and 1-dimensional velocity:



$$F_x = ma_x, \quad v^2 = v_0^2 + 2a_x d.$$

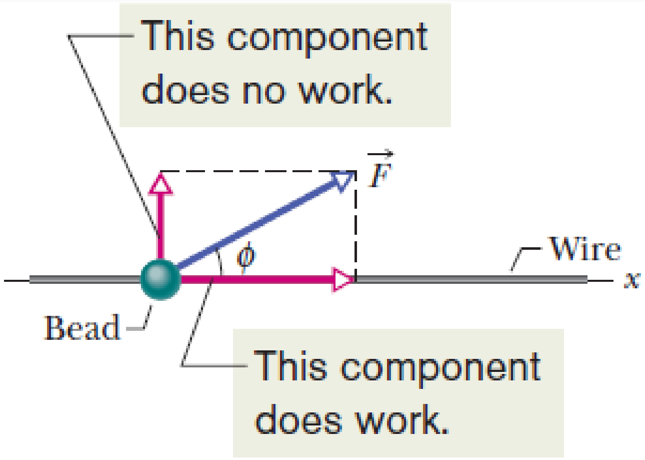
- Rearrange into kinetic energies:

$$\underbrace{\frac{1}{2}mv_f^2}_{K_f} - \underbrace{\frac{1}{2}mv_0^2}_{K_i} = \underbrace{F_x}_{ma_x} d.$$

$$W = F_x d.$$

- K_f *is the* kinetic energy *of the bead* ***at the end*** *of the displacement d*
- K_i *is the kinetic energy of the bead* ***at the start*** *of the displacement d*

- To calculate the work a force ***F*** does on an object as the object moves through some displacement ***d***, we use *only the force component along the object's displacement*.
- The force component perpendicular to the displacement direction does zero work. $F \uparrow \longrightarrow \vec{d}$ $\quad Fd\cos 90 = 0$



- The only component of force taken into account here is the x-component.
- A constant force directed at angle φ to the displacement (in the x-direction) of a bead does work on the bead.
- For a constant force ***F***, the work done W is:

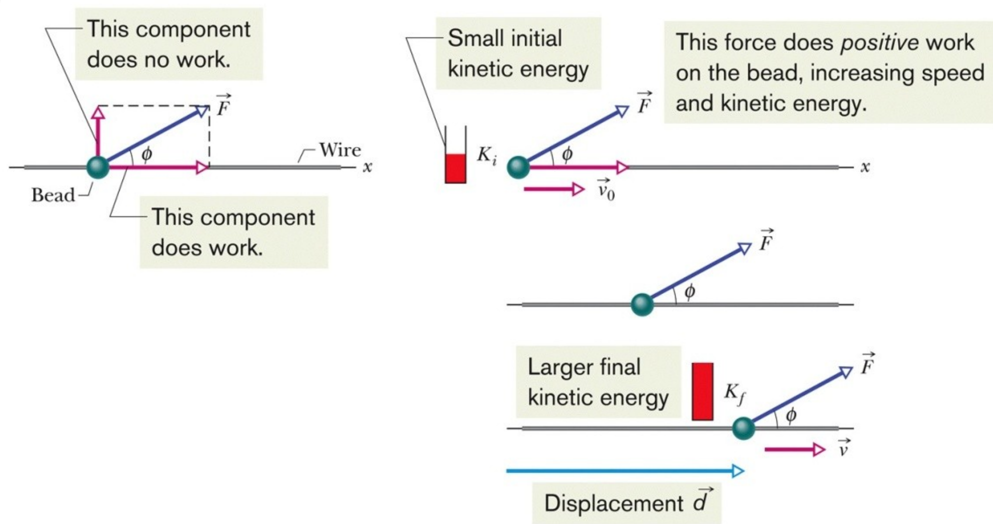
$$W = Fd\cos\phi$$

90° 0°

$$W = \vec{F} \cdot \vec{d}$$

- Notes on these equations:
  - Force is **constant**
  - Object is **particle-like** (rigid) COM
  - Work can be **positive** or **negative**

  Transfer of energy to/from the system



- A force
  - does **positive work** when it has a vector component in the **same direction** as the displacement,
  - does **negative work** when it has a vector component in the **opposite direction**,
  - does **zero work** when it has no such vector component.
- For two or more forces, the **net work** is the sum of the works done by all the individual forces. $\sum \vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_N$
- Two methods to calculate net work: $W = W_1 + W_2 + \ldots + W_N$
  1. We can find all the works and sum the individual work terms.
  2. We can take the vector sum of forces (F_{net}) and calculate the net work once. $\vec{F}_{net} \cdot \Delta\vec{x} = W$

- The theorem says that the **change in kinetic energy** of a particle is the **net work done** on the particle.

$$\begin{pmatrix}\text{change in the kinetic}\\ \text{energy of a particle}\end{pmatrix} = \begin{pmatrix}\text{net work done on}\\ \text{the particle}\end{pmatrix}$$

$\Delta K = +W$
$+\Delta U = -W$
$K_f - K_i = -(U_f + U_i)$
$K_f + U_f = K_i + U_i$

$$\Delta K = K_f - K_i = W,$$

$$\begin{pmatrix}\text{kinetic energy after}\\ \text{the net work is done}\end{pmatrix} = \begin{pmatrix}\text{kinetic energy}\\ \text{before the net work}\end{pmatrix} + \begin{pmatrix}\text{the net}\\ \text{work done}\end{pmatrix}.$$

$$K_f = K_i + W,$$

- The work-kinetic energy theorem holds for positive and negative work.

Example: If the kinetic energy of a particle is *initially* 5 J,

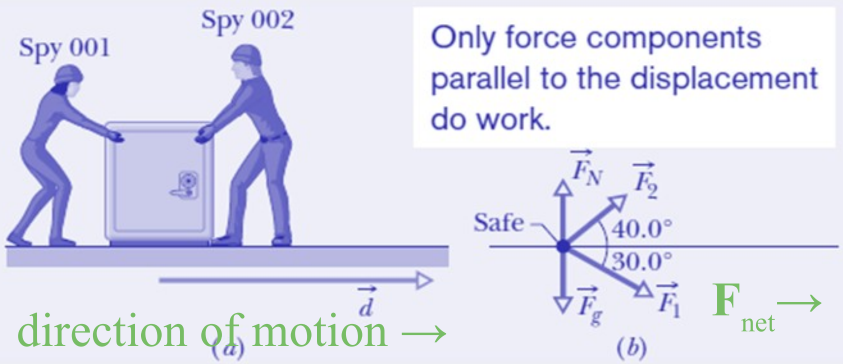
- A net transfer of 2 J to the particle (positive work): Final KE = 7 J

  inserted into the system

- A net transfer of 2 J from the particle (negative work): Final KE = 3 J

  removed from the system

Figure 7-4*a* shows two industrial spies sliding an initially stationary 225 kg floor safe a displacement $\vec{d}$ of magnitude 8.50 m, straight toward their truck. The push $\vec{F}_1$ of spy 001 is 12.0 N, directed at an angle of 30.0° downward from the horizontal; the pull $\vec{F}_2$ of spy 002 is 10.0 N, directed at 40.0° above the horizontal. The magnitudes and directions of these forces do not change as the safe moves, and the floor and safe make frictionless contact.



