



**İzmir Kâtip Çelebi University**  
**Department of Engineering Sciences**  
**Phy102 Physics II**  
**Final Examination**  
**January 09, 2023 17:00 – 18:30**  
**Good Luck!**

**NAME-SURNAME:**

**SIGNATURE:**

**ID:**

**DEPARTMENT:**

**INSTRUCTOR:**

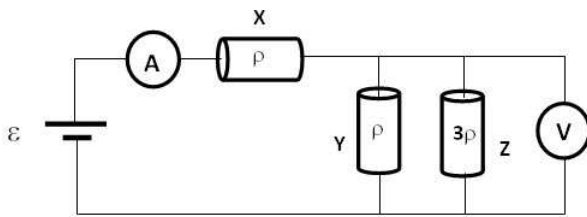
**DURATION:** 90 minutes

- ◇ Answer all the questions.
- ◇ Write the solutions explicitly and clearly.  
Use the physical terminology.
- ◇ You are allowed to use Formulae Sheet.
- ◇ Calculator is allowed.
- ◇ You are not allowed to use any other electronic equipment in the exam.
- ◇ I declare hereby that I fulfilled the requirements for the attendance according to the University regulations and I accept that my examination will not be valid otherwise.

Question	Grade	Out of
1A		15
1B		15
2		20
3		20
4		20
5		20
<b>TOTAL</b>		110

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1. A) The circuit containing three cylindrical resistors, namely X, Y and Z, which obey Ohm's Law is shown in the figure below. The resistors which have length of  $L$  and cross-sectional area of  $A$  are connected to an ideal battery of emf  $\varepsilon$ . As shown an ammeter is connected in series while voltmeter is connected to ends of resistor Z. The resistors X and Y have a resistivity  $\rho$  and the resistor Z has a resistivity  $3\rho$ .



- i) Find the current  $i$  through the ammeter.  
ii) Find the reading of voltmeter.

Express your result in terms of given quantities and constants ( $\varepsilon$ ,  $A$ ,  $\rho$ ,  $L$ ). (**Hint:** Resistance is related to resistivity;  $R = \rho \frac{L}{A}$ )

$$i) \frac{1}{R_{yz}} = \frac{1}{R_y} + \frac{1}{R_z} \rightarrow R_{yz} = \frac{R_y R_z}{R_y + R_z} \Rightarrow R_{eq} = R_x + R_{yz} = R_x + \frac{R_y R_z}{R_y + R_z}$$

where  $R_x = R_y = \rho \frac{L}{A}$  &  $R_z = 3\rho \frac{L}{A} \Rightarrow R_{eq} = \rho \frac{L}{A} + \frac{\rho \frac{L}{A} \cdot 3\rho \frac{L}{A}}{\rho \frac{L}{A} + 3\rho \frac{L}{A}} = \frac{7}{4} \rho \frac{L}{A}$

$$\varepsilon = i R_{eq} \rightarrow i = \frac{\varepsilon}{R_{eq}} = \frac{4}{7} \frac{\varepsilon A}{\rho L} = i_x \rightarrow i_x = i_y + i_z$$

ii)

Loop 1:  $\varepsilon - i_x R_x - i_y R_y = 0 \rightarrow i_y = \frac{\varepsilon - i_x R_x}{R_y}$

Loop 2:  $i_y R_y - i_z R_z = 0 \rightarrow i_z = i_y \frac{R_y}{R_z}$

$$\Rightarrow i_y = \frac{\varepsilon - i_x R_x}{R_y} \text{ \& } i_z = \left( \frac{\varepsilon - i_x R_x}{R_y} \right) \frac{R_y}{R_z} = \frac{\varepsilon - i_x R_x}{R_z}$$

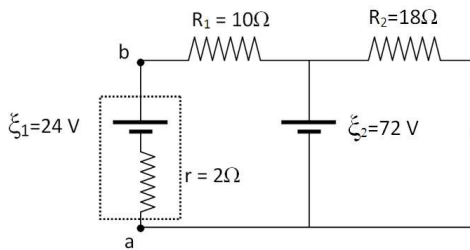
$$\Rightarrow V = i_z R_z = \left( \frac{\varepsilon - i_x R_x}{R_z} \right) R_z = \varepsilon - i_x R_x = \varepsilon - \left( \frac{4}{7} \frac{\varepsilon A}{\rho L} \right) \rho \frac{L}{A} = \varepsilon - \frac{4\varepsilon}{7}$$

$$\boxed{V = \frac{3\varepsilon}{7}}$$

- B) What uniform magnetic field, applied perpendicular to a beam of electrons moving at  $1.30 \times 10^6 \text{ m/s}$ , is required to make the electrons travel in a circular arc of radius of 0.35 m? (Hint: Centripetal Force;  $F_c = m \frac{v^2}{R}$ )

$$\begin{aligned} v &= 1.3 \times 10^6 \text{ m/s} & F_c &= m \frac{v^2}{R} \text{ \& } F_B = |q|vB \sin \theta \\ R &= 0.35 \text{ m} \\ e &= 1.602 \times 10^{-19} \text{ C} (\equiv |q|) & |q|vB \sin 90^\circ &= m_e v^2 / R \quad (S) \\ m_e &= 9.109 \times 10^{-31} \text{ kg} & \Rightarrow B &= \frac{m_e v}{e R} \quad (S) \\ B &=? & &= \frac{(9.109 \times 10^{-31} \text{ kg})(1.3 \times 10^6 \text{ m/s})}{(1.602 \times 10^{-19} \text{ C})(0.35 \text{ m})} \\ & & &= \boxed{2.11 \times 10^{-5} \text{ T}} \quad (S) \end{aligned}$$

2. Consider circuit as shown in figure which consists of two batteries. One of the following batteries has an internal resistance  $r$ , while the other battery is an ideal battery.



Calculate;

- Currents through each battery,
- Total power dissipated by resistors.
- Potential difference between points  $a$  and  $b$ ,  $V_{ab}$ ,

i Currents through each battery,

ii Potential difference between points  $a$  and  $b$ ,  $V_{ab}$ ,

iii Total power supplied by batteries,

iv Total power dissipated by resistors.

