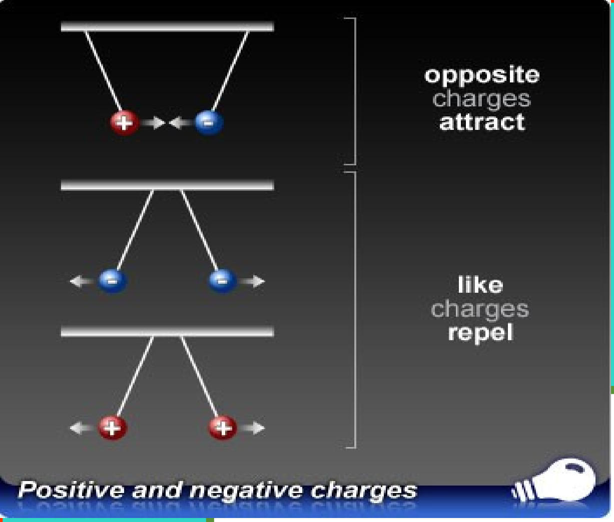
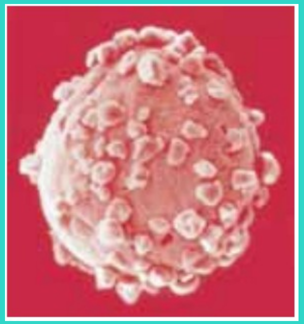
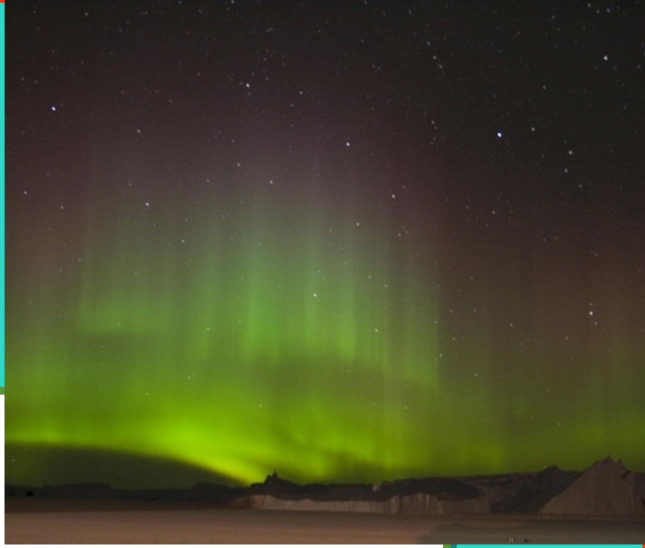






Chapter 21

Electric Charge

İZMİR KÂTİP ÇELEBİ ÜNİVERSİTESİ

21 ELECTRIC CHARGE 561

For Your Information (FYI)

Physics of electromagnetism → Combination of electric and magnetic phenomena

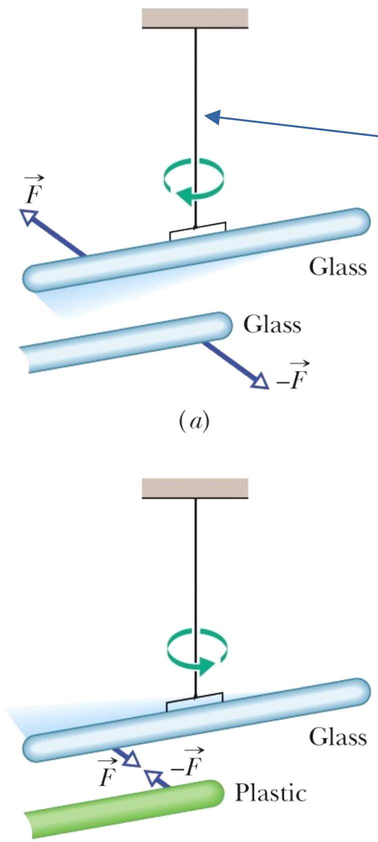
Begin with electrical phenomena; first step is to discuss the nature of electric charge and electric force.

Every object contains a vast amount of electric charge; intrinsic characteristic

21-2 Electric Charge

- There are two kinds of charge: **positive** and **negative**.
- An object with equal amounts of the two kinds of charge is electrically **neutral**, whereas one with an imbalance is electrically **charged**.

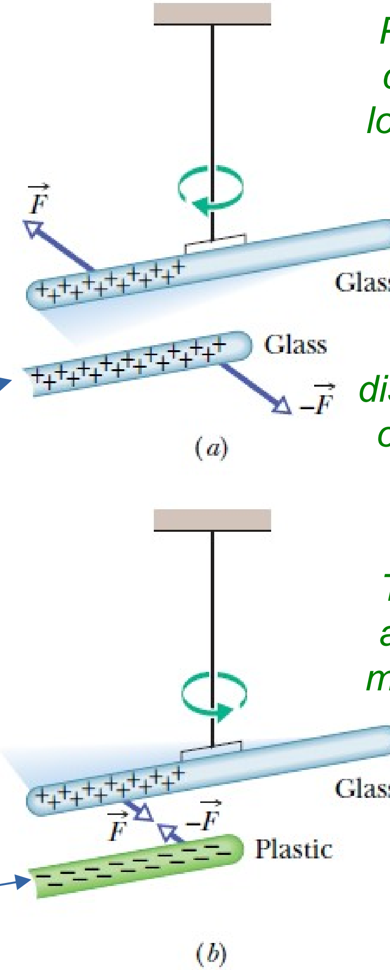
Charges with the same electrical sign repel each other, and charges with opposite electrical signs attract each other.



(a) The two glass rods were each rubbed with a silk cloth (positively charged) and one was suspended by thread. When they are close to each other, they repel each other.

(b) The plastic rod was rubbed with fur (negatively charged). When brought close to the glass rod, the rods attract each other.

Atoms consist of positively charged protons, negatively charged electrons, and electrically neutral neutrons. The protons and neutrons are packed tightly together in a central nucleus.



Positive charge localized!

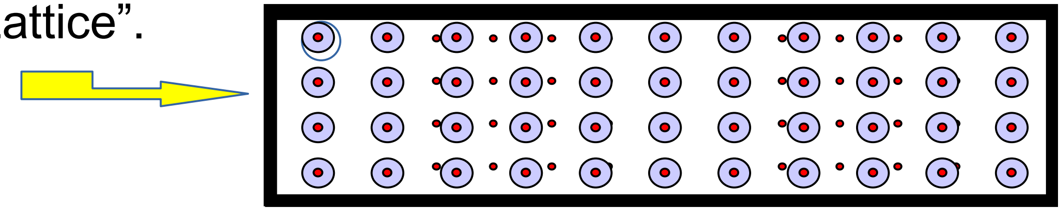
Not distributed over the object

Think about metals!

Fig. 21-2 (*a*) Two charged rods of the same sign repel each other. (*b*) Two charged rods of opposite signs attract each other. Plus signs indicate a positive net charge, and minus signs indicate a negative net charge.

Electric Charges in Solids

- *When atoms of a conductor come together to form the solid, some of their outermost (and so most loosely held) electrons become free, leaving behind positively charged atoms.*
- In macroscopic solids, nuclei often arrange themselves into a regular pattern called a "Crystal Lattice".



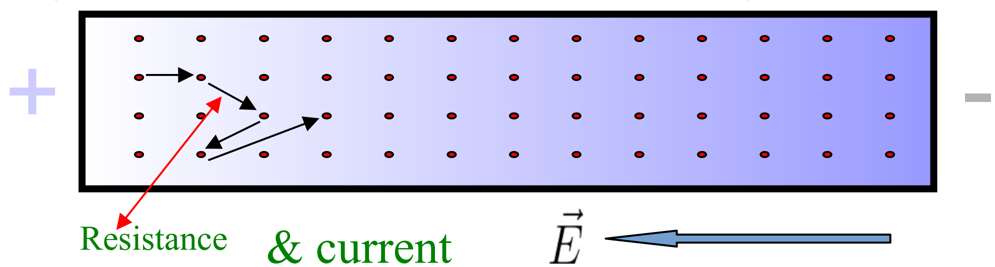
Electrons move around this lattice.

Red Circles = Static Positive Charge (Nuclei)

Depending on "How They Move" the solid can be classified by its "Electrical Properties" as an Insulator or a Conductor. We call the mobile electrons as conduction electrons.

- **Conductors** are materials through which charge can move **freely**; examples include metals (such as copper in common lamp wire), the human body, and tap water.

Forming a "Sea" of electrons. This is why metals conduct electricity.



Under Applied Potential Difference V

Blue Background = Mobile Electrons

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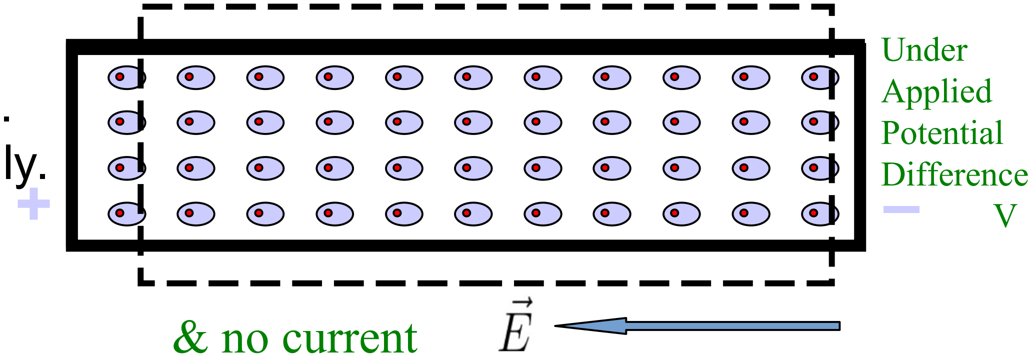
21-3 Conductors and Insulators FYI

Materials classified based on their ability to move charge (Conductance)

- **Nonconductors - ** also called **insulators - ** are materials through which charge cannot move freely; examples include wood, rubber, plastic, glass, and chemically pure water.
- There are few (if any) free electrons in a nonconductor.

