İzmir Kâtip Çelebi University
Department of Engineering Sciences
Phy102 Physics II
Midterm Examination
April 22, 2024 17:45 – 19:15
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

INSTRUCTOR:

DURATION: 90 minutes

◇ **Answer all the questions.**
◇ **Write the solutions explicitly and clearly. Use the physical terminology.**
◇ **You are allowed to use Formulae Sheet.**
◇ **Calculator is allowed.**
◇ **You are not allowed to use any other electronic equipment in the exam.**

Question	Grade	Out of
1A		10
1B		10
2		20
3		20
4		20
5		20
TOTAL		100

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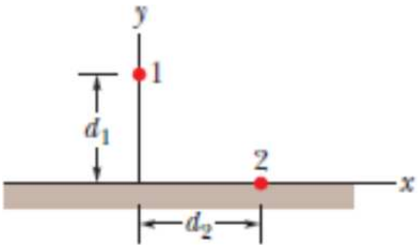
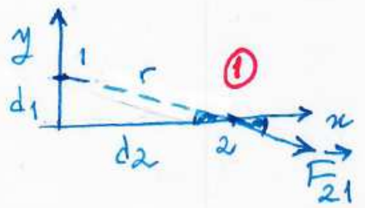
1. A) In figure given below, particle 1 of charge $q_1 = 4e$ is above a floor by distance $d_1 = 2.00\ mm$ and particle 2 of charge $q_2 = 6e$ is on the floor, at distance $d_2 = 6.00\ mm$ horizontally from particle 1.



What is the **x component of the electrostatic force on particle 2 due to particle 1**?



(1)

$$\vec F_{21} = F_{21,x}\,\hat i + F_{21,y}\,\hat j = |\vec F_{21}|\cos\theta\,\hat i - |\vec F_{21}|\sin\theta\,\hat j$$

$F_{21,x} = ?$ (2) $\frac{1}{4\pi\varepsilon_0}\frac{|q_2||q_1|}{r^2}$; $\frac{d_2}{r} = \frac{d_2}{\sqrt{d_1^2+d_2^2}}$ (2)

$$\Rightarrow F_{21,x} = k\frac{|q_2||q_1|}{d_1^2+d_2^2}\,\frac{d_2}{\sqrt{d_1^2+d_2^2}} = \frac{k(6e)(4e)\,d_2}{(d_1^2+d_2^2)^{3/2}}$$ (2)

$$= 8.99\times10^{9}\,\frac{Nm^2}{C^2}\,\frac{24\times(1.602\times10^{-19}C)^2\,(6\times10^{-3}m)}{\left((2\times10^{-3}m)^2+(6\times10^{-3}m)^2\right)^{3/2}} = \boxed{1.31\times10^{-22}\,N}$$ (2) (1)

B) An electrometer is a device used to measure static charge-an unknown charge is placed on the plates of the meter's capacitor, and the potential difference is measured. What minimum charge can be measured by an electrometer with a capacitance of 50 pF and a voltage sensitivity of 0.15 V?

$C = 50\,pF$

$V = 0.15\,V = V_{min}$

$q_{min} = ?$

$q_{min} = V_{min} C = (0.15\,V)(50 \times 10^{-12}\,F) =$ (5) (3)

$\boxed{q_{min} = 7.5\,pC}$ (2)

2. At some instant the velocity components of an electron moving between two charged parallel plates are $v_x = 3 \times 10^5\ m/s$ and $v_y = 5.0 \times 10^3\ m/s$. Suppose the electric field between the plates is given by $\vec{E} = (180N/C)\hat{j}$. In unit-vector notation, what are

   i the electron's acceleration in that field

   ii the electron's velocity when its x coordinate has changed by 2.4 cm?

$\bar{e}$: electron

$v_x = 3\times10^5\ m/s = v_{0x}$

$v_y = 5\times10^3\ m/s = v_{0y}$

$\vec{E} = 180\ N/C\ \hat{j}$

(2) i) $a = ?$ (2) $\vec{F}_E = q\vec{E} = (1.6\times10^{-19}C)(180\ N/C)(-\hat{j}) = 288\times10^{-19}N\,(-\hat{j})$ (1)(1)

$\Rightarrow m_e\vec{a} = \vec{F}_E$ (2) $\rightarrow \vec{a} = \dfrac{288\times10^{-19}N}{9.109\times10^{-31}kg}(-\hat{j}) = \boxed{3.16\times10^{13}\ m/s^2\,(-\hat{j})}$ (1)(1)