1) loop 1:  $-i_1 r + \mathcal{E}_1 - i_1 R_1 - \mathcal{E}_2 = 0 \Rightarrow -2i_1 + 24 - 10i_1 - 72 = 0$  (2)

2) loop 2:  $\mathcal{E}_2 - i_3 R_2 = 0 \Rightarrow 72 - 18i_3 = 0$  (2)  $-12i_3 = 48$

3)  $i_1 + i_2 = i_3 \Rightarrow -4A + i_2 = 4A$  (1)  $i_3 = 4A$  (1)  $i_1 = -4A$  (1)

Three unknowns ( $i_1, i_2, i_3$ ), three equations

$i_1 = -4A$ : Through battery 1

$i_2 = 8A$ : Through battery 2

$i_3 = 4A$ : Through Resistor 3

ii)  $V_{ab} = V_b - V_a$  (2)

$V_a + i_1 r + \mathcal{E}_1 = V_b$

$V_b - V_a = 4A \cdot 2\Omega + 24V = 32V$  (1)

iii)  $P = i \mathcal{E}$

Battery 1:  $P_1 = i_1 \mathcal{E}_1 = (4A)(24V) = -96W$  (1.5)

Battery 2:  $P_2 = i_2 \mathcal{E}_2 = (8A)(72V) = 576W$  (1.5)

$P_1 + P_2 = 480W$

iv)  $P = i^2 R$

Resistor 1:  $P_1' = i_1^2 R_1 = (4A)^2 (10\Omega) = 160W \sim R_1$  (1)

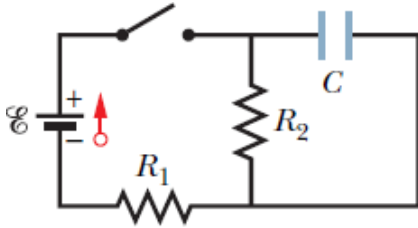
Resistor 2:  $P_2' = i_3^2 R_2 = (4A)^2 (18\Omega) = 288W \sim R_2$  (1)

Internal Resistor:  $P_r' = i_1^2 r = (4A)^2 (2\Omega) = 32W \sim r$  (1)

$480W = 480W$



3. In Figure given below,  $R_1 = 8.0 \times 10^3 \Omega$ ,  $R_2 = 10.0 \times 10^3 \Omega$ ,  $C = 6 \times 10^{-7} F$ , and the ideal battery has emf  $\epsilon = 12.0 V$ . First, the switch is closed a long time so that the steady state is reached. Then the switch is opened at time  $t = 0$ .



What is the current in resistor 2 at  $t = 2.00 \times 10^{-3} s$ ?

$R_1 = 8 \text{ k}\Omega$   
 $R_2 = 10 \text{ k}\Omega$   
 $C = 0.6 \mu\text{F}$   
 $\mathcal{E} = 12 \text{ V}$

initially, C acts as a connecting wire ①  
 after a long time, C acts as a broken wire ②

① Charging stage ② fully charged

$\Rightarrow V_C = V_{R_2} = iR_2$   
 $i = \frac{\mathcal{E}}{R_1 + R_2}$  ②

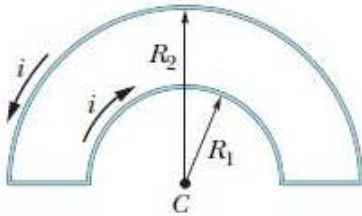
$\rightarrow V_{R_2} = \left(\frac{\mathcal{E}}{R_1 + R_2}\right) R_2 = \frac{12 \text{ V}}{(8 \text{ k}\Omega + 10 \text{ k}\Omega)} \cdot 10 \text{ k}\Omega = \frac{20 \text{ V}}{3} (= V_C)$  when fully charged ②

③ switch is opened  $\rightarrow t = 0$  &  $V_C = V_0 = \frac{20}{3} \text{ V}$  ②  
 discharging through  $R_2$ ,  $V = V_0 \exp(-t/RC)$  ③

$\rightarrow V = \left(\frac{20}{3} \text{ V}\right) \exp\left(-\frac{2 \text{ ms}}{(10 \text{ k}\Omega)(0.6 \mu\text{F})}\right) = \left(\frac{20}{3} \text{ V}\right) \exp\left(-\frac{2 \times 10^{-3} \text{ s}}{10 \times 10^3 \Omega \cdot 0.6 \times 10^{-6} \text{ F}}\right)$

$V(t = 2 \text{ ms}) = 4.78 \text{ V}$  ②  $\rightarrow i_{R_2} = \frac{V}{R_2} = \frac{4.78 \text{ V}}{10 \text{ k}\Omega} = 4.77 \times 10^{-4} \text{ A}$  ② ①

4. In Figure, two semicircular arcs have radii  $R_2 = 2.6 \text{ cm}$  and  $R_1 = 1.05 \text{ cm}$ , carry current  $i = 0.0937 \text{ A}$ , and share the same center of curvature  $C$ .



What are the

i magnitude

ii direction (into or out of the page, why?)

of the net magnetic field at  $C$ ?

**Hint:** Use Biot-Savart Law.

Biot-Savart law:  $d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{s} \times \hat{r}}{r^2} \rightarrow$

$$dB = \frac{\mu_0}{4\pi} \frac{i ds \sin 90^\circ}{R^2} = \frac{\mu_0}{4\pi} \frac{i ds}{R^2} \quad \left\{ \begin{array}{l} \text{where} \\ ds = R d\phi \end{array} \right\} \rightarrow B = \int dB$$

$$\rightarrow B = \frac{\mu_0}{4\pi} i \int_0^\phi \frac{R d\theta}{R^2} = \frac{\mu_0 i}{4\pi R} \phi \quad \left\{ \begin{array}{l} \text{where} \\ \phi \text{ is arc angle} \end{array} \right.$$

$R_1 = 1.05 \times 10^{-2} \text{ m}$   
 $R_2 = 2.6 \times 10^{-2} \text{ m}$   
 $i = 0.0937 \text{ A}$   
 $\phi = 180^\circ \equiv \pi$

i)  $B = B_1 + B_2 = \frac{\mu_0 i}{4\pi} \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \pi$  (4)