(a) What is the net work done on the safe by forces $\vec{F}_1$ and $\vec{F}_2$ during the displacement $\vec{d}$?

Calculations: From Eq. 7-7 and the free-body diagram for the safe in Fig. 7-4*b*, the work done by $\vec{F}_1$ is

$$W_1 = F_1 d \cos \phi_1 = (12.0\text{ N})(8.50\text{ m})(\cos 30.0°)$$
$$= 88.33\text{ J},$$ (+) positive work

and the work done by $\vec{F}_2$ is

$$W_2 = F_2 d \cos \phi_2 = (10.0\text{ N})(8.50\text{ m})(\cos 40.0°)$$
$$= 65.11\text{ J}.$$ (+) positive work

Thus, the net work W is

$$W = W_1 + W_2 = 88.33\text{ J} + 65.11\text{ J}$$
$$= 153.4\text{ J} \approx 153\text{ J}. \quad \text{(Answer)}$$

Sample problem: Industrial spies

(b) During the displacement, what is the work W_g done on the safe by the gravitational force $\vec{F}_g$ and what is the work W_N done on the safe by the normal force $\vec{F}_N$ from the floor?

Calculations: Thus, with mg as the magnitude of the gravitational force, we write

$$W_g = mgd \cos 90° = mgd(0) = 0 \quad \text{(Answer)}$$

and $$W_N = F_N d \cos 90° = F_N d(0) = 0. \quad \text{(Answer)}$$

We should have known this result. Because these forces are perpendicular to the displacement of the safe, they do zero work on the safe and do not transfer any energy to or from it.

(c) The safe is initially stationary. What is its speed v_f at the end of the 8.50 m displacement?

Calculations: We relate the speed to the work done by combining Eqs. 7-10 and 7-1:

$$W = K_f - K_i = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2.$$

The initial speed v_i is zero, and we now know that the work done is 153.4 J. Solving for v_f and then substituting known data, we find that

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(153.4\text{ J})}{225\text{ kg}}}$$
$$= 1.17\text{ m/s}. \quad \text{(Answer)}$$

Sample problem: Constant force in unit vector notation

During a storm, a crate of crepe is sliding across a slick, oily parking lot through a displacement $\vec{d} = (-3.0\text{ m})\hat{i}$ while a steady wind pushes against the crate with a force $\vec{F} = (2.0\text{ N})\hat{i} + (-6.0\text{ N})\hat{j}$. The situation and coordinate axes are shown in Fig. 7-5.

(a) How much work does this force do on the crate during the displacement?

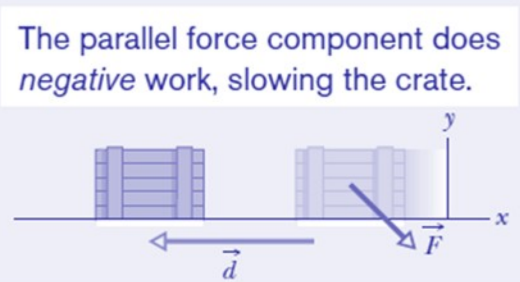


Fig. 7-5 Force $\vec{F}$ slows a crate during displacement $\vec{d}$.

Calculations: We write

$$W = \vec{F}\cdot\vec{d} = [(2.0\text{ N})\hat{i} + (-6.0\text{ N})\hat{j}]\cdot[(-3.0\text{ m})\hat{i}].$$

Of the possible unit-vector dot products, only $\hat{i}\cdot\hat{i}$, $\hat{j}\cdot\hat{j}$, and $\hat{k}\cdot\hat{k}$ are nonzero (see Appendix E). Here we obtain

$$W = (2.0\text{ N})(-3.0\text{ m})\hat{i}\cdot\hat{i} + (-6.0\text{ N})(-3.0\text{ m})\hat{j}\cdot\hat{i}$$
$$= (-6.0\text{ J})(1) + 0 = -6.0\text{ J}. \qquad \text{(Answer)}$$

Thus, the force does a negative 6.0 J of work on the crate, transferring 6.0 J of energy from the kinetic energy of the crate.

(b) If the crate has a kinetic energy of 10 J at the beginning of displacement $\vec{d}$, what is its kinetic energy at the end of $\vec{d}$?

Calculation: Using the work–kinetic energy theorem in the form of Eq. 7-11, we have

$$K_f = K_i + W = 10\text{ J} + (-6.0\text{ J}) = 4.0\text{ J}. \qquad \text{(Answer)}$$

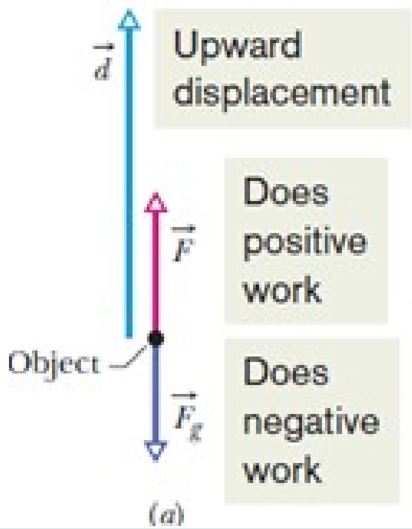
Less kinetic energy means that the crate has been slowed.

(-) negative work: removal of energy

$F_{a/g}.d = W_g = mgd\cos\phi$ (work done by gravitational force).

(a) An applied force lifts an object.

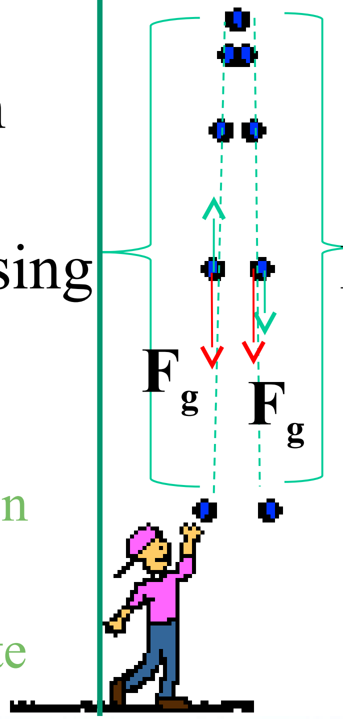
F_g and displacement are in the opposite direction



same direction

opposite

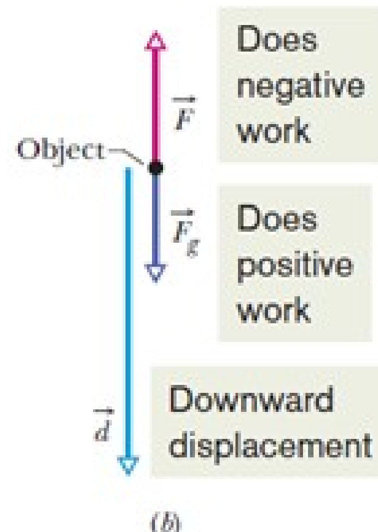
Rising



Falling

(b) An applied force lowers an object.