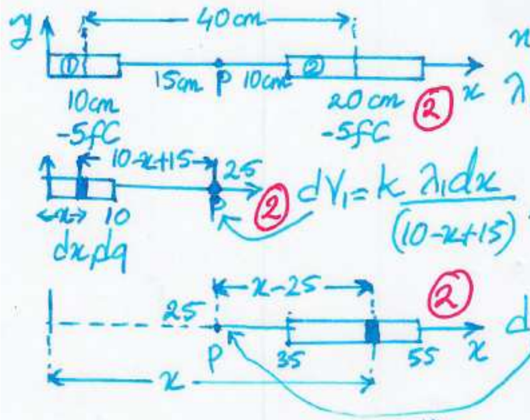
ii) $\Delta x = x - x_0 = 2.4\times10^{-2}m$ & no force acting on x direction (2)

$\Rightarrow v_x = v_{0x}$ (1) & $v_y = v_{0y} + at$ (1) $\Rightarrow \dfrac{\Delta x}{\Delta t} = v_x \rightarrow \dfrac{\Delta x}{v_x} = \Delta t$

$\Rightarrow \Delta t = \dfrac{2.4\times10^{-2}m}{3\times10^5 m/s} = 8\times10^{-8}s$ (1)(1) $\rightarrow v_y = 5\times10^3 m/s - (3.16\times10^{13} m/s^2)(8\times10^{-8}s) = \boxed{-2.52\times10^6 m/s}$ (1)(1)

$\Rightarrow \boxed{\vec{v} = 3\times10^5 m/s\ \hat{i} + 2.52\times10^6 m/s\,(-\hat{j})}$ (2)

3. Two non-conductive rods are located on x-axis. The first rod has a length of 10 cm and the second one has a length 20 cm. A charge of $q = -5 \times 10^{-15}$ C is uniformly distributed along the each length. The distance between the centres of the rods is 40 cm. Find the **magnitude of the electric potential** at the middle of the distance between the centres of the rods. (Hints: $\int dx/(A-x) = -ln|A-x| + C$ and $\int dx/(x-A) = ln|-A+x| + C$)



uniform distribution, $\lambda = Q/L$

$\lambda_1 = \frac{-5\times10^{-15}C}{10\times10^{-2}m}$, $\lambda_2 = \frac{-5\times10^{-15}C}{20\times10^{-2}m}$ (2)

$dV_1 = k\frac{\lambda_1 dx}{(10-x+15)}$ (2), $V_1 = \int dV_1 = k\lambda_1 \int_0^{10} \frac{dx}{(25-x)}$ (3)

(2) $dV_2 = k\frac{\lambda_2 dx}{x-25}$, $V_2 = k\lambda_2 \int_{35}^{55} \frac{dx}{(x-25)}$ (3)

$\Rightarrow V_1 = k\lambda_1\left(-ln|25-x|\Big|_0^{10}\right) = k\lambda_1(-ln15 + ln25) = k\lambda_1 \, ln(5/3)$

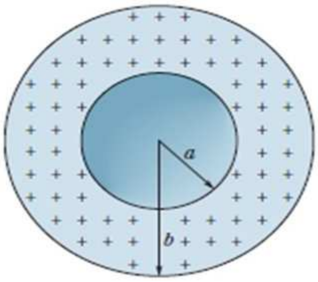
$= \left(8.99\times10^9 \frac{Nm^2}{C^2}\right)\left(\frac{-5\times10^{-15}C}{10\times10^{-2}m}\right) ln(5/3) = \underline{-2.30\times10^{-4}V}$ (2)

$V_2 = k\lambda_2\left(ln|-25+x|\Big|_{35}^{55}\right) = k\lambda_2(ln30 - ln10) = k\lambda_2 \, ln3$

$= \left(8.99\times10^9 \frac{Nm^2}{C^2}\right)\left(\frac{-5\times10^{-15}C}{20\times10^{-2}m}\right) ln3 = \underline{-2.47\times10^{-4}V}$ (2)

$\Rightarrow V_P = V_1 + V_2 = \boxed{-4.77\times10^{-4}V}$ (2)

4. Figure shows a spherical shell with uniform volume charge density $\rho = 1.56 \times 10^{-9}\ C/m^3$, inner radius $a = 10\ cm$, and outer radius $b = 2.00a$.

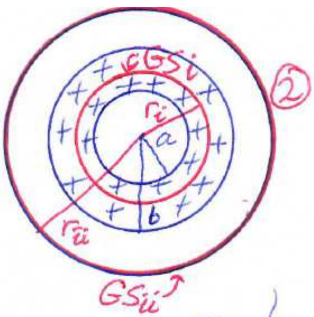


What is the magnitude of the electric field at radial distances

i $r = 1.5a$

ii $r = 3.00b$

Hints: Use Gauss' Law. Volume of the spherical shell: $\frac{4}{3}\pi(b^3 - a^3)$.