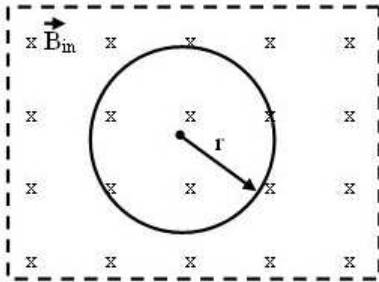
$$\rightarrow B = \frac{4\pi \times 10^{-7} \text{ Tm/A} \cdot 0.0937 \text{ A}}{4\pi} \left( \frac{1}{1.05 \times 10^{-2} \text{ m}} - \frac{1}{2.6 \times 10^{-2} \text{ m}} \right) \pi$$

$$= 1.67 \times 10^{-6} \text{ T} \quad (2)$$

ii) into the page (4)

$B_1$  (into the page) &  $|B_1| > |B_2|$   
 $B_2$  (out of the page)

5. In figure below, the magnetic flux through the circular loop of radius  $r = 2.0 \text{ m}$  increases according to the relation  $\Phi_B = 3t^2 + 3t$ , where  $\Phi_B$  is in Webers and  $t$  is in seconds.



- Find the magnitude of the induced  $emf, \xi$  in the circular loop at  $t = 2.0 \text{ s}$ .
- What is the magnitude and direction of the induced current in the circular loop at  $t = 2.0 \text{ s}$  if the loop has a total resistance of  $R = 30 \Omega$ ?

i)  $\Phi_B(t) = 3t^2 + 3t$  : increasing flux  $\Rightarrow$  induced  $\mathcal{E}, i$  should oppose

$\mathcal{E} = -N \frac{d\Phi_B}{dt} \xrightarrow{(5)} |\mathcal{E}| = \left. \frac{d(3t^2 + 3t)}{dt} \right|_{t=2s} = 6t + 3 = 15 \text{ V} \xrightarrow{(5)}$

ii)  $i = \frac{\mathcal{E}}{R} = \frac{15 \text{ V}}{30 \Omega} = 0.5 \text{ A} \xrightarrow{(2)}$

$\vec{B}_{applied} \otimes \sim$  into the page  $\xrightarrow{(2)}$   
 $\vec{B}_{induced} \odot \leftarrow$  since it should oppose  
 by Right Hand Rule  $\sim$  direction of induced current is ccw  $\xrightarrow{(3)}$

The diagram shows the circular loop with induced current flowing counter-clockwise (ccw). The induced magnetic field  $B_{induced}$  is directed out of the page, represented by 'o' marks within the loop's area.





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**Phy102 Physics II**  
**Final Examination**  
**January 14, 2022 11:00 – 12:30**  
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**NAME-SURNAME:**

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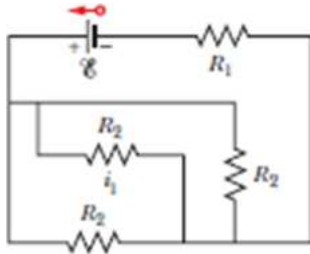
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<b>TOTAL</b>		110

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1. A) In Figure,  $R_1 = 2.0 \Omega$ ,  $R_2 = 6.0 \Omega$ , and the ideal battery has emf  $\varepsilon = 4.0 \text{ V}$ .



- What are the size and direction (left or right) of current  $i_1$ ?
- How much energy is dissipated by all four resistors in 3.00 minutes?

$R_1 = 6 \Omega$   
 $R_2 = 18 \Omega$   
 $\mathcal{E} = 12 \text{ V}$   
 $i_1 = ?$

$\frac{1}{R_{\text{eq}}} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$  (1.5)  
 $R_{\text{eq}} = 2 \Omega$   
 $i = \frac{V}{R} = \frac{4 \text{ V}}{4 \Omega} = 1 \text{ A}$  (1.5)  
 $i_1 = \frac{i}{3} = \frac{1 \text{ A}}{3} = 0.33 \text{ A}$  (Rightward) (1.5)  
 $P = i^2 R_{\text{eq}} = (1 \text{ A})^2 (4 \Omega) = 4 \text{ W}$  (1.5)  
 $P = \frac{\Delta U}{\Delta t} \Rightarrow \Delta U = (4 \text{ W})(180 \text{ sec}) = 720 \text{ J}$  (1.5)

- B) A  $15.0 \text{ k}\Omega$  resistor and a capacitor are connected in series and then a  $12.0 \text{ V}$  potential difference is suddenly applied across them. The potential difference across the capacitor rises to  $5.0 \text{ V}$  in  $1.30 \mu\text{s}$ .
- Calculate the time constant of the circuit.
  - Find the capacitance of the capacitor.

Charging capacitor:  $q = C \mathcal{E} (1 - e^{-t/RC})$  &  $\tau = RC$  (2)

$$V(t) = \mathcal{E} (1 - e^{-t/RC})$$
 (3)

i)  $V(t) = \mathcal{E} (1 - e^{-t/RC}) \Rightarrow 5\text{V} = 12\text{V} \left(1 - e^{-\frac{1.3 \times 10^{-6} \text{ s}}{15 \times 10^3 \Omega C}}\right)$

$$e^{-1.3 \times 10^{-6} \text{ s}/\tau} = 1 - 5/12 \rightarrow \ln e^{-1.3 \times 10^{-6} \text{ s}/\tau} = \ln 7/12$$

$$\rightarrow -1.3 \times 10^{-6} \text{ s}/\tau = \ln 7/12 \rightarrow \tau = \frac{-1.3 \times 10^{-6} \text{ s}}{\ln 7/12} = \frac{-1.3 \times 10^{-6} \text{ s}}{-0.54}$$

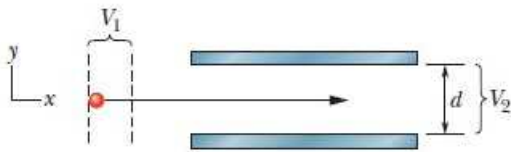
$$\Rightarrow \tau = 2.41 \mu\text{s}$$
 (2) (3)

ii)  $\tau = RC \rightarrow C = \frac{\tau}{R} = \frac{2.41 \times 10^{-6} \text{ s}}{15 \times 10^3 \Omega} = 1.61 \times 10^{-10} \text{ F}$

$$= 0.161 \text{ nF}$$

$$= 162 \text{ pF}$$
 (2)

2. In Figure, an electron accelerated from rest through potential difference  $V_1 = 1.00 \text{ kV}$  enters the gap between two parallel plates having separation  $d = 10.0 \text{ mm}$  and potential difference  $V_2 = 50 \text{ V}$ . The lower plate is at the lower potential. Neglect fringing and assume that the electron's velocity vector is perpendicular to the electric field vector between the plates.