F_g and displacement are in the same direction



opposite

same

$$\Delta K = K_f - K_i = W_a + W_g,$$

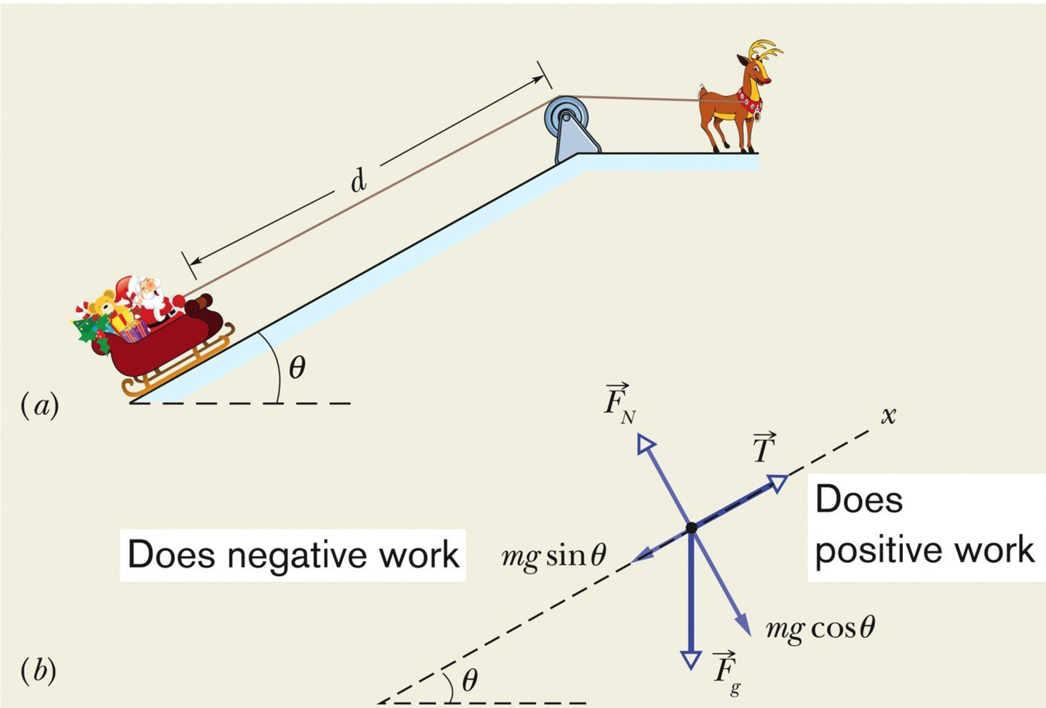
$v_f = v_i$!

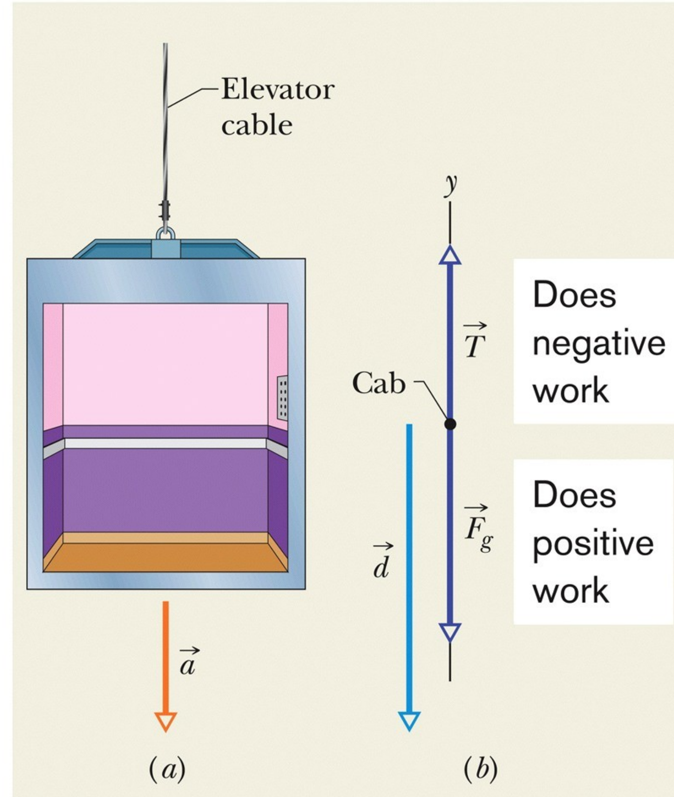
$W_a + W_g = 0$ ⟵ In one common situation, the object is stationary before and after the lift

$W_a = -W_g$ ⟶ $W_a = -mgd\cos\phi$ (work done in lifting and lowering; $K_f = K_i$).

Note that we get the same result if K_f and K_i are not zero but are still equal.

Example: You are a passenger:





- Being pulled up   state is changed
  - Tension does positive work, h ↑
  - gravity does negative work. h ↓

- Being lowered down in an elevator
  - Tension does negative work,
  - gravity does positive work.

h_2 : higher potential energy

Sample problem: Accelerating elevator cab

An elevator cab of mass $m = 500$ kg is descending with speed $v_i = 4.0$ m/s when its supporting cable begins to slip, allowing it to fall with constant acceleration $\vec{a} = \vec{g}/5$ (Fig. 7-8*a*).

(a) During the fall through a distance $d = 12$ m, what is the work W_g done on the cab by the gravitational force $\vec{F}_g$?

Calculation: From Fig. 7-8*b*, we see that the angle between the directions of $\vec{F}_g$ and the cab's displacement $\vec{d}$ is 0°. Then, from Eq. 7-12, we find

$$W_g = mgd\cos 0° = (500\text{ kg})(9.8\text{ m/s}^2)(12\text{ m})(1)$$
$$= 5.88 \times 10^4\text{ J} \approx 59\text{ kJ}. \quad \text{(Answer)}$$

(+) positive work

(b) During the 12 m fall, what is the work W_T done on the cab by the upward pull $\vec{T}$ of the elevator cable?