In an **Insulator**, each electron cloud is tightly bound to the protons in a nucleus. But the electron cloud can "Distort" locally.



but polarization & dipole moment

- **Semiconductors** are materials that are intermediate between conductors and insulators; examples include silicon and germanium in computer chips. on / off states
- **Superconductors** are materials that are perfect conductors, allowing charge to move without any hindrance (no resistance!).

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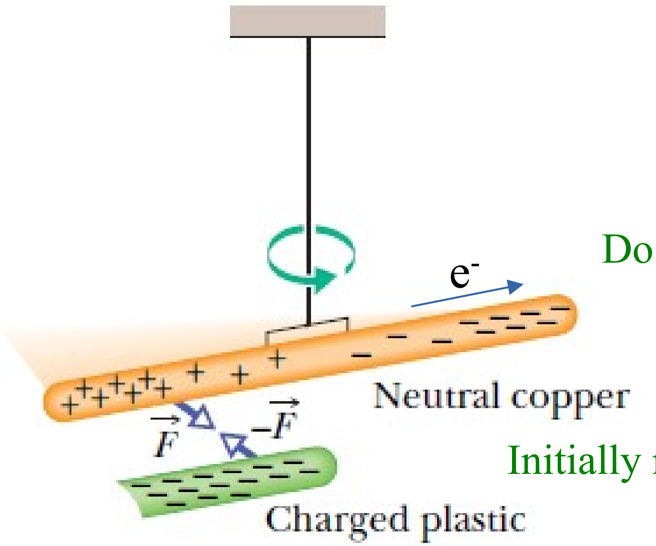
Induced Charge



Do we have a uniform charge distribution?

Initially neutral! Is still?

Fig. 21-4 A neutral copper rod is electrically isolated from its surroundings by being suspended on a nonconducting thread. Either end of the copper rod will be attracted by a charged rod. Here, conduction electrons in the copper rod are repelled to the far end of that rod by the negative charge on the plastic rod. Then that negative charge attracts the remaining positive charge on the near end of the copper rod, rotating the copper rod to bring that near end closer to the plastic rod.

Coulomb's Law

$$\vec{F} = k\frac{q_1 q_2}{r^2}\hat{\mathrm{r}} \qquad \text{(Coulomb's law)},$$

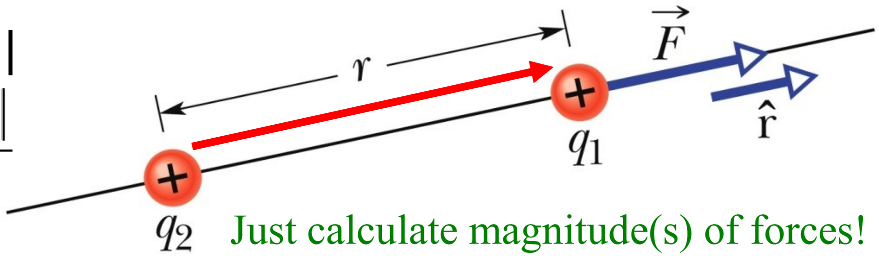
Charles-Augustin De Coulomb (1736–1806)

- The force of repulsion/attraction due to the charge properties of objects is called an **electrostatic force.** Why static? We study the snapshot of the system.
- Coulomb's law describes the **electrostatic force** (or electric force) between two charged particles.
  - The magnitude of the electric forces between charged objects is measured by "Torsion Balance" which was invented by Charles Augustin De Coulomb. http://www.dnatube.com/video/11874/Application-Of-Coulombs-Torsion-Balance
- If the particles have charges q_2 and q_1, are separated by distance r, and are *at rest (or moving only slowly)* relative to each other, then the magnitude of the force acting on each due to the other is given by

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2}$$

- Note that Newton's Third Law says $|F_{12}| = |F_{21}|$

$$F_{12} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2}$$



Just calculate magnitude(s) of forces!

$$\vec{F} = |\vec{F}|\hat{r}$$

First make analysis of magnitudes of the forces then introduce (unit) vector

where $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/N.m^2$ is the constant for permittivity of free space.

- Response of the medium to the external $\vec{E}$
- If not air/vacuum: $\varepsilon_0 \rightarrow \varepsilon$

- The ratio $1/4\pi\varepsilon_0$ is often replaced with the electrostatic constant (or Coulomb constant) $k=8.99\times10^9\ N.m^2/C^2$.

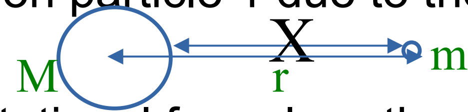
- **Multiple Forces:** If multiple electrostatic forces act on a particle, the net force is the vector sum (not scalar sum) of the individual forces

Interacting independently in pairs

Many charges but target is one
Vector Algebra

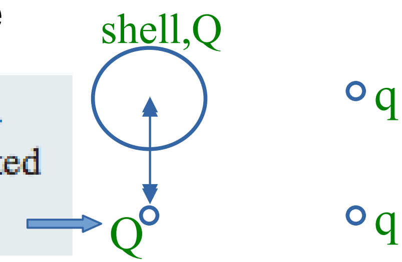
$$\vec{F}_{1,net} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} + \cdots + \vec{F}_{1n},$$

in which, $F_{1,4}$ is the force acting on particle 1 due to the presence of particle 4, etc.

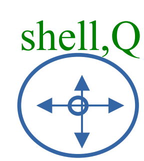


- **Shell Theories:** As with gravitational force law, the shell theorem has analogs in electrostatics. There are two shell theories for electrostatic force

Shell theory 1. A charged particle outside a shell with charge uniformly distributed on its surface is attracted or repelled as if the shell's charge were concentrated as a particle at its center.



Shell theory 2. A charged particle inside a shell with charge uniformly distributed on its surface has no net force acting on it due to the shell.



Let's say q is at the center: net force is zero.

- **Spherical Conductors:** If excess charge is placed on a spherical shell that is made of conducting material, they repel each other and the excess charge spreads uniformly over the (external) surface. Now, first shell theorem works!

21-4 Coulumb's Law

- The coulomb unit is derived from SI unit ampere for electric current i.
- Current is the rate Δq/Δt at which charge moves past a point or through a region

$$i = \frac{dq}{dt} \qquad \text{(electric current)},$$

in which i is the current (in amperes) and dq (in coulombs) is the amount of charge moving past a point or through a region in time dt (in seconds).

- Therefore, $1\text{ C} = (1\text{ A})(1\text{ s}).$

Example, The net force due to two other particles:

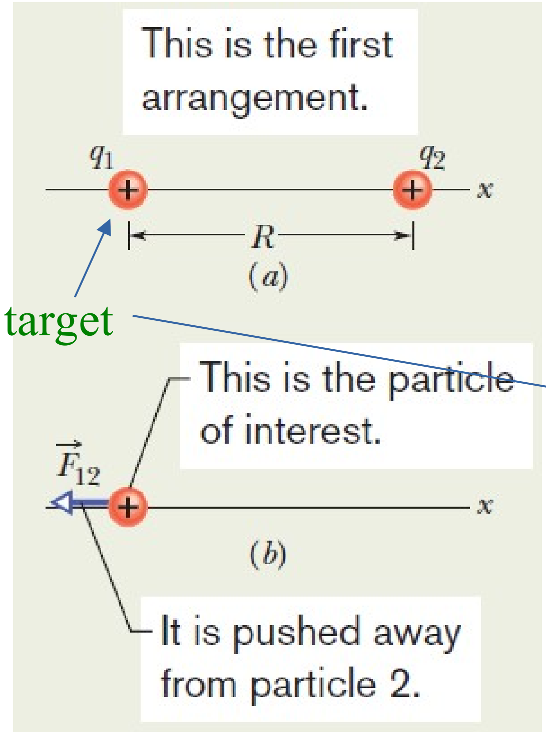
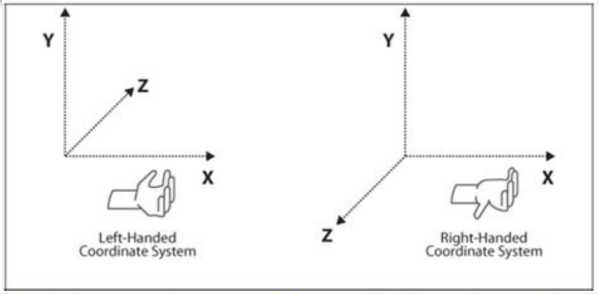


Fig. 21-8 *(a)* Two charged particles of charges q_1 and q_2 *are fixed in* place on an *x axis*. *(b) The free-body* diagram for particle 1, showing the electrostatic force on it from particle 2.

$$\vec{F}_{1,net} = |\vec{F}_{1,net}|\hat{r}$$



(a) Figure 21-8*a* shows two positively charged particles fixed in place on an *x* axis. The charges are $q_1 = 1.60 \times 10^{-19}$ C and $q_2 = 3.20 \times 10^{-19}$ C, and the particle separation is $R = 0.0200$ m. What are the magnitude and direction of the electrostatic force $\vec{F}_{12}$ on particle 1 from particle 2?

$$F_{12} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{R^2}$$

$$= (8.99 \times 10^9\ \mathrm{N\cdot m^2/C^2}) \times \frac{(1.60 \times 10^{-19}\ \mathrm{C})(3.20 \times 10^{-19}\ \mathrm{C})}{(0.0200\ \mathrm{m})^2}$$

$$= 1.15 \times 10^{-24}\ \mathrm{N}.$$

First magnitude then direction

Thus, force $\vec{F}_{12}$ in unit-vector notation is

$$\vec{F}_{12} = -(1.15 \times 10^{-24}\ \mathrm{N})\hat{\mathrm{i}}. \qquad \text{(Answer)}$$

$$|\vec{F}_{12}|(-\hat{i})$$

Example, The net force due to two other particles, cont.:

(b) Figure 21-8*c* is identical to Fig. 21-8*a* except that particle 3 now lies on the *x* axis between particles 1 and 2. Particle 3 has charge $q_3 = -3.20 \times 10^{-19}$ C and is at a distance $\frac{3}{4}R$ from particle 1. What is the net electrostatic force $\vec{F}_{1,\text{net}}$ on particle 1 due to particles 2 and 3?

