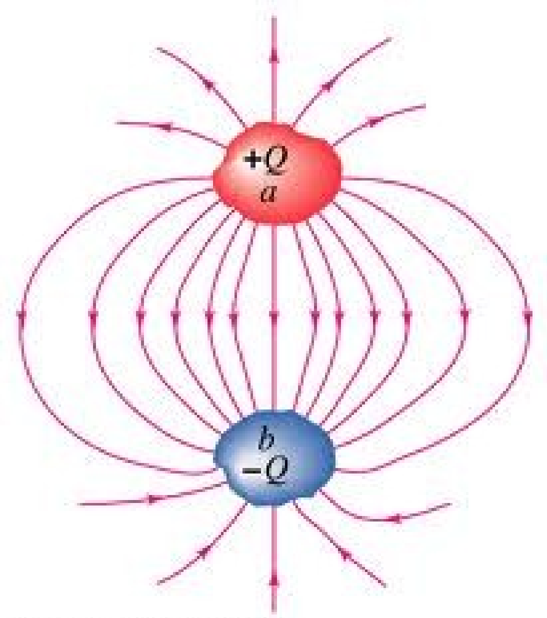
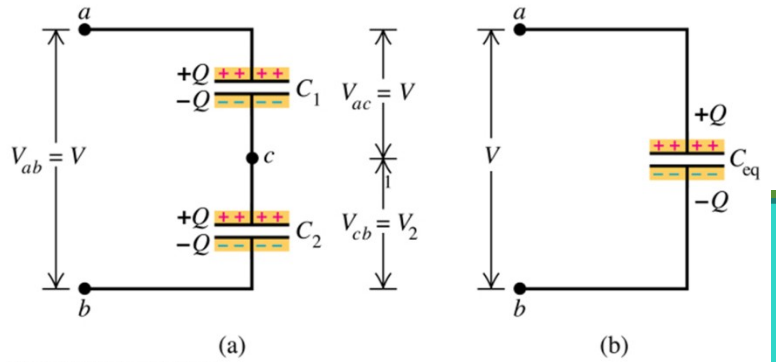
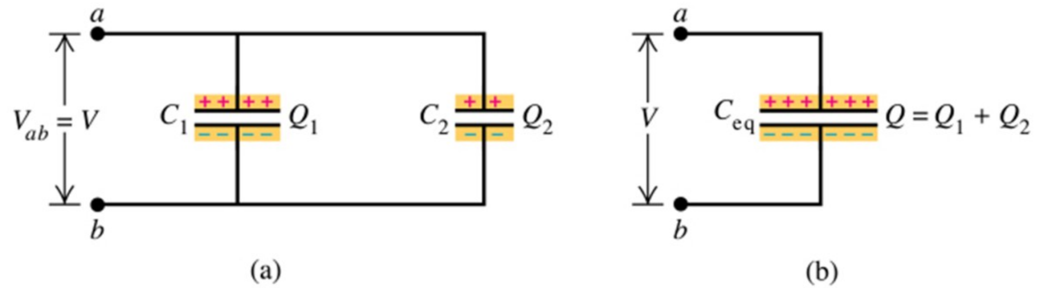
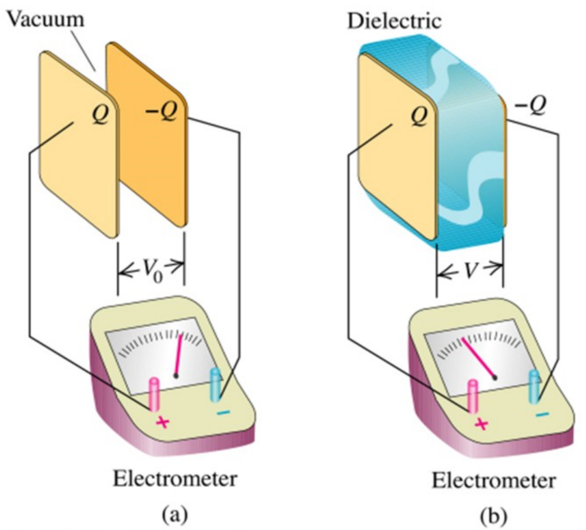
Chapter 25