Calculations: We get

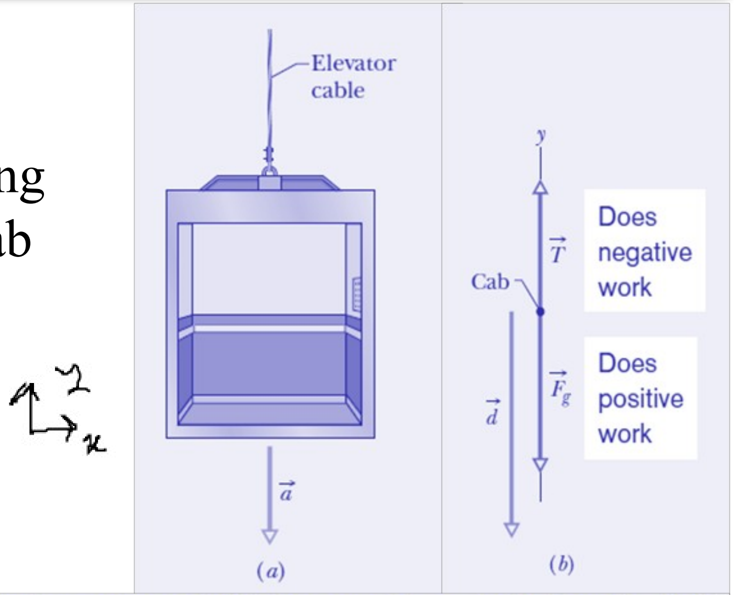
$$T - F_g = ma.$$

$$W_T = Td\cos\phi = m(a+g)d\cos\phi.$$

Next, substituting $-g/5$ for the (downward) acceleration a and then 180° for the angle ϕ between the directions of forces $\vec{T}$ and $m\vec{g}$, we find

$$W_T = m\left(-\frac{g}{5} + g\right)d\cos\phi = \frac{4}{5}mgd\cos\phi$$
$$= \frac{4}{5}(500\text{ kg})(9.8\text{ m/s}^2)(12\text{ m})\cos 180°$$
$$= -4.70 \times 10^4\text{ J} \approx -47\text{ kJ}. \quad \text{(Answer)}$$

(-) negative work



(c) What is the net work W done on the cab during the fall?

Calculation: The net work is the sum of the works done by the forces acting on the cab:

$$W = W_g + W_T = 5.88 \times 10^4\text{ J} - 4.70 \times 10^4\text{ J}$$
$$= 1.18 \times 10^4\text{ J} \approx 12\text{ kJ}. \; (+) \quad \text{(Answer)}$$

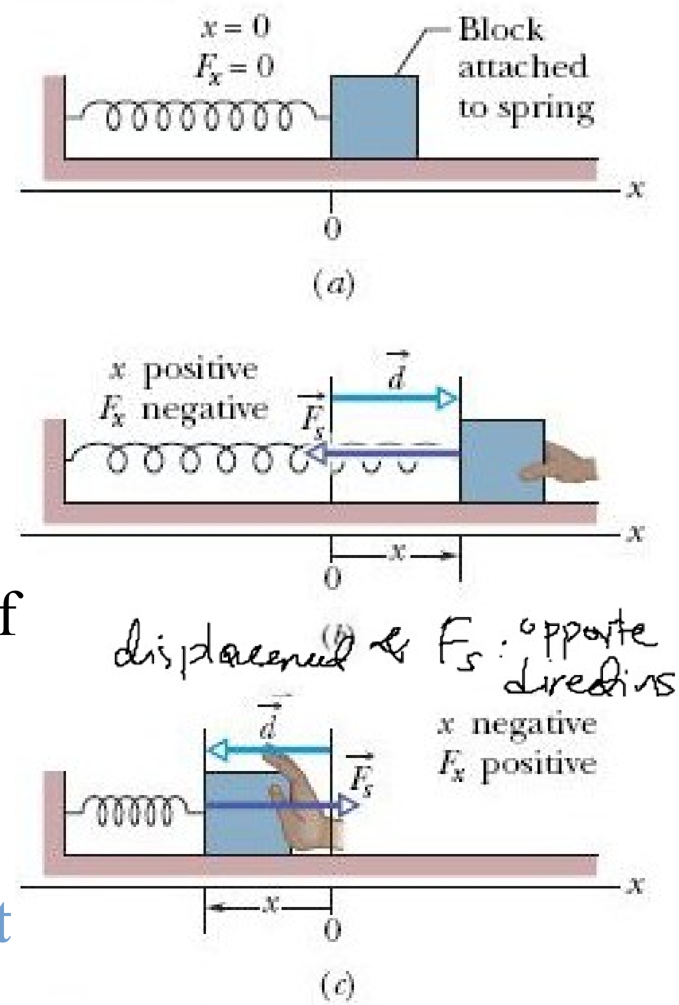
(d) What is the cab's kinetic energy at the end of the 12 m fall?

Calculation: From Eq. 7-1, we can write the kinetic energy at the start of the fall as $K_i = \frac{1}{2}mv_i^2$. We can then write Eq. 7-11 as

$$K_f = K_i + W = \tfrac{1}{2}mv_i^2 + W$$
$$= \tfrac{1}{2}(500\text{ kg})(4.0\text{ m/s})^2 + 1.18 \times 10^4\text{ J}$$
$$= 1.58 \times 10^4\text{ J} \approx 16\text{ kJ}. \uparrow \quad \text{(Answer)}$$

- **Hooke's Law:** To a good approximation for many springs, the force from a spring is proportional to the displacement of the free end from its position when the spring is in the **relaxed state**. F → F(x)

relaxed state: no change



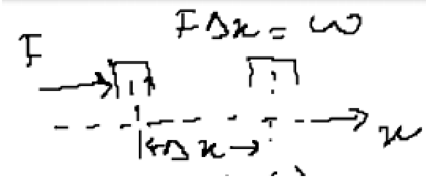
- The spring force is the *variable force* from a spring and is given by

$$\vec{F}_s = \ominus k\vec{d} \qquad \text{(Hooke's law),}$$

- The minus sign indicates that the direction of the spring force is always opposite the direction of the displacement of the spring's free end.
- The constant k is called the **spring constant (or force constant) and is a measure** of the stiffness of the spring.