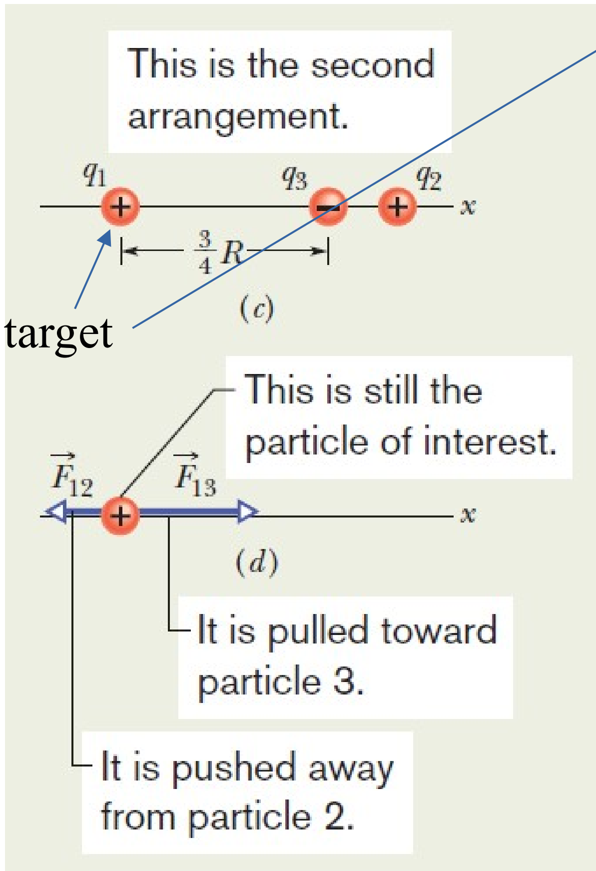


Fig. 21-8 (c) Particle 3 included. *(d) Free-body diagram* for particle 1.

Screening effect!

Superposition principle → NO Screening

$$\vec{F}_{1,net} = \vec{F}_{12} + \vec{F}_{13} \qquad \vec{F}_{12} \checkmark$$

$$F_{13} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_3|}{(\frac{3}{4}R)^2}$$

$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) \times \frac{(1.60 \times 10^{-19}\ \text{C})(3.20 \times 10^{-19}\ \text{C})}{(\frac{3}{4})^2(0.0200\ \text{m})^2}$$

$$= 2.05 \times 10^{-24}\ \text{N}.$$

+x-direction

We can also write $\vec{F}_{13}$ in unit-vector notation:

$$\vec{F}_{13} = (2.05 \times 10^{-24}\ \text{N})\hat{\text{i}}.$$

The net force $\vec{F}_{1,\text{net}}$ on particle 1 is the vector sum of $\vec{F}_{12}$ and $\vec{F}_{13}$; that is, from Eq. 21-7, we can write the net force $\vec{F}_{1,\text{net}}$ on particle 1 in unit-vector notation as

$$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{13}$$

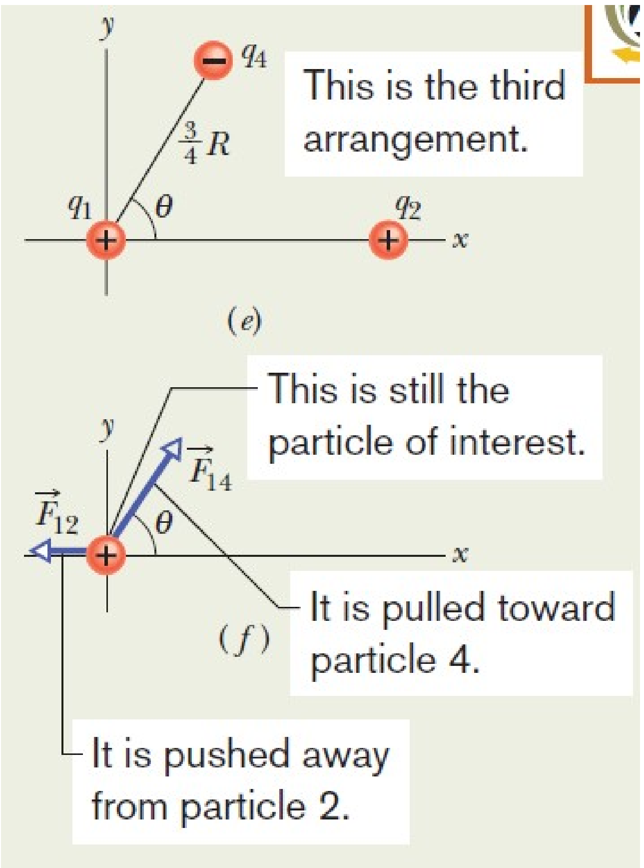
$$= -(1.15 \times 10^{-24}\ \text{N})\hat{\text{i}} + (2.05 \times 10^{-24}\ \text{N})\hat{\text{i}}$$

$$= (9.00 \times 10^{-25}\ \text{N})\hat{\text{i}}. \qquad \text{(Answer)}$$

Example, The net force due to two other particles, cont.:

(c) Figure 21-8*e* is identical to Fig. 21-8*a* except that particle 4 is now included. It has charge $q_4 = -3.20 \times 10^{-19}$ C, is at a distance $\frac{3}{4}R$ from particle 1, and lies on a line that makes an angle $\theta = 60°$ with the x axis. What is the net electrostatic force $\vec{F}_{1,\text{net}}$ on particle 1 due to particles 2 and 4?

$$\vec{F}_{1,net} = \vec{F}_{12} + \vec{F}_{14}$$



$\vec{F}_{12}$ ✓

$\vec{F}_{14} \Rightarrow x$ & y componets

$$F_{14} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_4|}{(\frac{3}{4}R)^2}$$
$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) \times \frac{(1.60 \times 10^{-19}\ \text{C})(3.20 \times 10^{-19}\ \text{C})}{(\frac{3}{4})^2(0.0200\ \text{m})^2}$$
$$= 2.05 \times 10^{-24}\ \text{N}.$$

$$\vec{F}_{14} = (F_{14}\cos\theta)\hat{i} + (F_{14}\sin\theta)\hat{j}.$$

Substituting 2.05×10^{-24} N for F_{14} and 60° for θ, this becomes

$$\vec{F}_{14} = (1.025 \times 10^{-24}\ \text{N})\hat{i} + (1.775 \times 10^{-24}\ \text{N})\hat{j}.$$

$F_{14,x}$ $\quad F_{14,y}$

Then we sum:

$$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{14}$$
$$= -(1.15 \times 10^{-24}\ \text{N})\hat{i} + (1.025 \times 10^{-24}\ \text{N})\hat{i} + (1.775 \times 10^{-24}\ \text{N})\hat{j}$$
$$\approx (-1.25 \times 10^{-25}\ \text{N})\hat{i} + (1.78 \times 10^{-24}\ \text{N})\hat{j}.$$

in vectorial form (Answer)

Fig. 21-8 (*e*) Particle 4 included. (*f*) Freebody diagram for particle 1.

Example, The net force due to two other particles, cont.:

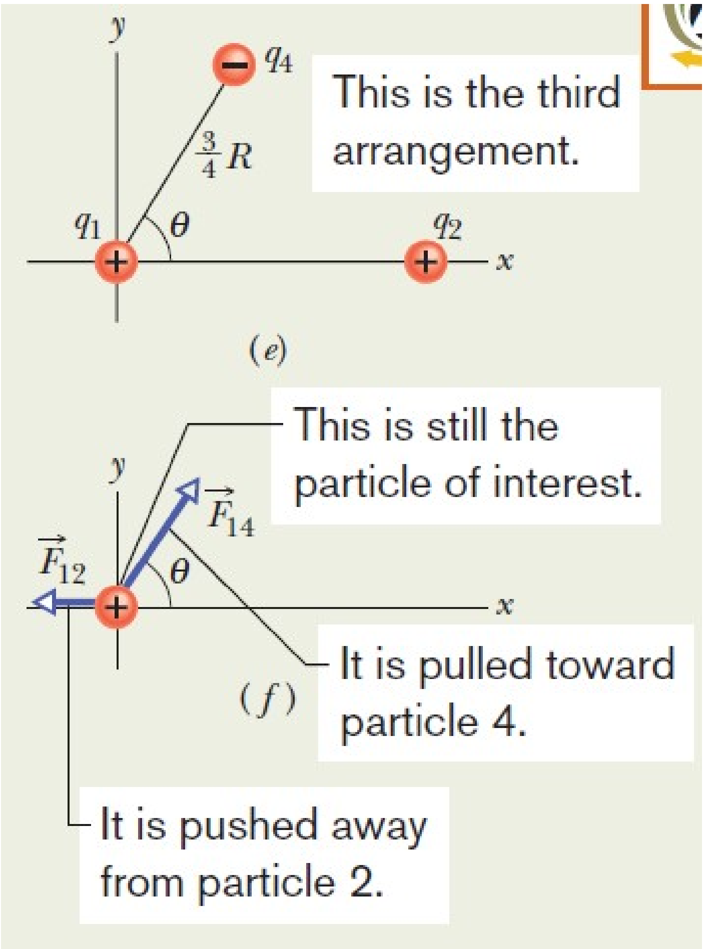


Fig. 21-8 *(e)* Particle 4 included. *(f)* Freebody diagram for particle 1.