Capacitance









25 CAPACITANCE 656

25-2 Capacitance

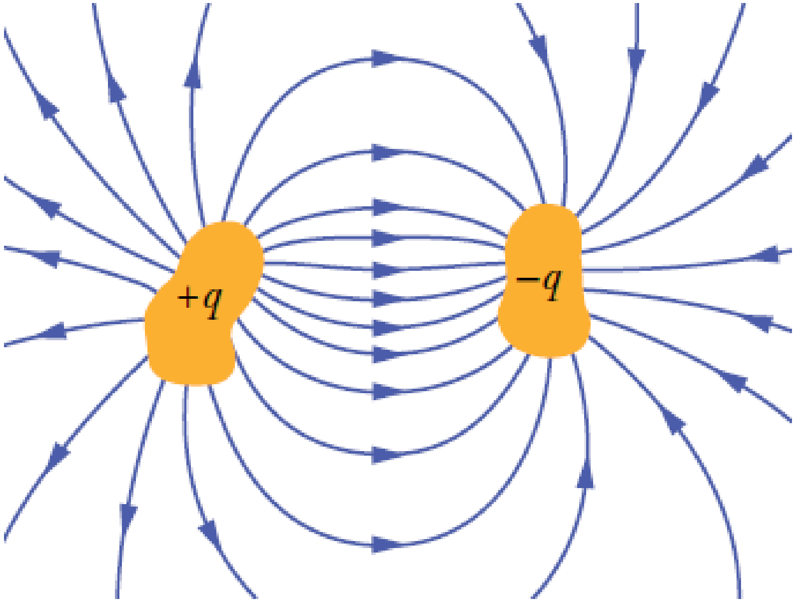
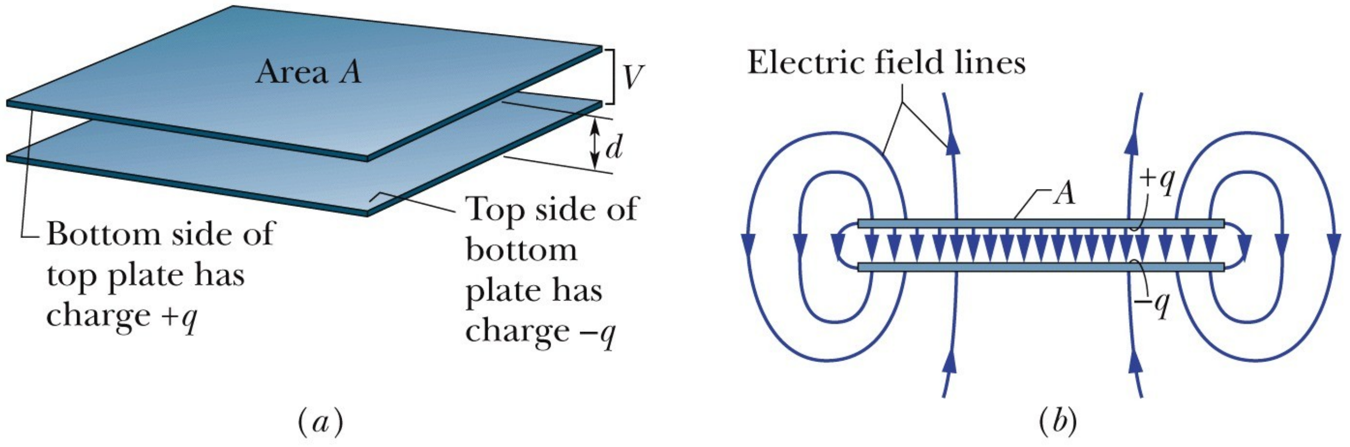