In one dimension (along x axis):

$$F_x = -kx \qquad \text{(Hooke's law),}$$



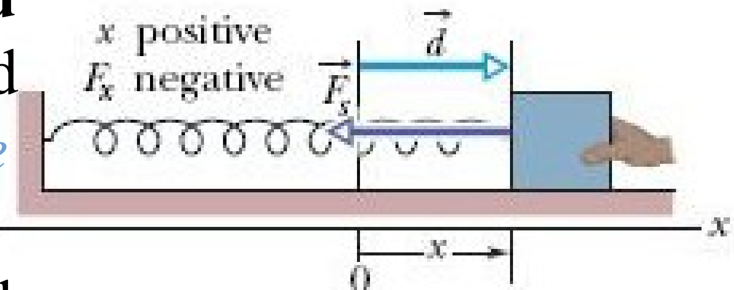
now F → F(x)

$$W_s = \sum -F_{xj}\,\Delta x,$$

Δx: getting smaller

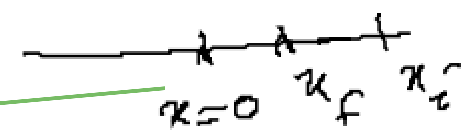
$$W_s = \int_{x_i}^{x_f} -F_x\,dx.$$

- If we **stretch** or **extend** the spring it resists, and exerts a *restoring force* that attempts to return the spring to its relaxed state.



$$W_s = \int_{x_i}^{x_f} -kx\,dx = -k\int_{x_i}^{x_f} x\,dx \qquad = (-\tfrac{1}{2}k)[x^2]_{x_i}^{x_f} = (-\tfrac{1}{2}k)(x_f^2 - x_i^2).$$

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 \qquad \text{(work by a spring force).}$$



Work W_s is positive if the block ends up closer to the relaxed position ($x = 0$) than it was initially. It is negative if the block ends up farther away from $x = 0$. It is zero if the block ends up at the same distance from $x = 0$.

x_i=0 & x_f=x :generally used in solutions

$$W_s = -\tfrac{1}{2}kx^2 \qquad \text{(work by a spring force).}$$

ΔK= W
ΔU= -W

$$\Delta K = K_f - K_i = W_a + W_s,$$

If $\Delta K = 0$ then $W_a = -W_s.$ $\quad v_f = v_i$ (=0)

If a block that is attached to a spring is stationary before and after a displacement, then the work done on it by the applied force displacing it is the negative of the work done on it by the spring force.

Sample problem: Work done by spring

In Fig. 7-10, a cumin canister of mass $m = 0.40$ kg slides across a horizontal frictionless counter with speed $v = 0.50$ m/s. It then runs into and compresses a spring of spring constant $k = 750$ N/m. When the canister is momentarily stopped by the spring, by what distance d is the spring compressed? $v_f = 0$

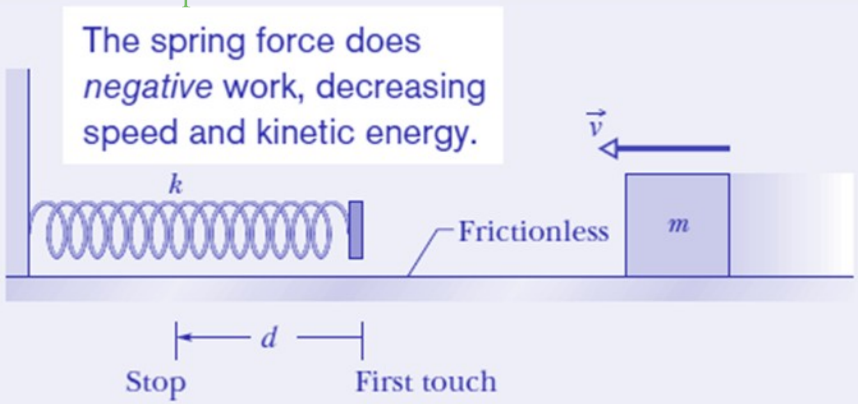


Fig. 7-10 A canister of mass m moves at velocity $\vec{v}$ toward a spring that has spring constant k.

Calculations: Putting the first two of these ideas together, we write the work–kinetic energy theorem for the canister as

$$K_f - K_i = -\tfrac{1}{2}kd^2. \qquad \Delta K = W$$

Substituting according to the third key idea gives us this expression

$$0 - \tfrac{1}{2}mv^2 = -\tfrac{1}{2}kd^2.$$

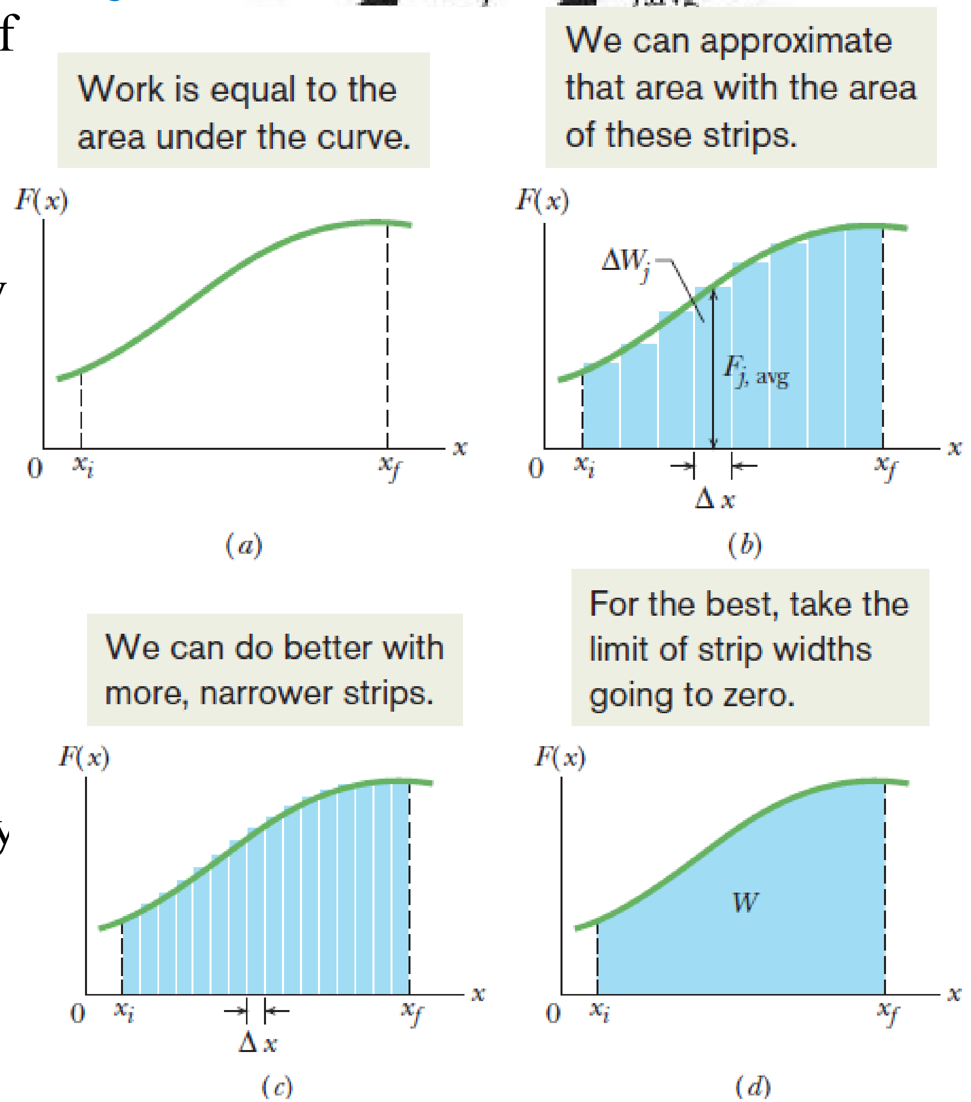
Simplifying, solving for d, and substituting known data then give us

$$d = v\sqrt{\frac{m}{k}} = (0.50\ \text{m/s})\sqrt{\frac{0.40\ \text{kg}}{750\ \text{N/m}}}$$
$$= 1.2 \times 10^{-2}\ \text{m} = 1.2\ \text{cm}. \qquad \text{(Answer)}$$