$$F_{1,\text{net},x} = F_{12,x} + F_{14,x} = F_{12} + F_{14}\cos 60°$$
$$= -1.15 \times 10^{-24}\text{ N} + (2.05 \times 10^{-24}\text{ N})(\cos 60°)$$
$$= -1.25 \times 10^{-25}\text{ N}.$$

$$F_{1,\text{net},y} = F_{12,y} + F_{14,y} = 0 + F_{14}\sin 60°$$
$$= (2.05 \times 10^{-24}\text{ N})(\sin 60°)$$
$$= 1.78 \times 10^{-24}\text{ N}.$$

The net force $\vec{F}_{1,\text{net}}$ has the magnitude

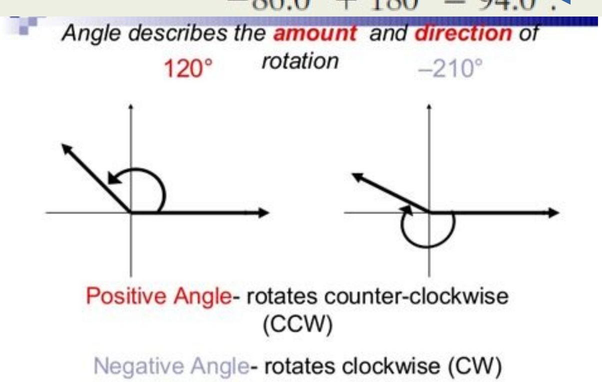
$$F_{\text{net}} = \sqrt{F^2_{1,\text{net},x} + F^2_{1,\text{net},y}} = 1.78 \times 10^{-24}\text{ N}.$$

To find the direction of $\vec{F}_{1,\text{net}}$, we take

$$\theta = \tan^{-1}\frac{F_{1,\text{net},y}}{F_{1,\text{net},x}} = -86.0°.$$ ← cw (from -x)

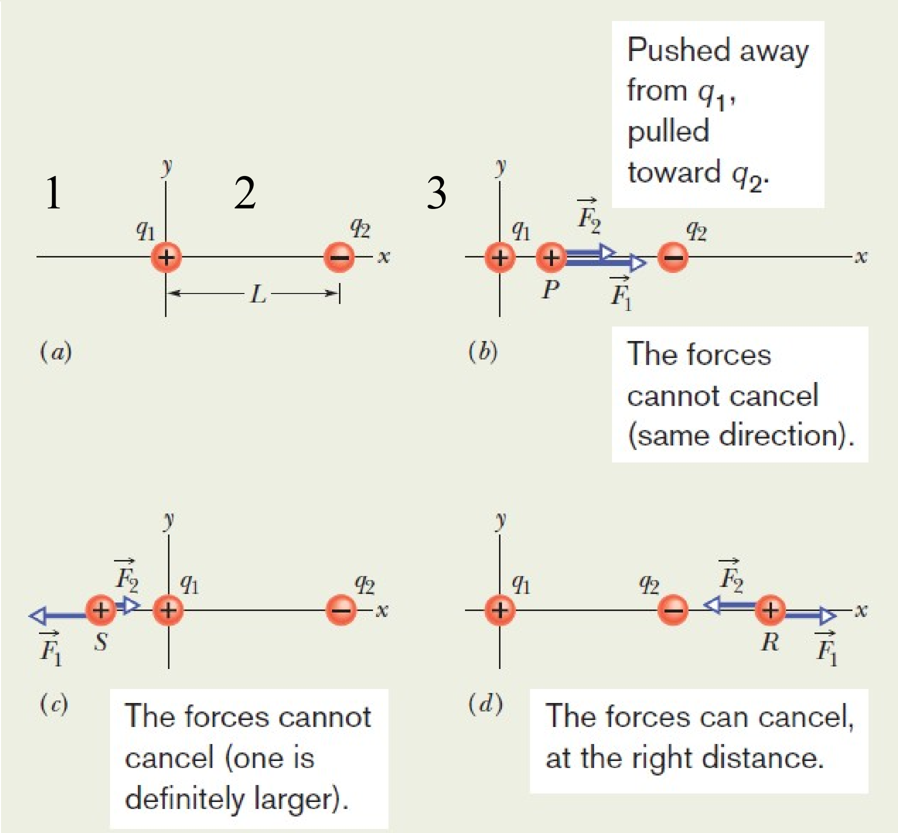
However, this is an unreasonable result because $\vec{F}_{1,\text{net}}$ must have a direction between the directions of $\vec{F}_{12}$ and $\vec{F}_{14}$. To correct θ, we add 180°, obtaining

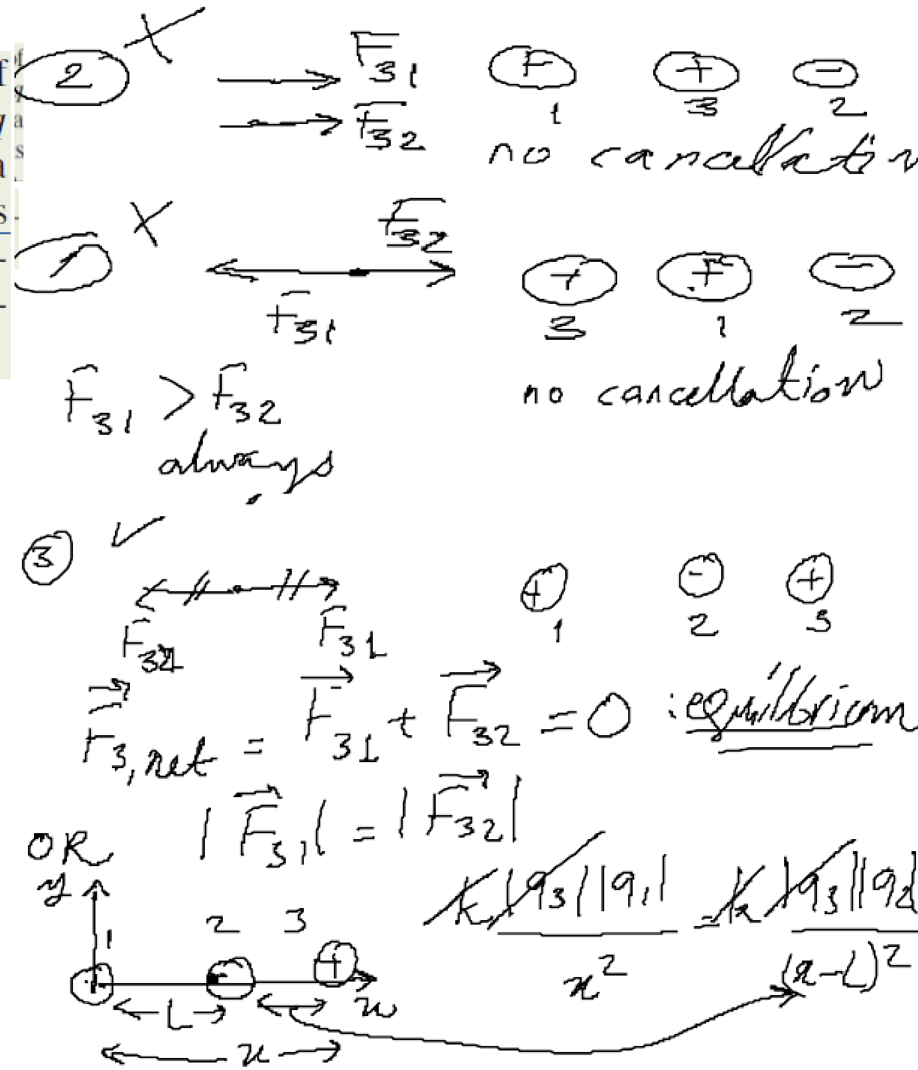
$$-86.0° + 180° = 94.0°.$$ ← ccw (from +x) (Answer)



Example, Equilibrium of two forces:

Figure 21-9*a* shows two particles fixed in place: a particle of charge $q_1 = +8q$ at the origin and a particle of charge $q_2 = -2q$ at $x = L$. At what point (other than infinitely far away) can a proton be placed so that it is in *equilibrium* (the net force on it is zero)? Is that equilibrium *stable* or *unstable*? (That is, if the proton is displaced, do the forces drive it back to the point of equilibrium or drive it farther away?)





Example, Equilibrium of two forces, cont.:

$$\frac{1}{4\pi\varepsilon_0}\frac{8qq_p}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{2qq_p}{(x-L)^2}.$$

$$\left(\frac{x-L}{x}\right)^2 = \frac{1}{4}.$$

$$\frac{x-L}{x} = \frac{1}{2},$$

$$x = 2L.$$

Instant stability

The equilibrium at x=2L is unstable that is, if the proton is displaced leftward from point *R,* then F_1 *and* F_2 both increase but F_2 increases more (because q_2 is closer than q_1), and a net force will drive the proton farther leftward. If the proton is displaced rightward, both F_1 and F_2 decrease but F_2 decreases more, and a net force will then drive the proton farther rightward. In a stable equilibrium, if the proton is displaced slightly, it returns to the equilibrium position.

Example, Charge Sharing:

In Fig. 21-10*a*, two identical, electrically isolated conducting spheres *A* and *B* are separated by a (center-to-center) distance *a* that is large compared to the spheres. Sphere *A* has a positive charge of $+Q$, and sphere *B* is electrically neutral. Initially, there is no electrostatic force between the spheres. (Assume that there is no induced charge on the spheres because of their large separation.)