Fig. 25-2 Two conductors, isolated electrically from each other and from their surroundings, form a *capacitor.* When the capacitor is charged, the charges on the conductors, or *plates* as they are called, have the same magnitude q but opposite signs. *(Paul Silvermann/Fundamental Photographs)*

25-2 Capacitance



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A parallel-plate capacitor, made up of two plates of area A separated by a distance *d*. The charges on the facing plate surfaces have the same magnitude *q* but opposite signs

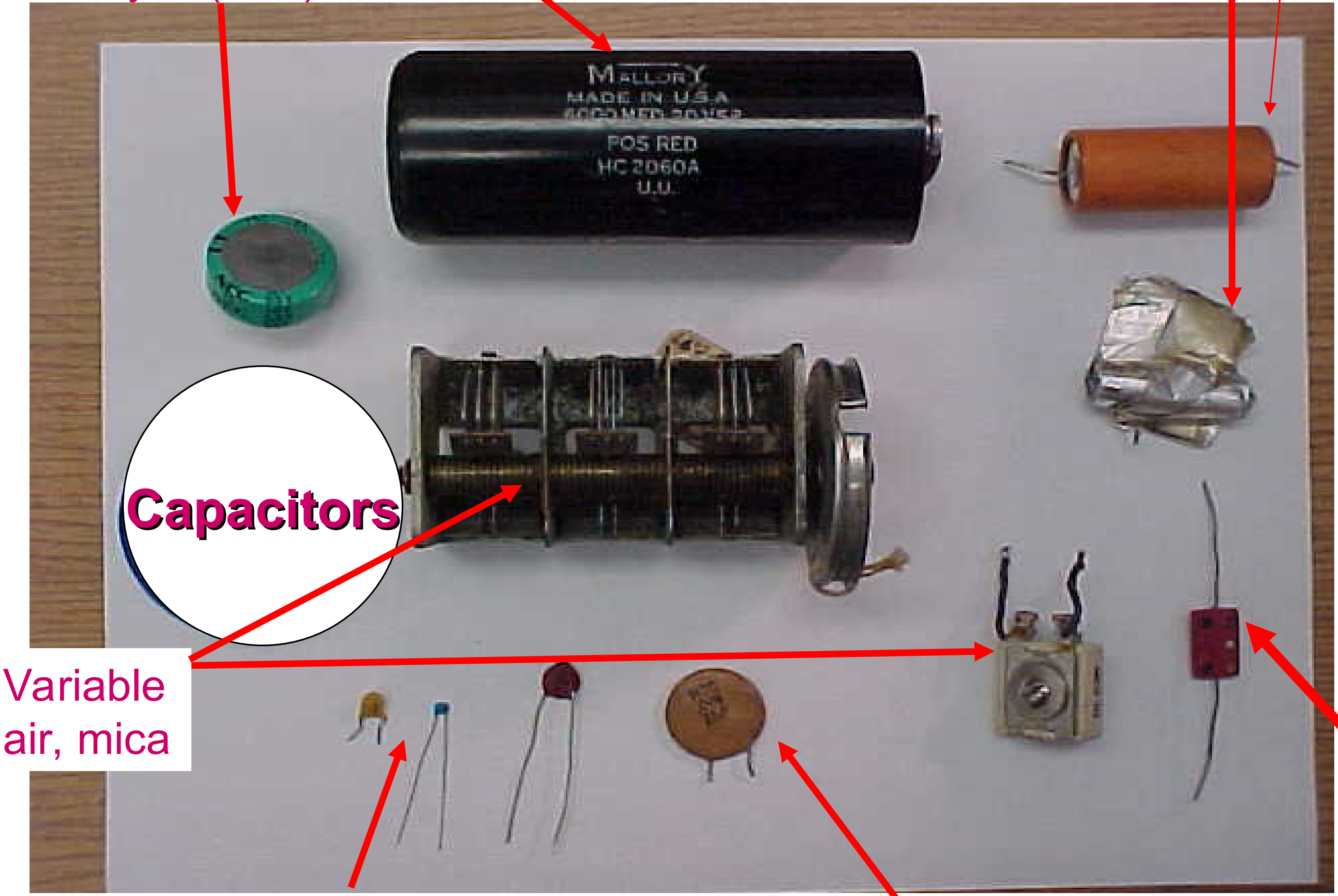
As the field lines show, the *electric field* due to the charged plates is uniform in the central region between the plates. The field is not uniform at the edges of the plates, as indicated by the "*fringing*" of the field lines there.