In unit-vector notation, what uniform magnetic field allows the electron to travel in a straight line in the gap?

$V_1 = 1 \text{ kV}$  &  $d = 10 \times 10^{-3} \text{ m}$ ,  $V_2 = 50 \text{ V}$ ,  $m_e = 9.11 \times 10^{-31} \text{ kg}$   
 higher potential  $\rightarrow$  straight line  $\Rightarrow |\vec{F}_B| = |\vec{F}_E|$   
 $|q|v_z B = |q|E$  (2)  
 $\sqrt{\frac{2qV_1}{m_e}} B = \frac{V_2}{d}$  (2)  
 $\rightarrow B = \frac{50 \text{ V}}{10 \times 10^{-3} \text{ m}} \sqrt{\frac{9.11 \times 10^{-31} \text{ kg}}{2 \times 1.6 \times 10^{-19} \text{ C} \times 1 \times 10^3 \text{ V}}}$   
 $B = 2.67 \times 10^{-4} \text{ T}$   
 $\vec{B} = 2.67 \times 10^{-4} \text{ T} (-\hat{y})$  (2) (2)

$\Delta U = qV_1 - 0$  (2)  
 $= (1.6 \times 10^{-19} \text{ C}) (1 \times 10^3 \text{ V})$   
 $\Delta U = \Delta K = \frac{1}{2} m_e v_z^2$

$V_2 = Ed$   
 $\Rightarrow E = \frac{V_2}{d}$  (2)

(8) lower potential (2)



3. A long wire carries a 10 A current from left to right. An electron 1.0 cm above the wire is traveling to the right at a speed of  $1.0 \times 10^7$  m/s. What are the magnitude and the direction of the magnetic force on the electrons?

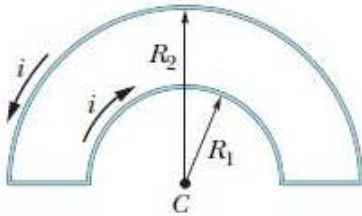
$$B = \frac{\mu_0 I}{2\pi d} = \frac{(4\pi \times 10^{-7} \text{ Tm/A}) 10 \text{ A}}{2\pi 1.0 \times 10^{-2} \text{ m}} = 2 \times 10^{-4} \text{ T}$$

$$\vec{F}_B = q \vec{v} \times \vec{B} \Rightarrow |\vec{F}_B| = (1.602 \times 10^{-19} \text{ C})(1.0 \times 10^7 \text{ m/s})(2 \times 10^{-4} \text{ T})$$

$$= 3.2 \times 10^{-16} \text{ N}$$

$$\vec{F}_B = 3.2 \times 10^{-16} \text{ N } \hat{j}$$

4. In Figure, two semicircular arcs have radii  $R_2 = 3.9 \text{ cm}$  and  $R_1 = 1.575 \text{ cm}$ , carry current  $i = 0.1405 \text{ A}$ , and share the same center of curvature  $C$ .




What are the

i magnitude

ii direction (into or out of the page, why?)

of the net magnetic field at  $C$ ?

**Hint:** Use Biot-Savart Law.

Biot-Savart law:  $d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{s} \times \hat{r}}{r^2} \rightarrow$  

$$dB = \frac{\mu_0}{4\pi} \frac{i ds \sin 90^\circ}{R^2} = \frac{\mu_0}{4\pi} \frac{i ds}{R^2} \quad \left\{ \begin{array}{l} \text{where} \\ ds = R d\phi \end{array} \right\} \rightarrow B = \int dB$$

$$\rightarrow B = \frac{\mu_0}{4\pi} i \int_0^\phi \frac{R d\theta}{R^2} = \frac{\mu_0 i}{4\pi R} \phi \quad \left\{ \begin{array}{l} \text{where} \\ \phi \text{ is arc angle} \end{array} \right.$$


$R_1 = 1.05 \times 10^{-2} \text{ m}$   
 $R_2 = 2.6 \times 10^{-2} \text{ m}$   
 $i = 0.0937 \text{ A}$   
 $\phi = 180^\circ \equiv \pi$

i)  $B = B_1 + B_2 = \frac{\mu_0 i}{4\pi} \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \pi$  (4)

$$\rightarrow B = \frac{4\pi \times 10^{-7} \text{ Tm/A} \cdot 0.0937 \text{ A}}{4\pi} \left( \frac{1}{1.05 \times 10^{-2} \text{ m}} - \frac{1}{2.6 \times 10^{-2} \text{ m}} \right) \pi$$

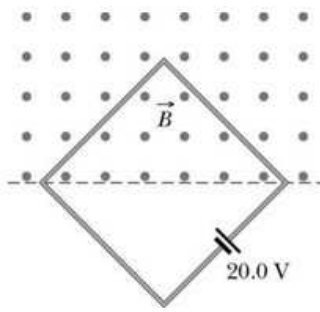
$$= 1.67 \times 10^{-6} \text{ T} \quad (2)$$

ii) into the page (4)



$B_1$  (into the page) &  $|B_1| > |B_2|$   
 $B_2$  (out of the page)

5. A square wire loop with  $3.00\text{ m}$  sides and resistance  $3\ \Omega$  is perpendicular to a uniform magnetic field, with half the area of the loop in the field as shown in figure. The loop contains an ideal battery with emf ( $\varepsilon$ )  $20.0\text{ V}$ . The magnitude of the field varies with time according to  $B = 0.0420 - 0.3870t$ , with  $B$  in teslas and  $t$  in second.