Shell theorem I

(a) Suppose the spheres are connected for a moment by a conducting wire. The wire is thin enough so that any net charge on it is negligible. What is the electrostatic force between the spheres after the wire is removed?

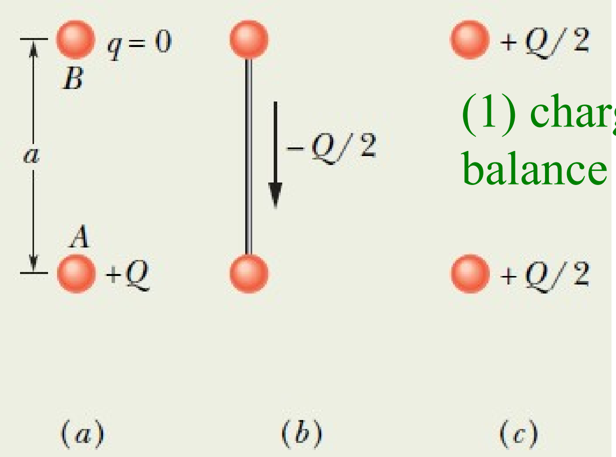


Fig. 21-10 *Two small conducting spheres A and B. (a) To start, sphere A is charged positively. (b) Negative charge is transferred from B to A through a connecting wire. (c) Both spheres are then charged* positively.

(1) charge balance & (2) charge conservation $Q_i=Q_f$

(1) Since the spheres are identical, connecting them means that they end up with identical charges (same sign and same amount). (2) The initial sum of the charges (including the signs of the charges) must equal the final sum of the charges.

Reasoning: When the spheres are wired together, the (negative) conduction electrons on B move away from one another (along the wire to positively charged A— Fig. 21-10b.)

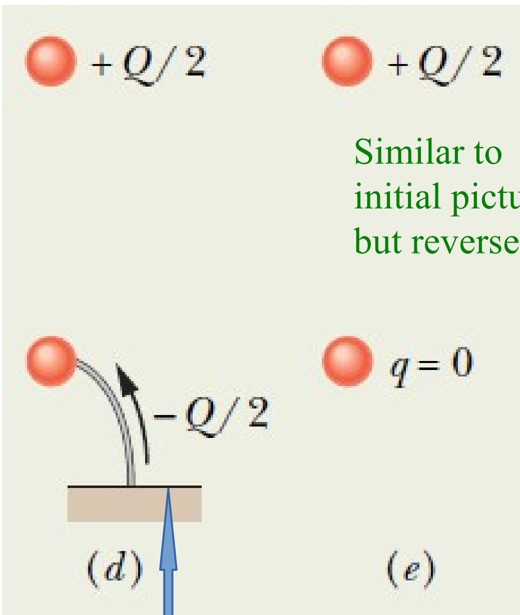
As B loses negative charge, it becomes positively charged, and as A gains negative charge, it becomes less positively charged. The transfer of charge stops when the charge on B has increased to Q/2 and the charge on A has decreased to Q/2, which occurs when Q/2 has shifted from B to A.

The spheres, now positively charged, repel each other.

$$F = \frac{1}{4\pi\varepsilon_0}\frac{(Q/2)(Q/2)}{a^2} = \frac{1}{16\pi\varepsilon_0}\left(\frac{Q}{a}\right)^2.$$

Example, Charge Sharing, cont.:

(b) Next, suppose sphere *A* is grounded momentarily, and then the ground connection is removed. What now is the electrostatic force between the spheres?



Similar to initial picture, but reversed

Fig. 21-10 *(d) Negative charge is transferred through a grounding wire* to sphere *A. (e) Sphere A is then neutral.*

Ground → neutral
(large enough reservoir)

Creating a pathway between object and Earth's surface

Reasoning: When we provide a conducting path between a charged object and the ground (which is a huge conductor), we neutralize the object.

Where sphere *A* negatively charged, the mutual repulsion between the excess electrons would cause them to move from the sphere to the ground.

However, because sphere *A* is positively charged, electrons with a total charge of *Q/2* move from the ground up onto the sphere (Fig. 21-10*d),* leaving the sphere with a charge of 0 (Fig. 21-10*e).* Thus, there is (again) no electrostatic force between the two spheres.

Example, with symmetry:

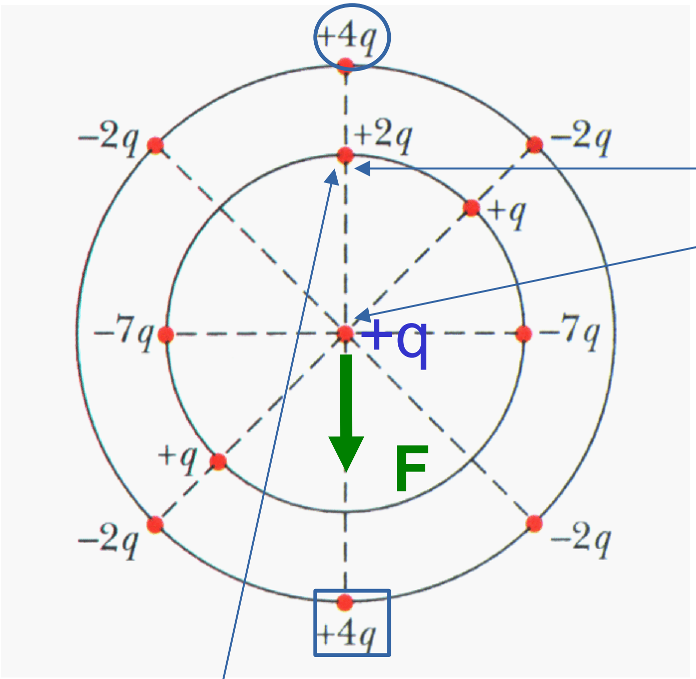
Charge +q placed at center

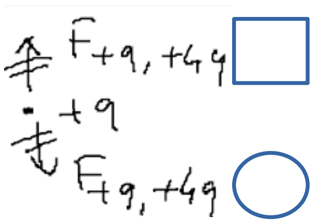
What is the Force on central particle?

target

All forces cancel except from +2q!

consider the symmetry!





without considering the symmetry:

$$\vec{F}_{+q,net} = \sum_{i=1}^{11} \vec{F}_{+q,i}$$

$$\left|\vec{F}\right| = \frac{k\ \left|+2q\right|\ \left|+q\right|}{r^2}$$

http://www.phys.lsu.edu/~jdowling/PHYS21132-SP15/lectures/index.html

- Electric charge is quantized (restricted to certain values). The total charge of any object is found to always be a multiple of a certain **elementary charge**, "e":

$$e = 1.602 \times 10^{-19}\ \text{C}.$$

- The value of this elementary charge is one of the fundamental constants of nature, and it is the magnitude of the charge of both the proton and the electron.

Table 21-1

The Charges of Three Particles

Particle	Symbol	Charge
Electron	e or e^-	$-e$
Proton	p	$+e$
Neutron	n	0

- Elementary particles either carry no charge, or carry a single elementary charge.
  - Since the days of Benjamin Franklin, our understanding of of the nature of electricity has changed from being a type of '*continuous fluid' (not correct!) to a collection of smaller charged particles*.
- The charge of a particle can be written as *ne*, where *n* is a positive or negative integer and *e* is the elementary charge.
- Any positive or negative charge *q* that can be detected can be written as

$$q = ne, \qquad n = \pm 1, \pm 2, \pm 3, \ldots,$$

Benjamin Franklin (1705–1790)

- It is possible, for example, to find a particle that has no charge at all, or a charge of +10e, or -6e, but not a particle with a charge of, say, 3.57e.
- One cannot ISOLATE FRACTIONAL CHARGE (e.g. –1/2 e, +1/3 e, etc.)

21-5 Charge is Quantized FYI

- Many descriptions of electric charge use terms that might lead you to the conclusion that charge is a substance. Phrases like:
  "Charge on a sphere"
  "Charge transferred"
  "Charge carried on the electron"
- However, charge is a ***property*** of **particles,** one of many properties, such as mass.

Mass is an intrinsic property as well as charge.

İZMİR KÂTİP ÇELEBİ ÜNİVERSİTESİ

Example, Mutual Electric Repulsion in a Nucleus:

Atomic sizes → 10^{-10} m (Angstrom or nanometer)

The nucleus in an iron atom has a radius of about 4.0×10^{-15} m and contains 26 protons.

(a) What is the magnitude of the repulsive electrostatic force between two of the protons that are separated by 4.0×10^{-15} m?

KEY IDEA

The protons can be treated as charged particles, so the magnitude of the electrostatic force on one from the other is given by Coulomb's law.

Calculation: Table 21-1 tells us that the charge of a proton is $+e$. Thus, Eq. 21-4 gives us

$$F = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

$$= \frac{(8.99 \times 10^9\ \mathrm{N\cdot m^2/C^2})(1.602 \times 10^{-19}\ \mathrm{C})^2}{(4.0 \times 10^{-15}\ \mathrm{m})^2}$$

$$= 14\ \mathrm{N}. \quad \text{(Answer)}$$

← Small or NOT?

No explosion: This is a small force to be acting on a macroscopic object like a cantaloupe, but an enormous force to be acting on a proton. Such forces should explode the nucleus of any element but hydrogen (which has only one proton in its nucleus). However, they don't, not even in nuclei with a great many protons. Therefore, there must be some enormous attractive force to counter this enormous repulsive electrostatic force. → need a strong force to balance!

(b) What is the magnitude of the gravitational force between those same two protons?

KEY IDEA

Because the protons are particles, the magnitude of the gravitational force on one from the other is given by Newton's equation for the gravitational force (Eq. 21-2).