- The charge q and the potential difference V for a capacitor are proportional to each other: $q = CV.$
- The proportionality constant C is called the capacitance of the capacitor.
  - Its value ***depends ONLY on the geometry of the plates*** A,d
  - not on their charge or potential difference. QXV
  - The SI unit is called the farad (F): 1 farad (1 F)= 1 coulomb per volt =1 C/V.
- When a capacitor is **charged**, its plates have charges of equal magnitudes but opposite signs: q+ and q-.
  - However, we refer to the charge of a capacitor as being q, the absolute value of these charges on the plates.

25-2 Capacitance

Electrolytic (1940-70)
Electrolytic (new)

Paper (1940-70)



Variable
air, mica

Tantalum (1980 on)

Ceramic (1930 on)

Mica (1930-50)

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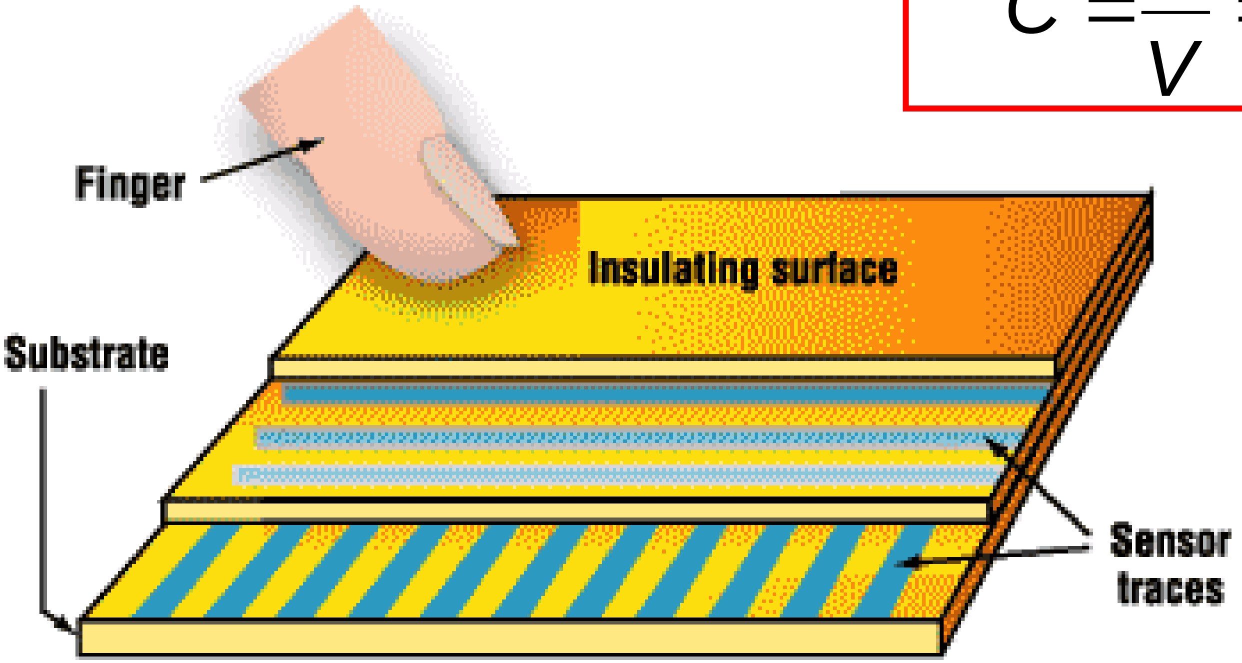
25-2 Capacitance

Capacitance and Your iPhone!

$$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$$



1. A capacitive sensor is a solid-state sensor made using standard pc-board or flex circuit technology. A finger on top of a grid of conductive traces changes the capacitance of the nearest traces. This change in trace capacitance can be measured, and finger position can be computed.