7. 8 Work Done by a General Variable Force

A. One-dimensional force, graphical analysis: $W = \sum \Delta W_j = \sum F_{j,avg}\,\Delta x.$

- We can divide the area under the curve of F(x) (Fig. a) into a number of narrow strips of width x (Fig. b).
- We choose Δx small enough to permit us to take the force F(x) as being reasonably constant over that interval.
- We let $F_{j,avg}$ be the average value of F(x) within the j^{th} interval.
- The work done by the force in the j^{th} interval is approximately $\Delta W_j = F_{j,avg}\ \Delta x$.
- ΔW_j is then equal to the area of the j^{th} rectangular, shaded strip.
- We can make the approximation better by reducing the strip width Δx and using more strips (Fig. c).
- In the limit, the strip width approaches zero, the number of strips then becomes infinitely large and we have, as an exact result (Fig. d).



$$W = \lim_{\Delta x \to 0} \sum F_{j,avg}\,\Delta x.$$ constant F

$$W = \int_{x_i}^{x_f} F(x)\,dx \quad \text{(work: variable force).}$$ variable F

7. 8 Work Done by a General Variable Force

B. Three dimensional force: $F(x,y,z)$ & $d\vec{r}$

If $$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k},$$

where F_x is the x-components of ***F*** and so on,

and $$d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}.$$

where *dx* is the *x*-component of the displacement vector ***dr*** and so on,

then $$dW = \vec{F}\cdot d\vec{r} = F_x\,dx + F_y\,dy + F_z\,dz.$$

$\hat{i}\cdot\hat{i} = 1$

$\hat{i}\cdot\hat{j} = 0$ $(|\hat{i}||\hat{j}|\cos 90)$

Finally, $$W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x\,dx + \int_{y_i}^{y_f} F_y\,dy + \int_{z_i}^{z_f} F_z\,dz.$$

- The work-kinetic energy theorem still applies!

Work – Kinetic Energy Theorem with a Variable Force

- A particle of mass *m* is moving along an x axis and acted on by a net force *F(x)* that is directed along that axis.
- The work done on the particle by this force as the particle moves from position x_i to position x_f is :

$$W = \int_{x_i}^{x_f} F(x)\,dx = \int_{x_i}^{x_f} ma\,dx,$$

Newton 2nd law

$$a(v(x)) = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}v,$$

$$ma\,dx = m\frac{dv}{dx}v\,dx = mv\,dv.$$

also replace integral limits

$$W = \int_{v_i}^{v_f} mv\,dv = m\int_{v_i}^{v_f} v\,dv$$

$$= \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2.$$

$$W = K_f - K_i = \Delta K,$$

The work-kinetic energy theorem still applies!

for both variable or not variable forces

FYI

Sample problem: Work from 2-D integration:

Force $\vec{F} = (3x^2\ \text{N})\hat{\text{i}} + (4\ \text{N})\hat{\text{j}}$, with x in meters, acts on a particle, changing only the kinetic energy of the particle. How much work is done on the particle as it moves from coordinates (2 m, 3 m) to (3 m, 0 m)? Does the speed of the particle increase, decrease, or remain the same?

Calculation: We set up two integrals, one along each axis:

$$W = \int_2^3 3x^2\,dx + \int_3^0 4\,dy = 3\int_2^3 x^2\,dx + 4\int_3^0 dy$$
$$= 3[\tfrac{1}{3}x^3]_2^3 + 4[y]_3^0 = [3^3 - 2^3] + 4[0 - 3]$$
$$= 7.0\ \text{J}.$$

(+) positive work (Answer)

The positive result means that energy is transferred to the particle by force $\vec{F}$. Thus, the kinetic energy of the particle increases and, because $K = \frac{1}{2}mv^2$, its speed must also increase. If the work had come out negative, the kinetic energy and speed would have decreased.

$F(x,y)$
$\int$ variable — constant!

i: (2,3) f: (3,0)

$w = \vec{F}\cdot\vec{d} = \int (F_x\hat{i} + F_y\hat{j})(dx\,\hat{i} + dy\,\hat{j})$

$\int_{x_i}^{x_f} F_x\,dx + \int_{y_i}^{y_f} F_y\,dy$

7. 9 Power

- The *time rate* at which work is done by a force is said to be the power due to the force. How fast for transfer? Δt,dt
  - The SI unit of power is the joule per second, or Watt.

K,U ~ Joule
W ~ Joule
P ~ Watt

- If a force does an amount of work W in an amount of time t, the **average power** due to the force during that time interval is

$$P_{\text{avg}} = \frac{W}{\Delta t} \quad \text{(average power).}$$

- The **instantaneous power** P is the instantaneous time rate of doing work, which we can write as

$$P = \frac{dW}{dt} \quad \text{(instantaneous power).}$$

- Solve for the instantaneous power using the definition of work:

$$P = \frac{dW}{dt} = \frac{F \cos\phi \, dx}{dt} = F \cos\phi \left(\frac{dx}{dt}\right),$$

$$P = Fv \cos\phi.$$

$$P = \vec{F} \cdot \vec{v} \quad \text{(instantaneous power).}$$

P ~ v

7. 9 Power

Figure 7-14 shows constant forces $\vec{F}_1$ and $\vec{F}_2$ acting on a box as the box slides rightward across a frictionless floor. Force $\vec{F}_1$ is horizontal, with magnitude 2.0 N; force $\vec{F}_2$ is angled upward by 60° to the floor and has magnitude 4.0 N. The speed v of the box at a certain instant is 3.0 m/s. What is the power due to each force acting on the box at that instant, and what is the net power? Is the net power changing at that instant?