Calculation: With m_p ($= 1.67 \times 10^{-27}$ kg) representing the mass of a proton, Eq. 21-2 gives us

$$F = G\frac{m_p^2}{r^2}$$

not effective in atomic sizes

$$= \frac{(6.67 \times 10^{-11}\ \mathrm{N\cdot m^2/kg^2})(1.67 \times 10^{-27}\ \mathrm{kg})^2}{(4.0 \times 10^{-15}\ \mathrm{m})^2}$$

$$= 1.2 \times 10^{-35}\ \mathrm{N}. \quad \text{(Answer)}$$

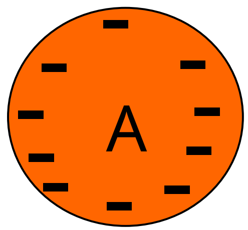
very small

CHECKPOINT 4

Initially, sphere A has a charge of $-50e$ and sphere B has a charge of $+20e$. The spheres are made of conducting material and are identical in size. If the spheres then touch, what is the resulting charge on sphere A?

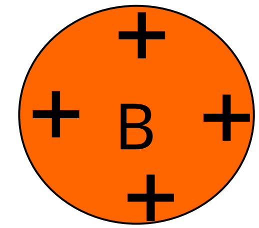
Charges are located at surface

Let "–" = –5e and "+" = +5e



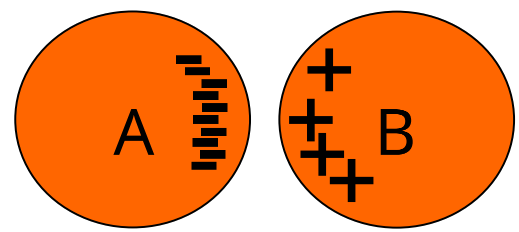
–50e

Long distance
No electrostatic force
No induction



+20e

- The net electric charge of any isolated system is always conserved.



–50e +20e

No contact. But induction: charges are relocated!

Total Charge on BOTH SPHERES is conserved:

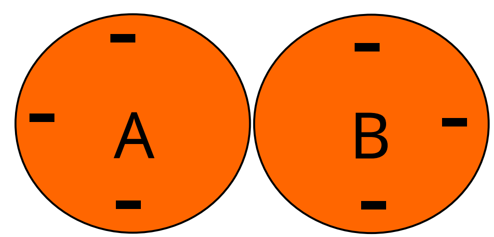
$-50e + 20e = -30e$ → Q/2 at both A & B

Since spheres are identical, total charge is split evenly.

Charge on sphere A ONLY :

$-30e/2 = -15e$

Charge Sharing & Charge Conservation



–15e –15e

Touch. Relocation of the charges

http://www.phys.lsu.edu/~jdowling/PHYS21132-SP15/lectures/index.html

1) What must be the distance between point charge $Q_1 = 26.0\ \mu C$ and point charge $Q_2 = -47.0\ \mu C$ for the electrostatic force between them to have a magnitude of 5.70 N?

should be in SI unit

$Q_1 = 26.0\ \mu C$

$Q_2 = -47.0\ \mu C$

$|\vec{F}_{12}| = |\vec{F}_{21}| = 5.70 N$

$F = k \frac{|q_1||q_2|}{d^2}$

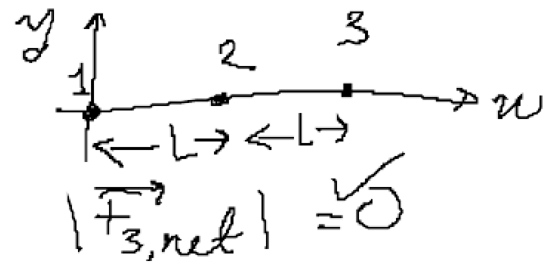
$d = ?$

$5.70 N = 8.99\times10^9 N m^2/C^2 \frac{|26.0\times10^{-6} C| |(-47\times10^{-6} C)|}{d^2}$

$d = 1.39\ m$

$$5.70\ N = |\vec{F}_{12}| = k\frac{|q_1||q_2|}{d^2}$$

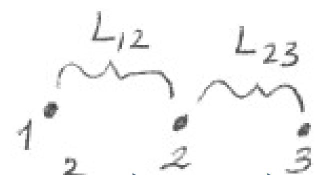
2) In Figure, three charged particles lie on an x axis. Particles 1 and 2 are fixed in place. Particle 3 is free to move, but the net electrostatic force on *it* from particles 1 and 2 happens to be zero. If $L_{23} = L_{12}$, what is the ratio q_1/q_2?



equal distances

3 charged particles
1 & 2 are fixed
3 is free to move
$F_{3,net} = 0$
$L_{23} = L_{12}$

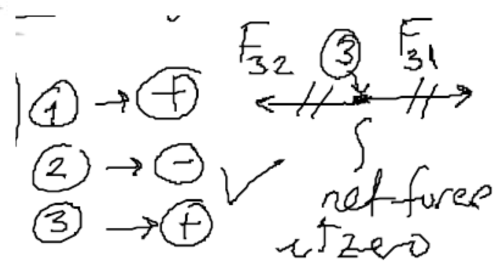
$$\frac{q_1}{q_2} = ?$$



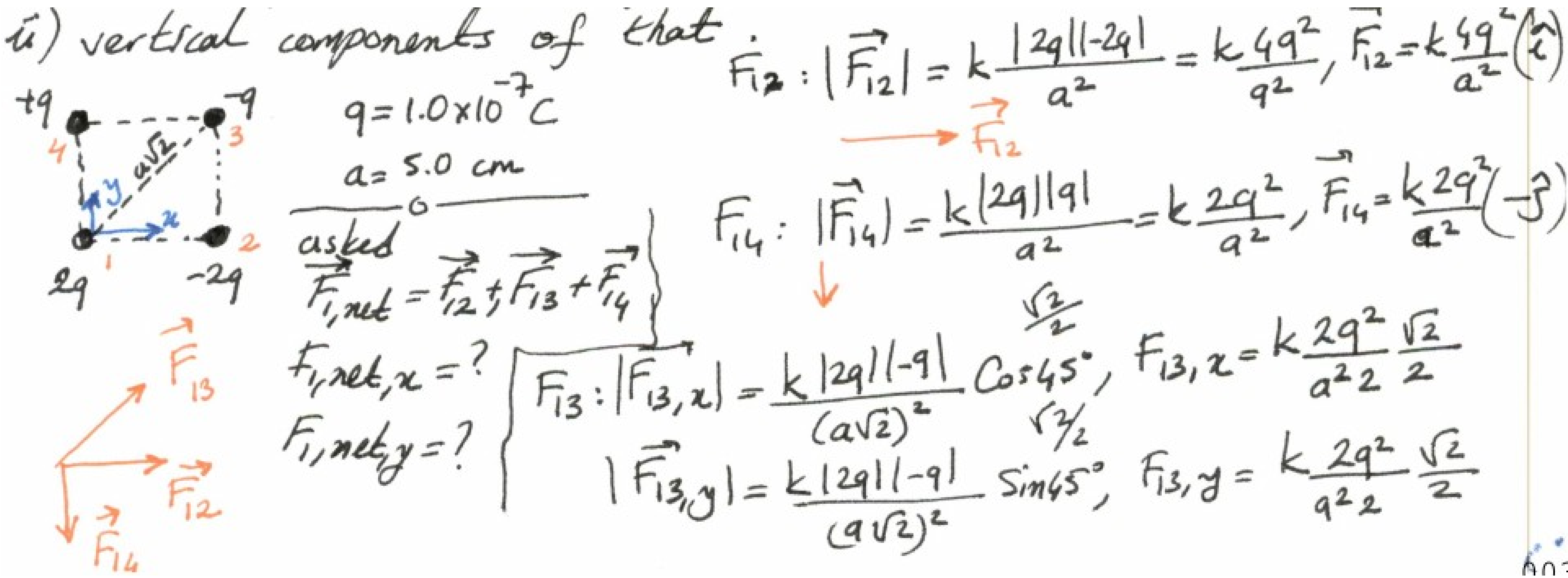
$$\vec{F}_{3,net} = \sum_{i=1}^{2} \vec{F}_{3i} = \vec{F}_{31} + \vec{F}_{32} = k\frac{|q_3||q_1|}{d_{31}^2} + k\frac{|q_3||q_2|}{d_{32}^2} = 0$$

$$\rightarrow \frac{q_1}{(2L)^2} + \frac{q_2}{L^2} = 0 \rightarrow \frac{q_1}{q_2} = -4$$

why minus sign?



3) In Figure shown, what are i) horizontal components of the net electrostatic force on the charged particle in the lower left corner of the system ii) vertical component of that.

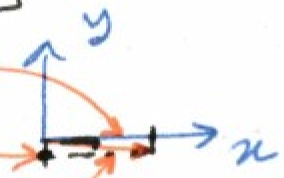


i) $F_{1,net,x} = F_{12,x} + F_{13,x} + F_{14,x} = k\frac{q^2}{a^2}\left(4+\frac{\sqrt{2}}{2}\right) = 8.99\times10^{9}\frac{(1.0\times10^{-7})^2}{0.050^2}\left(4+\frac{\sqrt{2}}{2}\right) = \boxed{0.17\,N}$

ii) $F_{1,net,y} = F_{12,y} + F_{13,y} + F_{14,y} = k\frac{q^2}{a^2}\left(-2+\frac{\sqrt{2}}{2}\right) = \boxed{-0.046\,N}$

$$\vec{F}_{1,net} = k\frac{q^2}{a^2}\left[\left(4+\frac{\sqrt{2}}{2}\right)\hat{i} + \left(-2+\frac{\sqrt{2}}{2}\right)\hat{j}\right]$$

$$\vec{F}_{1,net} = 0.17\,N\,\hat{i} + 0.046\,(-\hat{j})$$



$$\tan\theta = \frac{-0.046}{0.17}$$

4) Two identical conducting spheres, fixed in place, attract each other with an electrostatic force of 0.108 N when their center-to-center separation is 50.0 cm. The spheres are then connected by a thin conducting wire. When the wire is removed, the spheres repel each other with an electrostatic force of 0.0360 N. Of the initial charges on the spheres, with a positive net charge, what

a. the negative charge on one of them?
b. the positive charge on the other?

attract

$$F = -k\frac{|q_1||q_2|}{(0.5\,m)^2} = 0.108\,N$$

repel

$$F = +k\frac{\left|\frac{q_1+q_2}{2}\right|\left|\frac{q_1+q_2}{2}\right|}{(0.5\,m)^2} = 0.036\,N$$

center-to-center ⇒ shell theorem I

charge conservation

charge sharing

$$Q_i = Q_f = q_1 + q_2 \Rightarrow (+)$$

d = ?