- Find the value and direction of the induced  $\varepsilon$ .
- What is the net emf in the circuit?
- Find the magnitude and the direction of the net current around the loop?

**Hint:** Magnetic field is decreasing.

$L = 3.00\text{ m}$   
 $R = 3\ \Omega$   
 $\varepsilon_B = 20.0\text{ V}$   
 $B = 0.0420 - 0.3870t$   
 $A = L^2/2$

$\varepsilon_i = -\frac{d\Phi_B}{dt} = -\frac{d(LBA)}{dt} = -\frac{L^2}{2} \frac{dB}{dt} = -\frac{L^2}{2} \frac{d(0.0420 - 0.3870t)}{dt}$   
 $= -\frac{L^2}{2} (-0.3870\text{ T/s}) = \frac{(3.00\text{ m})^2}{2} (0.3870\text{ T/s})$   
 $\varepsilon_i = 1.76\text{ V}$

$B$  is out of page and DECREASING.  
 $\rightarrow$  Induced emf should support the external magnetic field  $\rightarrow$  CCC: direction of induced emf (current,  $\Rightarrow$  some direction with the battery)

$\varepsilon_{\text{total}} = \varepsilon_B + \varepsilon_i = 20.0\text{ V} + 1.76\text{ V} = 21.76\text{ V}$

$i = \frac{V}{R} = \frac{\varepsilon_{\text{total}}}{R} = \frac{21.76\text{ V}}{3\ \Omega} = 7.23\text{ A}$

ii) Current is in the ccw.



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1. A) The magnitude  $J$  of the current density in a certain lab wire with a circular cross section of radius  $R=15.00$  mm is given by  $J = (6.00 \times 10^7)r^2$ , with  $J$  in amperes per square meter and radial distance  $r$  in meters. What is the current through the outer section bounded by  $r=0.200R$  and  $r=0.600R$ ?

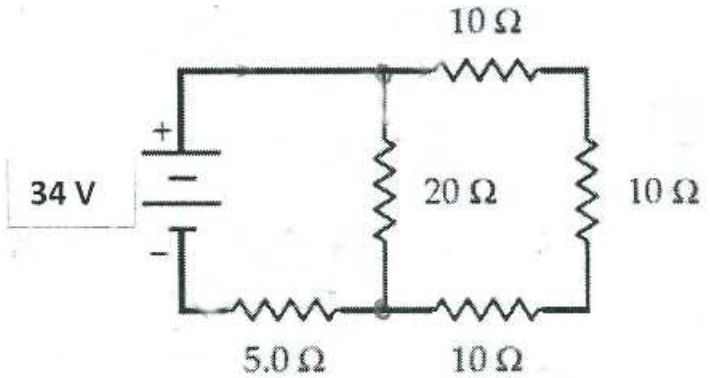
$R = 15 \times 10^{-3} \text{ m}$   
 $J(r) = 6 \times 10^7 r^2 \text{ A/m}^2$   
 $i = ?$  from  $r = 0.2R$   
 to  $r = 0.6R$



$$\begin{aligned}
 i &= \int \vec{J} \cdot d\vec{A} = \int_{0.2R}^{0.6R} 6 \times 10^7 r^2 2\pi r dr \\
 &= 12\pi \times 10^7 \int_{0.2R}^{0.6R} r^3 dr = 12\pi \times 10^7 \left[ \frac{r^4}{4} \right]_{0.2R}^{0.6R} \\
 &= 3\pi \times 10^7 [(0.6R)^4 - (0.2R)^4] \\
 &= 3\pi \times 10^7 \times 0.128 R^4 = \underline{0.61 \text{ A}}
 \end{aligned}$$

B) For the circuit shown find

- i) the current delivered by the battery,
- ii) the potential difference across the  $20\ \Omega$  resistor.



Handwritten solution for problem B:

Step 1: Simplify the parallel resistors.  $\frac{1}{R_{eq}} = \frac{1}{20\Omega} + \frac{1}{30\Omega}$   
 $R_{eq} = \frac{(30\Omega)(20\Omega)}{20\Omega + 30\Omega} = 12\Omega$  (5)

Step 2: Find the current from the battery.  $I = \frac{34V}{17\Omega} = 2A$  (5)

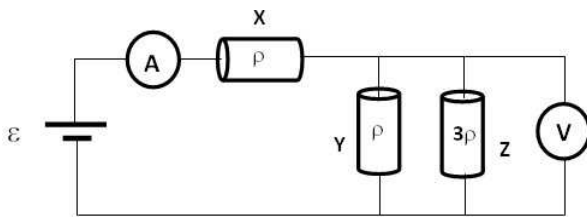
Step 3: Find the potential difference across the  $20\Omega$  resistor.  $V = IR = 12\Omega \times 2A = 24V$  (5)

Step 4: Find the current through the  $20\Omega$  resistor.  $I_{20\Omega} = \frac{24V}{20\Omega} = 1.2A$

Step 5: Find the current through the  $30\Omega$  resistor.  $I_{30\Omega} = \frac{24V}{30\Omega} = 0.8A$

Step 6: Total current  $I = I_1 + I_2$

2. The circuit containing three cylindrical resistors, namely  $X$ ,  $Y$  and  $Z$ , which obey Ohm's Law is shown in the figure below. The resistors which have length of  $L$  and cross-sectional area of  $A$  are connected to an ideal battery of emf  $\varepsilon$ . As shown an ammeter is connected in series while voltmeter is connected to ends of resistor  $Z$ . The resistors  $X$  and  $Y$  have a resistivity  $\rho$  and the resistor  $Z$  has a resistivity  $3\rho$ .



i Find the current  $i$  through the ammeter.

ii Find the reading of voltmeter. (Hint: Multi-loop circuit. Apply junction and loop rules.)