$\rho = 1.56\times10^{-9}\ C/m^3$

$a = 10\times10^{-2}\ m$

$b = 2a$

$r_i = 1.5a$

$r_{ii} = 3b$

$V_{ss} = \frac{4}{3}\pi(b^3 - a^3)$

$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$ (2), $\rho = \frac{q}{V} = \frac{q_{enc}}{V_{enc}}$ (2), $\vec{E} \parallel \vec{A}$ (2)

i) $\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \rightsquigarrow E 4\pi r_i^2 = \frac{q_{enc}}{\varepsilon_0}$ $\Big\}$ $q_{enc} = ?$

GS_i

$\rightsquigarrow q_{enc} = \rho V_{enc} = \rho \frac{4}{3}\pi(r_i^3 - a^3)$ (2)

$\rightsquigarrow E 4\pi r_i^2 = \rho \frac{4}{3}\frac{\pi(r_i^3 - a^3)}{\varepsilon_0}$ $\Big\}$ $r_i = 1.5a$

$\Rightarrow E = \frac{\rho\, 4/3\pi}{4\pi\varepsilon_0}\ \frac{((1.5a)^3 - a^3)}{1.5a^2} = \frac{\rho a}{3\varepsilon_0}\left(\frac{2.375}{2.25}\right)$ (2) (1)

$E(r = 1.5a) = \frac{(1.56\times10^{-9}\ C/m^3)(10\times10^{-2}\ m)}{3 \times 8.85\times10^{-12}\ C^2/(Nm^2)}\left(\frac{2.375}{2.25}\right) = \boxed{6.20\ N/C}$ (1)

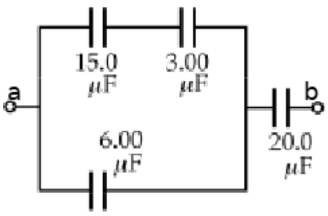
ii) $E 4\pi r_{ii}^2 = \frac{q_{enc}}{\varepsilon_0} \rightsquigarrow q_{enc} = \rho \frac{4}{3}\pi(b^3 - a^3)$ (2)

GS_{ii} $\rightsquigarrow E 4\pi r_{ii}^2 = \rho \frac{4}{3}\frac{\pi(b^3 - a^3)}{\varepsilon_0}$ $\Big\}$ $r_{ii} = 3b$

$\Rightarrow E = \frac{\rho\, 4/3\pi}{4\pi\varepsilon_0}\ \frac{b^3 - a^3}{(3b)^2} = \frac{\rho\, 7a}{108\,\varepsilon_0}$

(2) $E(r = 3b) = \frac{(1.56\times10^{-9}\ C/m^3)\, 7(10\times10^{-10}\ m)}{108 \times 8.85\times10^{-12}\ C^2/(Nm^2)} = \boxed{1.14\ N/C}$ (1) (1)

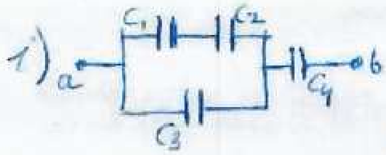
5. Four capacitors are connected as shown in Figure.



i Find the equivalent capacitance between points a and b.

ii Calculate the charge on each capacitor if $\Delta V_{ab} = 15.0\ V$.

1)



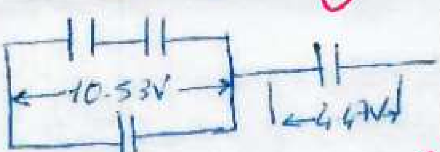
$C_{12} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(15\times10^{-6}F)(3\times10^{-6}F)}{18\times10^{-6}F} = 2.5\times10^{-6}F$ (3)

$C_{123} = C_{12} + C_3 = 2.5\times10^{-6}F + 6\times10^{-6}F = 8.5\times10^{-6}F$ (3)

$C_{eqv} = C_{1234} = \frac{C_{123} C_4}{C_{123} + C_4} = \frac{(8.5\times10^{-6}F)(20\times10^{-6}F)}{28.5\times10^{-6}F} = 5.97\times10^{-6}F = 5.97\mu F$ (3)

ii) $C = \frac{Q}{V} \rightarrow Q_{eqv} = Q_{1234} = C_{eqv} V = 5.97\times10^{-6}F \times 15V = 89.67\mu C$ (3)

$\rightarrow Q_4 = Q_{123} = Q_{eqv} = 89.67\mu C$ (1) $\rightarrow V_4 = \frac{89.67\mu C}{20\mu F} = 4.47V$ (1) ← (4)



10.53V ← (3) (1) $\rightarrow Q_3 = C_3 V_3 = 63.18\mu C$ (3)

$Q_{12} = C_{12} V = (2.5\times10^{-6}F)\ 10.53V = Q_1 = Q_2$

$\Rightarrow V_1 = \frac{Q_1}{C_1} = \frac{2.63\mu C}{15\mu F} = 1.75V$ (1) ← (1) $\rightarrow Q_1 = Q_2 = 2.63\mu C$ (1) (1)

$V_2 = \frac{Q_1}{C_2} = 8.78V$ ← (2) (1)