3 (9)

Two identical conducting sphere

- First attract each other

(1) $F = -0.108\,N$

$d = 50.0\,cm$

- Then connected by a conducting wire charge transfer happens.

⇒ Balance

- Wire is removed. Repulsion

(2) $F = +0.0360\,N$

Assumption: Positive net charge

$q_1 = ?$ $q_2 = ?$

Choose as q_2 is positive.

(1) $F = -k\dfrac{|q_1||q_2|}{d^2} \rightarrow \dfrac{-0.108N\,(0.5m)^2}{(8.99\times10^9 Nm^2/C^2)} = q_1 q_2$

$q_1 q_2 = -3.00\times10^{-12}\,C^2$

(2) charge balance. Initial total charge is shared

$F = +k\dfrac{\left(\frac{q_1+q_2}{2}\right)\left(\frac{q_1+q_2}{2}\right)}{d^2}$

$\dfrac{(0.036N)\,4\,(0.5m)^2}{8.99\times10^9 Nm^2/C^2} = (q_1+q_2)^2$

$q_1+q_2 = 2.00\times10^{-6}\,C$

Two unknowns; q_1 & q_2
Two eqns

$q_2 = -\dfrac{3\times10^{-12}C^2}{q_1}$ { $q_1 - \dfrac{3\times10^{-12}}{q_1}C^2 = 2\times10^{-6}C$

$q_1^2 - 2\times10^{-6}C\,q_1 - 3\times10^{-12} = 0$ | $q_1 = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

(+) sign : $q_1 = 3.00\times10^{-6}C$ & $q_2 = -1.00\times10^{-6}C$

(−) sign : $q_1 = -1.00\times10^{-6}C$ & $q_2 = 3.00\times10^{-6}C$

such that $q_1+q_2 > 0$

Initial assumption

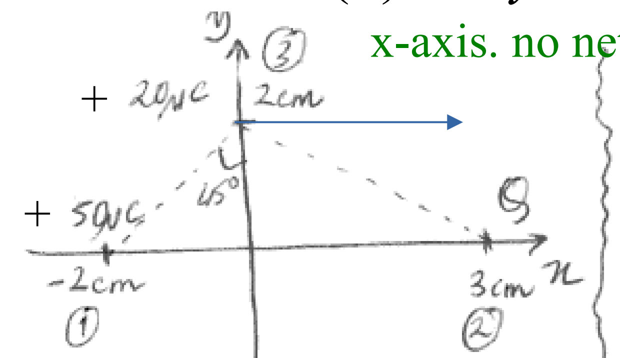
Quadratic equation:

$$ax^2 + bx + c = 0$$

$$x_{12} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

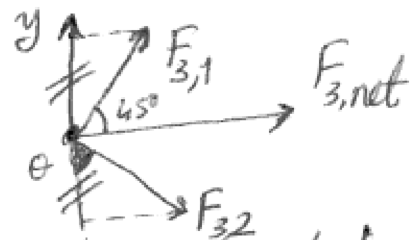
5) Two particles are fixed on an *x* axis. Particle 1 of charge 50 μC is located at <u>*x* = -2.0 cm</u>; particle 2 of charge *Q* is located at *x* = 3.0 cm. Particle 3 of charge magnitude 20 μC is released from rest on the *y* axis at *y* = 2.0 cm. What is the value of *Q* if the initial acceleration of particle 3 is in the positive direction of (a) the *x* axis and (b) the *y* axis (solution is not given)?

x-axis. no net force on the y-axis



① & ② are fixed
③ is released from rest

Q = ? if initial accelleration of ③ is in +x



$F_{31,y} = F_{32,y}$

$|F_{31}| = k \frac{|20\times10^{-6}C||50\times10^{-6}C|}{(-2\times10^{-2}m)^2 + (2\times10^{-2}m)^2}$

Q should be negative!

$|F_{32}| = k \frac{|20\times10^{-6}C||Q|}{(2\times10^{-2}m)^2 + (3\times10^{-2}m)^2}$

$|F_{31}| \cos 45 = |F_{32}| \cos\theta$, where $\theta = \tan^{-1}\frac{3\times10^{-2}m}{2\times10^{-2}m} = 56.31°$

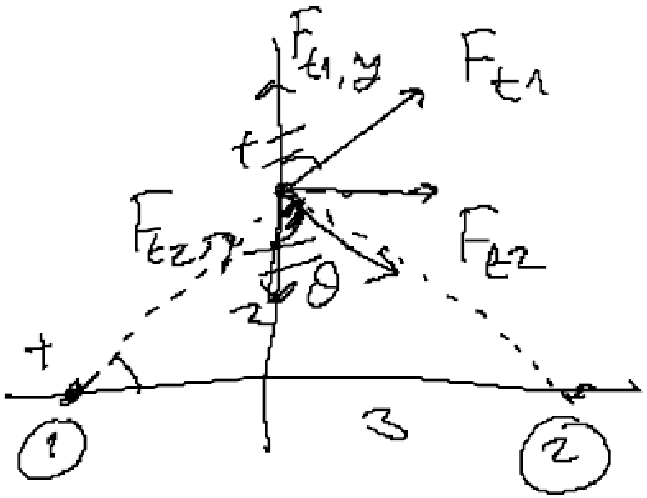
$\rightarrow k \frac{|20\times10^{-6}C||50\times10^{-6}C|}{8\times10^{-4}m^2} \cos 45 = k \frac{|20\times10^{-6}C||Q|}{13\times10^{-4}m^2} \cos 56.31°$

$\rightarrow |Q| = 104\ \mu C$

$Q = -104\ \mu C$

towards

21 Solved Problems



t : target

$$\vec{F}_{t,x,net} = \vec{F}_{t1,x} + \vec{F}_{t2,x}$$

$$|\vec{F}_{t1,y}| = |\vec{F}_{t2,y}| \quad \sim\theta$$

$$k\frac{|q_t||q_1|}{d_{1t}^2}\cos 45 = k\frac{|q_t||q_2|}{d_{2t}^2}\cos\theta$$

$$\tan\theta = \frac{3}{2}$$

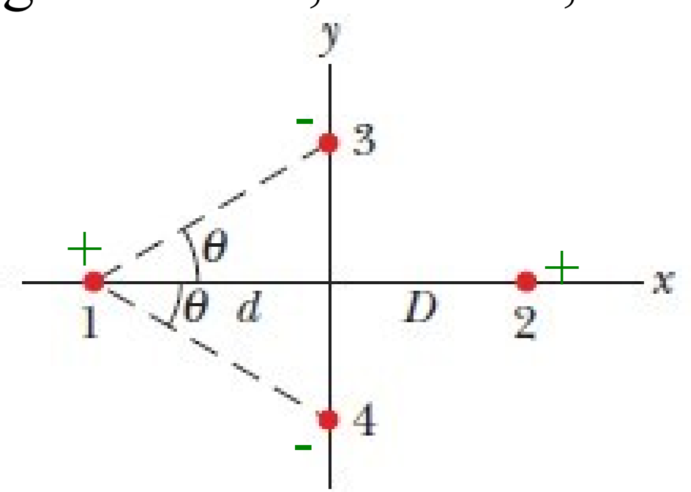
$$\theta = \arctan\frac{3}{2}$$

6) Figure shows an arrangement of four charged particles, with angle θ=35.0° and distance d=2.00 cm. Particle 2 has charge q_2= +8.00x10^{-19} C; particles 3 and 4 have charges $q_3 = q_4$ = -1.60x10^{-19} C.