Express your result in terms of given quantities and constants ( $\rho$ ,  $\varepsilon$ ,  $A$ ,  $L$ ). (Hint: Resistance is related to resistivity.)

$$i) \frac{1}{R_{yz}} = \frac{1}{R_y} + \frac{1}{R_z} \rightarrow R_{yz} = \frac{R_y R_z}{R_y + R_z} \Rightarrow R_{eq} = R_x + R_{yz} = R_x + \frac{R_y R_z}{R_y + R_z}$$

where  $R_x = R_y = \rho \frac{L}{A}$  &  $R_z = 3\rho \frac{L}{A} \Rightarrow R_{eq} = \rho \frac{L}{A} + \frac{\rho \frac{L}{A} \cdot 3\rho \frac{L}{A}}{\rho \frac{L}{A} + 3\rho \frac{L}{A}} = \frac{7}{4} \rho \frac{L}{A}$

$$\varepsilon = i R_{eq} \rightarrow i = \frac{\varepsilon}{R_{eq}} = \frac{4}{7} \frac{\varepsilon A}{\rho L} = i_x \rightarrow i_x = i_y + i_z$$

$$ii) \text{ Loop 1: } \varepsilon - i_x R_x - i_y R_y = 0 \rightarrow i_y = \frac{\varepsilon - i_x R_x}{R_y}$$

$$\text{ Loop 2: } i_y R_y - i_z R_z = 0 \rightarrow i_z = i_y \frac{R_y}{R_z}$$

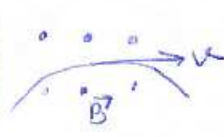
$$\Rightarrow i_y = \frac{\varepsilon - i_x R_x}{R_y} \text{ \& } i_z = \left( \frac{\varepsilon - i_x R_x}{R_y} \right) \frac{R_y}{R_z} = \frac{\varepsilon - i_x R_x}{R_z}$$

$$\Rightarrow V = i_z R_z = \left( \frac{\varepsilon - i_x R_x}{R_z} \right) R_z = \varepsilon - i_x R_x = \varepsilon - \left( \frac{4}{7} \frac{\varepsilon A}{\rho L} \right) \rho \frac{L}{A} = \varepsilon - \frac{4\varepsilon}{7}$$

$$\boxed{V = \frac{3\varepsilon}{7}}$$

3. A proton of kinetic energy 2.10 keV circles in a plane perpendicular to a uniform magnetic field. The orbit radius is 25.0 cm. Find
- the proton's speed,
  - the magnetic field magnitude,
  - the circling frequency,
  - the period of the motion.

proton

$$\begin{cases}
 m \frac{v^2}{R} = qvB \sin 90 \\
 v = \frac{qRB}{m} \rightsquigarrow R = \frac{mv}{qB}
 \end{cases}$$


i)  $\frac{1}{2} m_p v^2 = 2.1 \times 10^3 \text{ eV} \rightsquigarrow v^2 = \frac{2(2.1 \times 10^3 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})}{1.67 \times 10^{-27} \text{ kg}}$

$$\rightsquigarrow v = \boxed{0.634 \times 10^6 \text{ m/s}} \quad (3)$$

ii)  $B = \frac{m_p v}{qR} = \frac{(1.67 \times 10^{-27} \text{ kg})(0.634 \times 10^6 \text{ m/s})}{(1.6 \times 10^{-19} \text{ C})(25 \times 10^{-2} \text{ m})} = \boxed{0.027 \text{ T}} \quad (3)$

iii)  $T = \frac{1}{f} = \frac{2\pi R}{v} \rightsquigarrow f = \frac{v}{2\pi R} = \frac{0.634 \times 10^6 \text{ m/s}}{2\pi(25 \times 10^{-2} \text{ m})} = \boxed{0.404 \times 10^6 \text{ Hz}} \quad (2)$

iv)  $T = \frac{1}{f} = \boxed{2.48 \times 10^{-6} \text{ s}} \quad (2)$

4. A long wire carries a 10 A current from left to right. An electron 1.0 cm above the wire is traveling to the right at a speed of  $1.0 \times 10^7$  m/s. What are the magnitude and the direction of the magnetic force on the electrons?

$$B = \frac{\mu_0 I}{2\pi d} = \frac{(4\pi \times 10^{-7} \text{ Tm/A}) 10 \text{ A}}{2\pi 1.0 \times 10^{-2} \text{ m}} = 2 \times 10^{-4} \text{ T}$$

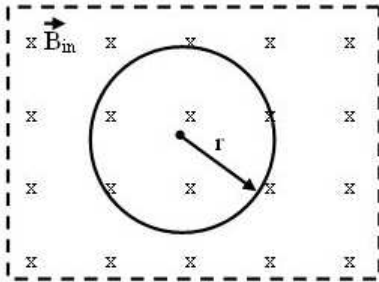
$$\vec{F}_B = q \vec{v} \times \vec{B} \Rightarrow |\vec{F}_B| = (1.602 \times 10^{-19} \text{ C})(1.0 \times 10^7 \text{ m/s})(2 \times 10^{-4} \text{ T})$$

$$= 3.2 \times 10^{-16} \text{ N}$$

$$\vec{F}_B = 3.2 \times 10^{-16} \text{ N } \hat{j}$$



5. In figure below, the magnetic flux through the circular loop of radius  $r = 2.0\text{ m}$  increases according to the relation  $\Phi_B = 6t^2 + 6t$ , where  $\Phi_B$  is in Webers and  $t$  is in seconds.



- Find the magnitude of the induced  $emf, \xi$  in the circular loop at  $t = 2.0\text{ s}$ .
- What is the magnitude and direction of the induced current in the circular loop at  $t = 2.0\text{ s}$  if the loop has a total resistance of  $R = 60\ \Omega$ ?

$i) \phi_B(t) = 3t^2 + 3t$  : increasing flux  $\Rightarrow$  induced  $\mathcal{E}, i$  should oppose  
 $\mathcal{E} = -N \frac{d\phi_B}{dt} \xrightarrow{(5)} |\mathcal{E}| = \left. \frac{d(3t^2 + 3t)}{dt} \right|_{t=2s} = 6t + 3 = 15\text{ V} \xrightarrow{(5)}$   
 $ii) i = \frac{\mathcal{E}}{R} = \frac{15\text{ V}}{30\ \Omega} = 0.5\text{ A} \xrightarrow{(2)}$   
 $\vec{B}_{applied} \otimes \sim$  into the page  
 $\vec{B}_{induced} \odot \leftarrow$  since it should oppose  
 by Right Hand Rule  $\sim$  direction of induced current is ccw  $(3)$

The diagram shows the circular loop with induced current flowing counter-clockwise (ccw) and induced magnetic field  $B_{induced}$  pointing out of the page, represented by 'o' marks.