Sample problem: Power, force, velocity

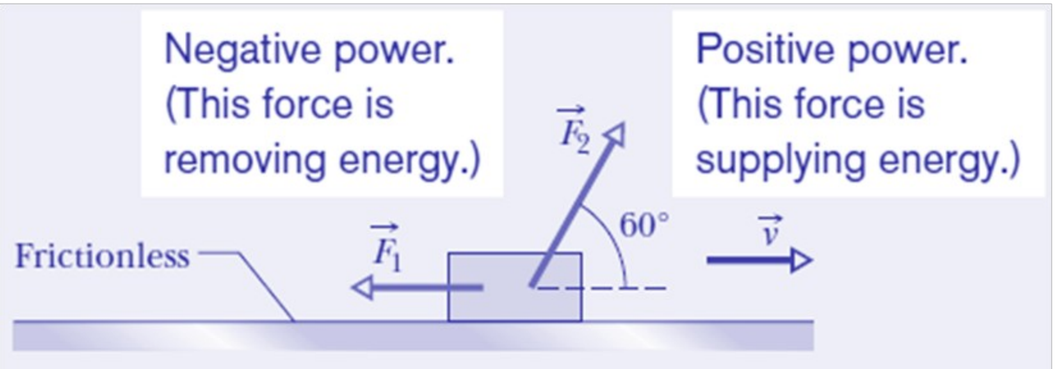


Fig. 7-14

Calculation: We use Eq. 7-47 for each force. For force $\vec{F}_1$, at angle $\phi_1 = 180°$ to velocity $\vec{v}$, we have

$$P_1 = F_1 v \cos\phi_1 = (2.0\text{ N})(3.0\text{ m/s})\cos 180° = -6.0\text{ W}. \quad \text{(Answer)}$$

This negative result tells us that force $\vec{F}_1$ is transferring energy *from* the box at the rate of 6.0 J/s.

For force $\vec{F}_2$, at angle $\phi_2 = 60°$ to velocity $\vec{v}$, we have

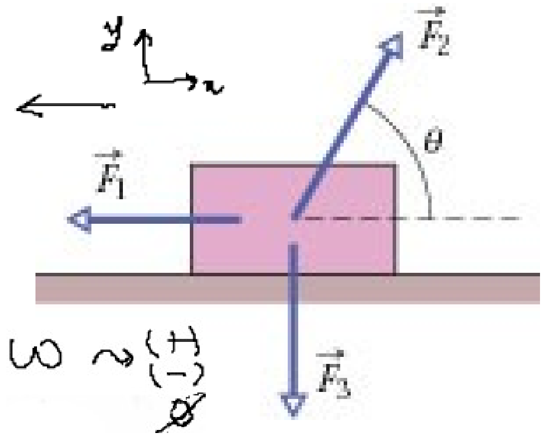
$$P_2 = F_2 v \cos\phi_2 = (4.0\text{ N})(3.0\text{ m/s})\cos 60° = 6.0\text{ W}. \quad \text{(Answer)}$$

This positive result tells us that force is transferring energy to the box at the rate of 6.0 J/s. The net power is the sum of the individual powers:

$$P_{net} = P_1 + P_2 = -6.0\text{ W} + 6.0\text{ W} = 0,$$

which means that the net rate of transfer of energy to or from the box is zero. Thus, the kinetic energy of the box is not changing, and so the speed of the box will remain at 3.0 m/s. Therefore both P_1 and P_2 are constant and thus so is P_{net}.

1. Figure shows three forces applied to a trunk that moves leftward by 3.00 m over a frictionless floor. The force magnitudes are , F_1=5.00 N, F_2=9.00 N, and F_3=3.00 N, and the indicated angle is θ=60.0°,. During the displacement, (a) what is the net work done on the trunk by the three forces and (b) does the kinetic energy of the trunk increase or decrease?



1s) leftward & frictionless

$|\vec{F}_1| = 5\,N$

$|\vec{F}_2| = 9\,N$ & $\theta = 60°$

$|\vec{F}_3| = 3\,N$

$\vec{d} = 3\,m$

$W = \vec{F}\cdot\vec{d}$

i) $W_1 = F_1 d \cos\theta_1 = (5N)(3m)\cos 0 = 15\,J$ (+)

$W_{2\parallel} = (9N)(3m)\cos(180° - 60°) = -13.5\,J$

$W_{2\perp} = (9N)(3m)\cos 90° = 0$

$W_3 = (3N)(3m)\cos 90 = 0$

→ Net work : $W = W_1 + W_2 + W_3 = \underline{1.5\,J}$ (+)

ii) $W = \Delta K$: work is found as positive ⇒ KE increases

2. A helicopter lifts a 72 kg astronaut 15 m vertically from the ocean by means of a cable. The acceleration of the astronaut is g/10. How much work is done on the astronaut by (a) the force from the helicopter and (b) the gravitational force on her? Just before she reaches the helicopter, what are her (c) kinetic energy and (d) speed?

17) $m = 72\ kg$, $h = 15\ m = d$, $a = g/10$

$\uparrow g/10$; F ~ does (+)W; F_g ~ does (−)W

↑ motion or $\vec{d}$

$F - F_g = ma$

$F = mg + ma = m\left(\frac{11}{10}g\right)$

i) $W_F = Fd = m\left(\frac{11}{10}g\right)d = (72\ kg)\left(\frac{11}{10}9.8\ m/s^2\right)(15\ m) = 1.16\times10^4\ J$ (+) ~ given into the system

ii) $W_{F_g} = -F_g d = -mgd = -(72\ kg)(9.8\ m/s^2)15\ m = -1.06\times10^4\ J$ ↳ removal of the energy

iii & iv) $W_{net} = W_F + W_{F_g} = (1.16-1.06)\times10^4\ J = 0.1\times10^4\ J$ { $W = \Delta K$ } Work-KE theorem

$\underbrace{0.1\times10^4\ J}_{KE} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$ ($v_i = 0$) $\rightarrow v_f = \sqrt{\frac{2\times(0.1\times10^4\ J)}{72\ kg}} = 5.4\ m/s$

another solution → $(mg + ma - mg)d = \frac{1}{2}mv^2 \rightarrow \sqrt{2ad} = v$: independent of mass

conservation of energy

3.The only force acting on a 2.0 kg body as it moves along a positive axis has an x component F_x= -6x N, with x in meters. The velocity at x=3.0 m is 8.0 m/s. (a) What is the velocity of the body at x=4.0 m? (b) At what positive value of x will the body have a velocity of 5.0 m/s?