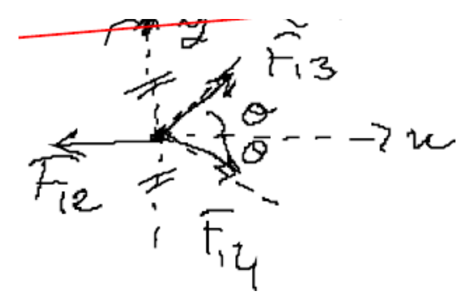
a. What is distance D between the origin and particle 2 <u>if the net electrostatic force on particle 1</u> due to the other particles is zero? target

b. If particles 3 and 4 were moved closer to the x axis but maintained their symmetry about that axis, would the required value of D be greater than, less than, or the same as in part (a)?

$$\vec{F}_{1,net} = 0 = \sum_{i=1}^{3} \vec{F}_{1i}$$



What about the sign of particle 1? Let say it is (+)



2d → x&y-components

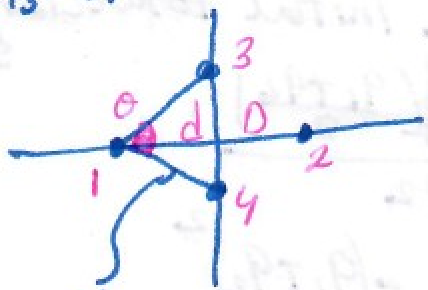
7(22) Four charged particles

$\theta = 35.0^\circ$

$d = 2.00\,cm$

$q_2 = +8.00\times10^{-19}\,C$

$q_3 = q_4 = -1.60\times10^{-19}\,C$



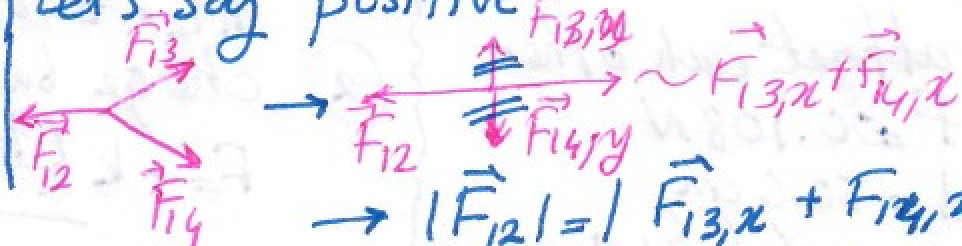
$\frac{d}{\cos\theta}$ $\quad -1 \le \cos\theta \le +1$

i) $D = ?$ if $|F_{1,net}| = 0$

$\vec{F}_{1,net} = \vec{F}_{1,2} + \vec{F}_{13} + \vec{F}_{14}$ Superposition principle

what about the sign of the charge q_1?

Lets say "positive"



$\rightarrow |\vec{F}_{12}| = |\vec{F}_{13,x} + \vec{F}_{14,x}|$

$$k\frac{|q_1||q_2|}{(d+D)^2} = k\frac{|q_1||q_3|}{\frac{d^2}{\cos^2\theta}} + \frac{|q_1||q_4|k}{\frac{d^2}{\cos^2\theta}}$$

$|q_3| = |q_4|$

$$\Rightarrow \frac{|q_2|}{(0.02m+D)^2} = \frac{2|q_3|}{d^2}\cos^2\theta$$

$$\frac{8.00\times10^{-19}C}{(0.02m+D)^2} = \frac{2\times1.6\times10^{-19}C}{(0.02m)^2}\cos^2 35$$

$$D = -0.02m + \sqrt{\frac{(0.02m)^2}{2\times1.6\times10^{-19}C}\,\frac{8.00\times10^{-19}C}{\cos^2 35}}$$

$= 0.0186\,m = 1.86\,cm$!!

ii) 3 & 4 moved closer $\Rightarrow \theta \downarrow$

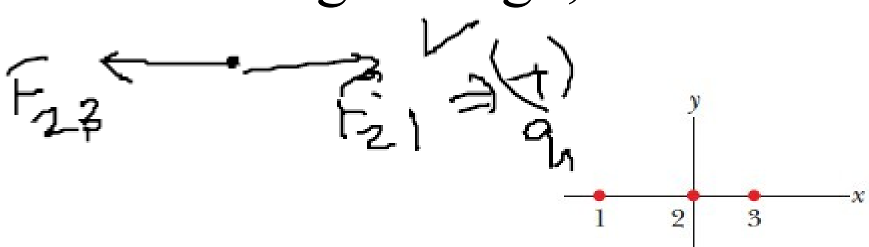
$\Rightarrow \cos\theta$ increases $\quad \cos 0 = 1$ as max

$$\Rightarrow D = \sqrt{\frac{x}{x\,\cos^2\theta}} - 0.02m \downarrow$$

D decreases

$\theta \downarrow \cdots \cos\theta \uparrow \cdots \frac{1}{\cos\theta} \downarrow$

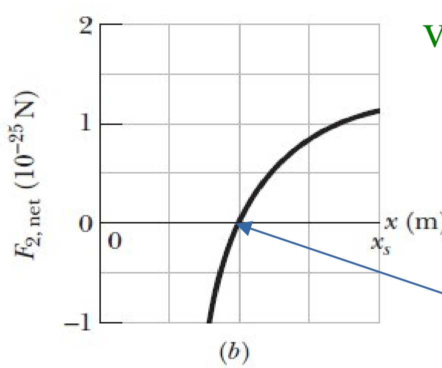
7) Figure (a) shows charged particles 1 and 2 that are fixed in place on an x axis. Particle 1 has a charge with a magnitude of $|q_1| = 8.00e$. Particle 3 of charge $q_3 = +8.00e$ is initially on the x axis near particle 2. Then particle 3 is gradually moved in the positive direction of the x axis. As a result, the magnitude of the net electrostatic force $\boldsymbol{F}_{2,net}$ on particle 2 due to particles 1 and 3 changes. Figure *(b)* gives the x component of that net force as a function of the position x of particle 3. The scale of the x axis is set by x_s=0.80 m. The plot has an asymptote of $F_{2,net}=1.5\times10^{-25}$ N as $x\to\infty$. As a multiple of e and including the sign, what is the charge q_2 of particle 2?



$$F_{2,net} = F_{21} \ at \ x \to \infty$$

$$1.5 \times 10^{-25} \ N = k\frac{|q_2||q_1|}{r_{12}^2}$$

(a)



(b)

variation of x_3

x=0.40m

F(x=0.40)=0

Fig. 21-33

11 (32) q_1 & q_2: fixed in place

$|q_1| = 8.00e$

$q_3 = 8.00e$

particle 3 gradualy moved in $+x$-axis

$F_{2,net}(x_3)$ is given

$x \to \infty$ $F_{2,net} = 1.5 \times 10^{-25}$ N

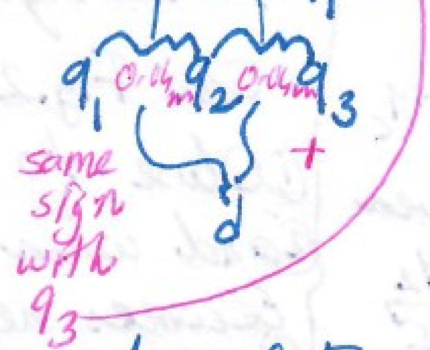
$q_2 = ?$

$k = 8.99 \times 10^9 \, Nm^2/C^2$

$e = 1.602 \times 10^{-19}$ C

at $x_3 = 0.04$ m $F(x_3) = 0$.

This shows that q_1 has positive charge.



Asymptotic value of $F_{2,net}$ as $x_3 \to \infty$.

Take this as a maximum force btw q_1 & q_2

$$1.5 \times 10^{-25} N = k \frac{|q_1||q_2|}{d^2} \to q_2 = \frac{1.5 \times 10^{-25} N \, d^2}{k \, q_1}$$

$$q_2 = \frac{1.5 \times 10^{-25} N \, (0.04 m)^2}{8.99 \times 10^9 \, Nm^2/C^2 \, (8 \times 1.602 \times 10^{-19} C)} = 2.08 \times 10^{-18} C$$

$$\Rightarrow \frac{2.08 \times 10^{-18} C}{1.602 \times 10^{-19} C} = 13e$$

Electric Charge

- The strength of a particle's electrical interaction with objects around it depends on its electric charge, which can be either positive or negative.

Conductors and Insulators

- Conductors are materials in which a significant number of electrons are free to move. The charged particles in nonconductors (insulators) are not free to move.

Conservation of Charge

- The net electric charge of any isolated system is always conserved.

Coulomb's Law

- The magnitude of the electrical force between two charged particles is proportional to the product of their charges and inversely proportional to the square of their separation distance.
- .

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1|\,|q_2|}{r^2}$$ **Eq. 21-4**

The Elementary Charge

- Electric charge is quantized (restricted to certain values).
- *e* is the elementary charge

$$e = 1.602 \times 10^{-19}\ \mathrm{C}.$$ **Eq. 21-12**

Additional Materials

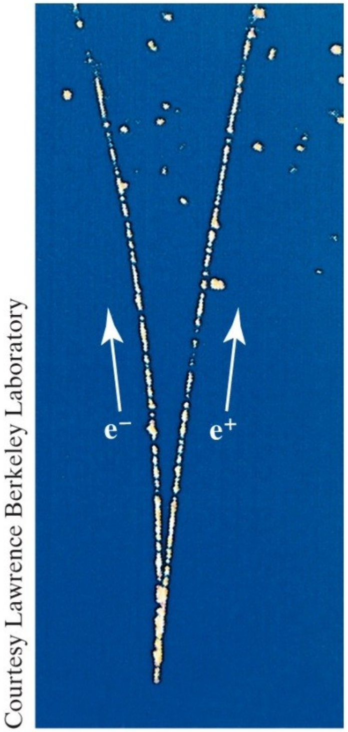
21-6 Charge is Conserved

- The net electric charge of any isolated system is always conserved.
- If two charged particles undergo an annihilation process, they have equal and opposite signs of charge.

$$e^- + e^+ \rightarrow \gamma + \gamma$$

- If two charged particles appear as a result of a pair production process, they have equal and opposite signs of charge.

$$\gamma \rightarrow e^- + e^+$$



- A photograph of trails of bubbles left in a bubble chamber by an electron and a positron. The pair of particles was produced by a gamma ray that entered the chamber directly from the bottom. Being electrically neutral, the gamma ray did not generate a telltale trail of bubbles along its path, as the electron and positron did.
- ***Radioactive decay of nuclei***, in which a nucleus transforms into (becomes) a different type of nucleus.
- A uranium-238 nucleus (^{238}U) transforms into a thorium- 234 nucleus (^{234}Th) by emitting an *alpha particle.* An alpha particle has the same makeup as a helium-4 nucleus, it has the symbol ^{4}He. Here the net charge is 238.

$$^{238}\text{U} \rightarrow {}^{234}\text{Th} + {}^{4}\text{He},$$