**İzmir Kâtip Çelebi University**  
**Department of Engineering Sciences**  
**Phy102 Physics II**  
**Final Examination**  
**January 09, 2018 14:30 – 16:30**  
**Good Luck!**

**NAME-SURNAME:**

**SIGNATURE:**

**ID:**

**DEPARTMENT:**

**DURATION:** 120 minutes

- ◇ Answer all the questions.
- ◇ Write the solutions explicitly and clearly.  
Use the physical terminology.
- ◇ You are allowed to use Formulae Sheet.
- ◇ Calculator is allowed.
- ◇ You are not allowed to use any other electronic equipment in the exam.
- ◇ I declare hereby that I fulfilled the requirements for the attendance according to the University regulations and I accept that my examination will not be valid otherwise.

Question	Grade	Out of
1A		15
1B		15
2		20
3		20
4		20
5		20
<b>TOTAL</b>		110

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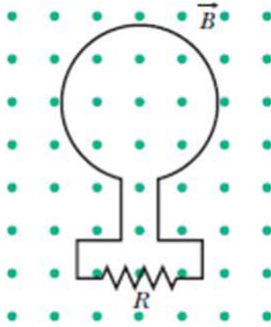
1. A) A parallel-plate air-filled capacitor has a capacitance of  $50 \text{ pF}$ .
- If each of its plates has an area of  $0.35 \text{ m}^2$ , what is the separation?
  - If the region between the plates is now filled with material having  $k=5.6$ , what is the capacitance?

i)  $C = \epsilon_0 \frac{A}{d} \sim 50 \times 10^{-12} \text{ F} = 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2} \frac{0.35 \text{ m}^2}{d}$

$\rightarrow d = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2)(0.35 \text{ m}^2)}{50 \times 10^{-12} \text{ F}} = 0.062 \text{ m}$

ii)  $C_1 = kC_0 = (5.6)(50 \times 10^{-12} \text{ F}) = 280 \text{ pF}$

B) In Figure given below, the magnetic flux through the loop increases according to the relation  $\Phi_B = 6.0t^2 + 7.0t$ , where  $\Phi_B$  is in milliwebers and  $t$  is in seconds.



- i) What is the magnitude of the emf ( $\varepsilon$ ) induced in the loop when  $t = 2.0$  s?
- ii) Is the direction of the current through  $R$  to the right or left?

Increasing magnetic flux  $\rightarrow$  induced emf in the loop

$$i) |\varepsilon| = \left| \frac{d\Phi_B}{dt} \right| \rightarrow \varepsilon = \left. \frac{d}{dt} (6.0t^2 + 7.0t) \right|_{t=2s} = 12t + 7 \Big|_{t=2s}$$

$$\rightarrow \boxed{\varepsilon = 31 \text{ mV}}$$


ii) Increasing flux  $\leftrightarrow$  induced emf should create a magnetic flux to oppose (to decrease external field)  
 To have an inward (induced)  $B$ , we should have a clockwise current at the loop.

$\rightarrow$   $\boxed{\text{Left through } R}$



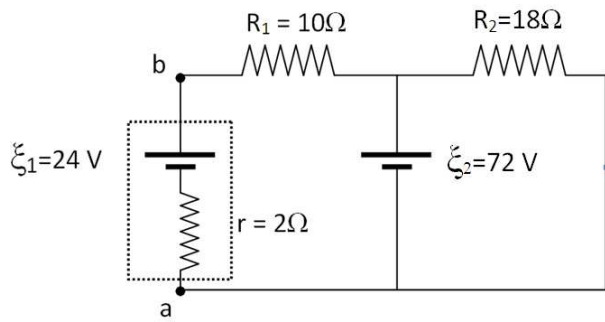
2. The magnitude  $J$  of the current density in a certain lab wire with a circular cross section of radius  $R=5.00$  mm is given by  $J = (2.00 \times 10^7)r^2$ , with  $J$  in amperes per square meter and radial distance  $r$  in meters. What is the current through the outer section bounded by  $r=0.800R$  and  $r=R$ ?

$R = 5 \times 10^{-3} \text{ m}$   
 $J(r) = 6 \times 10^7 r^2 \text{ A/m}^2$   
 $i = ?$  from  $r = 0.2R$   
 to  $r = 0.6R$

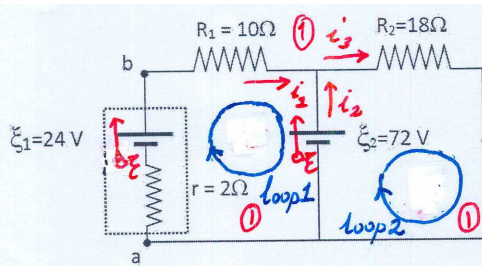


$$\begin{aligned}
 i &= \int \vec{J} \cdot d\vec{A} = \int_{0.2R}^{0.6R} 6 \times 10^7 r^2 2\pi r dr \\
 &= 12\pi \times 10^7 \int_{0.2R}^{0.6R} r^3 dr = 12\pi \times 10^7 \left[ \frac{r^4}{4} \right]_{0.2R}^{0.6R} \\
 &= 3\pi \times 10^7 [(0.6R)^4 - (0.2R)^4] \\
 &= 3\pi \times 10^7 \times 0.128 R^4 = \underline{\underline{0.61 \text{ A}}}
 \end{aligned}$$

3. Consider circuit as shown in figure which consists of two batteries. One of the following batteries has an internal resistance  $r$ , while the other battery is an ideal battery. Calculate;



- i Currents through each battery,
- ii Potential difference between points  $a$  and  $b$ ,  $V_{ab}$ ,
- iii Total power supplied by batteries,
- iv Total power dissipated by resistors.