F(x). variable force

x	v
3	8
4	?
?	5

(a) ~ 6.6 m/s
(b)

31) $m = 2\,kg$, moving $+x$

$F_x = -6x\,N$

$v_{x=3m} = 8\,m/s$

i) $v_{x=4} = ?$ $x_i = 3m \rightarrow x_f = 4m$

$$W = \int_{x_i}^{x_f} F(x)\,dx = -\int_3^4 6x\,dx = -6\frac{x^2}{2}\Big|_3^4 = -21\,J$$

$$W = -21\,J = \Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}m(8\,m/s)^2$$

($v_f = v_{x=4}$, $8\,m/s = v_i$)

$$\Rightarrow v_f = \sqrt{\frac{(-21\,J)\,2}{m} + 64\,m^2/s^2} \quad (m \sim 2\,kg) \Rightarrow v_f = \sqrt{43\,m^2/s^2} = 6.6\,m/s$$

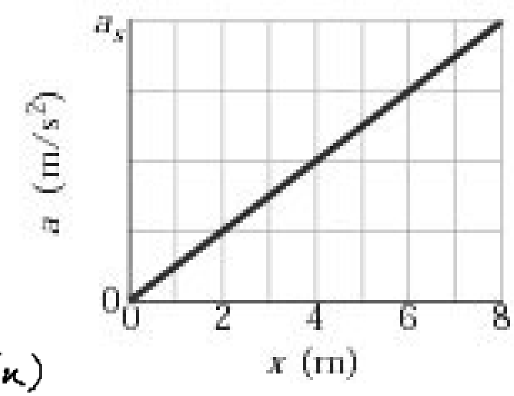
$W = \Delta K$

F_x → $K_f - K_i$

$\int_{x_i}^{x_f} F(x)\,dx$

v ↓ as expected since $W \sim (-)$, $F \sim (-)$

4. A 10 kg brick moves along an x axis. Its acceleration as a function of its position is shown in Figure. The scale of the figure's vertical axis is set by a_s=20.0 m/s². What is the net work performed on the brick by the force causing the acceleration as the brick moves from x=0 to x=8.0 m?



$F(x) = m\, a(x)$

4) $m = 10\,kg$

Moving $+x$

$$\text{slope} = \frac{20\,m/s^2 - 0}{8\,m - 0\,m} = \frac{a}{x} = 2.5\,\frac{1}{s^2} : \text{constant}$$

$a = \text{slope} \cdot x$

a(x)

$$W = \int_{x_i}^{x_f} F\,dx = \int m a\,dx = \int_0^8 m(\text{slope}\, x)\,dx$$

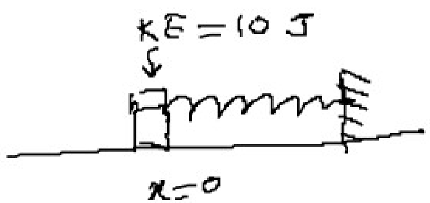
$$= 10\,kg\left(2.5\,\frac{1}{s^2}\right)\int_0^8 x\,dx = 25\,\frac{kg}{s^2}\,\frac{x^2}{2}\Big|_0^8$$

$$= 800\,J$$

5. A 0.30 kg ladle sliding on a horizontal frictionless surface is attached to one end of a horizontal spring (k=500 N/m) whose other end is fixed. The ladle has a kinetic energy of 10 J as it passes through its equilibrium position (the point at which the spring force is zero). relaxed state → x=0 position

(a) At what rate is the spring doing work on the ladle as the ladle passes through its equilibrium position? rate of doing work → power

(b) At what rate is the spring doing work on the ladle when the spring is compressed 0.10 m and the ladle is moving away from the equilibrium position?



$k = 500\ N/m$

$F = -kx$

$m = 0.3$

$W = \Delta K$ work-KE Theorem

$-\frac{1}{2}kx^2 = K_f - K_i$ (K_i = 10 J) → $K_i = K_f + \frac{1}{2}kx^2$

i) equilibrium position → $x = 0$

$W = 0$

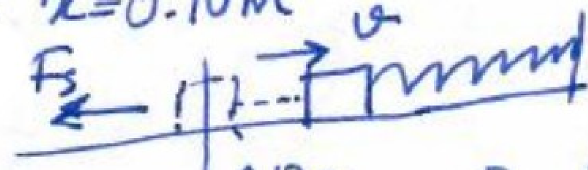
ii) $x \sim \Delta x = 0.1\ m$

48) $m = 0.30\ kg$
frictionless
$k = 500\ N/m$, spring
$K_i = 10\ J$ at relaxed state ↑ ⟹ $F = 0$ spring

i) ω at relaxed state position $= ?$ which is zero at that instant ($F = 0$)

ii) Rate of doing work → $\frac{dW}{dt}$ = Power = $\vec{F} \cdot \vec{v}$

when $x = 0.10\ m$



$K = 10\ J$

Conservation of energy

$(E_{mech} =)\ K_i + U_i = 10\ J = K_f + U_f$ (K_i)

$= \frac{1}{2} m v_f^2 - \frac{1}{2} k x^2$

$10\ J = \frac{1}{2}(0.30\ kg)\, v_f^2 - \frac{1}{2}(500\ N/m)(0.10\ m)^2$

$\rightarrow v_f = 7.1\ m/s$ ↑ at $x = 0.1\ m$

$P = F_s \frac{dx}{dt} = \vec{F}_s \cdot \vec{v} =$

$|\vec{F}_s| = |-kx| = 500\ \frac{N}{m}\ 0.10\ m = 50\ N$

$P = |\vec{F}_s||\vec{v}| \cos 180° = -(50\ N)(7.1\ m/s)$ ⟹ $P = -353.6$ $W \sim J/s$

magnitude

the work-KE theorem instead of E_{mech}

Kinetic Energy

- The energy associated with

$$K = \frac{1}{2}mv^2$$ **Eq. (7-1)**

Work Done by a Constant Force

$$W = Fd\cos\phi$$ **Eq. (7-7)**

$$W = \vec{F}\cdot\vec{d}$$ **Eq. (7-8)**

- The **net work** is the sum of individual works

Work

- Energy transferred to or from an object via a force
- Can be positive or negative

Work and Kinetic Energy

$$\Delta K = K_f - K_i = W,$$ **Eq. (7-10)**

$$K_f = K_i + W,$$ **Eq. (7-11)**

Work Done by the Gravitational Force

$$W_g = mgd\cos\phi$$ **Eq. (7-12)**

Spring Force

- Relaxed state: applies no force
- Spring constant *k* measures stiffness

$$\vec{F}_s = -k\vec{d}$$ **Eq. (7-20)**

Work Done in Lifting and Lowering an Object

$$W_a + W_g = 0$$

$$W_a = -W_g. \qquad \textbf{Eq. (7-16)}$$

Spring Force

- For an initial position $x = 0$:

$$W_s = -\tfrac{1}{2}kx^2 \qquad \textbf{Eq. (7-26)}$$

Work Done by a Variable Force

- Found by integrating the constant-force work equation

$$W = \int_{x_i}^{x_f} F(x)\,dx \qquad \textbf{Eq. (7-32)}$$

Power

- The rate at which a force does work on an object
- Average power:

$$P_{\text{avg}} = \frac{W}{\Delta t} \qquad \textbf{Eq. (7-42)}$$

- Instantaneous power:

$$P = \frac{dW}{dt} \qquad \textbf{Eq. (7-43)}$$

- For a force acting on a moving object:

$$P = Fv\cos\phi. \qquad \textbf{Eq. (7-47)}$$

$$P = \vec{F}\cdot\vec{v} \qquad \textbf{Eq. (7-48)}$$