- i Currents through each battery,
- ii Potential difference between points a and b,  $V_{ab}$ ,
- iii Total power supplied by batteries,
- iv Total power dissipated by resistors.

i) ① loop 1:  $-i_1 r + E_1 - i_1 R_1 - E_2 = 0 \Rightarrow -2i_1 + 24 - 10i_1 - 72 = 0$  ②

② loop 2:  $E_2 - i_3 R_2 = 0 \Rightarrow 72 - 18i_3 = 0$  ②  $-12i_1 = 48$

③  $i_1 + i_2 = i_3 \Rightarrow -4A + i_2 = 4A \Rightarrow i_2 = 8A$  ①  $i_3 = 4A$  ①

Three unknowns ( $i_1, i_2, i_3$ ), three equations

$i_1 = -4A$ : Through battery 1  
 $i_2 = 8A$ : Through battery 2  
 $i_3 = 4A$ : Through Resistor 3

$i_1 = -4A$  ①  
*opposite direction*

ii)  $V_{ab} = V_b - V_a$  ②

$V_a + i_1 r + E_1 = V_b$

$V_b - V_a = 4A \cdot 2\Omega + 24V = 32V$  ①

iii)  $P = iE$

Battery 1:  $P_1 = i_1 E_1 = (4A)(24V) = -96W$  ①.5

Battery 2:  $P_2 = i_2 E_2 = (8A)(72V) = 576W$  ①.5

$P_1 + P_2 = 480W$

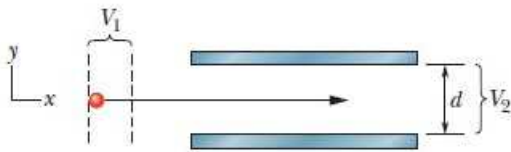
iv)  $P = i^2 R$

Resistor 1:  $P_1' = i_1^2 R_1 = (4A)^2 (10\Omega) = 160W$  ①  $P_1 + P_2 =$

Resistor 2:  $P_2' = i_3^2 R_2 = (4A)^2 (18\Omega) = 288W$  ①  $P_1' + P_2' + P_r'$

Internal Resistor:  $P_r' = i_1^2 r = (4A)^2 (2\Omega) = 32W$  ①  $480W = 480W$

4. In Figure, an electron accelerated from rest through potential difference  $V_1 = 1.00 \text{ kV}$  enters the gap between two parallel plates having separation  $d = 20.0 \text{ mm}$  and potential difference  $V_2 = 100 \text{ V}$ . The lower plate is at the lower potential. Neglect fringing and assume that the electron's velocity vector is perpendicular to the electric field vector between the plates.



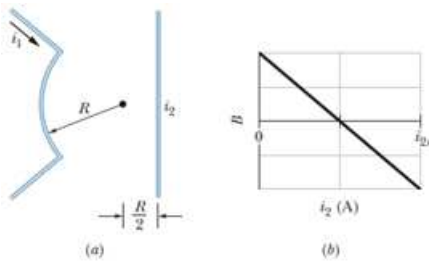
In unit-vector notation, what uniform magnetic field allows the electron to travel in a straight line in the gap?

$V_1 = 1 \text{ kV}$  &  $d = 10 \times 10^{-3} \text{ m}$ ,  $V_2 = 50 \text{ V}$ ,  $m_e = 9.11 \times 10^{-31} \text{ kg}$   
 higher potential  $\rightarrow$  straight line  $\Rightarrow |\vec{F}_B| = |\vec{F}_E|$   
 $|q|v_z B = |q|E$  (2)  
 $\sqrt{\frac{2qV_1}{m_e}} B = \frac{V_2}{d}$  (2)  
 $\rightarrow B = \frac{50 \text{ V}}{10 \times 10^{-3} \text{ m}} \sqrt{\frac{9.11 \times 10^{-31} \text{ kg}}{2 \times 1.6 \times 10^{-19} \text{ C} \times 1 \times 10^3 \text{ V}}}$   
 $B = 2.67 \times 10^{-4} \text{ T}$   
 $\vec{B} = 2.67 \times 10^{-4} \text{ T} (-\hat{k})$  (2) (2)

$\Delta U = qV_1 - 0$  (2)  
 $= (1.6 \times 10^{-19} \text{ C}) (1 \times 10^3 \text{ V})$   
 $\Delta U = \Delta K = \frac{1}{2} m_e v_z^2$  (2)  
 $v_z = Ed$   
 $\Rightarrow E = \frac{V_2}{d}$  (2)

(8) lower potential (2)

5. Figure(a) shows two wires, each carrying a current. Wire 1 consists of a circular arc of radius  $R$  and two radial lengths; it carries current  $i_1 = 3.0 \text{ A}$  in the direction indicated. Wire 2 is long and straight; it carries a current  $i_2$  that can be varied; and it is at distance  $R/2$  from the center of the arc. The net magnetic field  $B$  due to the two currents is measured at the center of curvature of the arc.



Figure(b) is a plot of the component of  $B$  in the direction perpendicular to the figure as a function of current  $i_2$ . The horizontal scale is set by  $i_{2s} = 2.00 \text{ A}$ . What is the angle subtended by the arc?

$i_1 = 3 \text{ A}, R$   
 $i_2 = \text{variable}, R/2$

net magnetic field at point P  
 $B_p = \frac{\mu_0 i_1 \phi}{4\pi R} - \frac{\mu_0 i_2}{2\pi R/2}$

(5)  $\frac{\mu_0 i_1 \phi}{4\pi R}$  (circular arc out of page) (5)  $\frac{\mu_0 i_2}{2\pi R/2}$  (straight wire into page)

at  $i_2 = 1 \text{ A} \Rightarrow B_p = 0 \Rightarrow \frac{\mu_0 3 \text{ A}}{4\pi R} \phi = \frac{\mu_0 1 \text{ A}}{\pi R}$

$\Rightarrow \phi = \frac{4}{3} \text{ radians} = 76.4^\circ$  (2)  
 (3.14 rad  $\rightarrow$   $180^\circ$ ) (3)