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Parallel Plate Capacitor — Example

- A huge parallel plate capacitor consists of two square metal plates of side 50 cm, separated by an air gap of 1 mm. What is the capacitance?

$\boxed{C = \varepsilon_0 A/d}$: only geometrical factors : A & d

$= (8.85 \times 10^{-12}\ \mathrm{F/m})(0.25\ \mathrm{m}^2)/(0.001\ \mathrm{m})$

$= 2.21 \times 10^{-9}\ \mathrm{F}$

(Very Small!!)

Lesson: difficult to get large values of capacitance without special tricks!

~ increase surface A
~ decrease distance d
~ insert a dielectric material in btw plates

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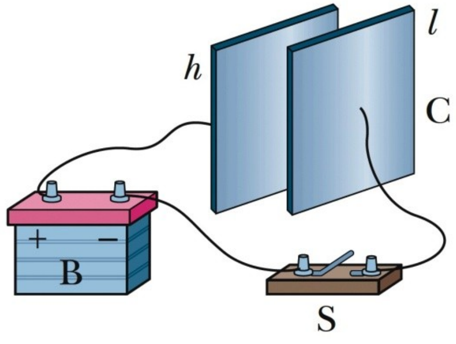
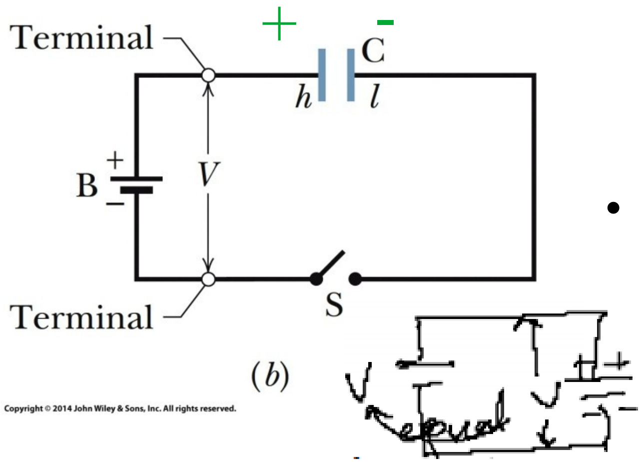
25-2 Charging a Capacitor

- When a circuit with a battery, an open switch, and an uncharged capacitor is completed by closing the switch, conduction electrons shift, leaving the capacitor plates with opposite charges.



(a)



(b)

- In Fig. a, a battery B, a switch S, an uncharged capacitor C, and interconnecting wires form a circuit.
- The same circuit is shown in the schematic diagram of Fig. b, in which the symbols for a battery, a switch, and a capacitor represent those devices.
- The battery maintains potential difference V between its terminals.
- The terminal of higher potential is labeled + and is often called the positive terminal; the terminal of lower potential is labeled - and is often called the negative terminal.

- When the switch is closed, electrically connecting those wires, *the circuit is complete and charge can then flow through the switch and the wires.*
- As the plates become oppositely charged, that potential difference increases **until it equals the potential difference V** between the terminals of the battery.
- The capacitor is then said to be **fully charged**, with a potential difference V and charge q.

$C=Q/V$

25-3 Calculating the Capacitance

- To relate the electric field E between the plates of a capacitor to the charge q on either plate, we use Gauss' law:

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q.$$

  - Here q is the charge enclosed by a Gaussian surface
  - $\oint \vec{E} \cdot d\vec{A}$ is the net electric flux through that surface. In our special case in the figure,

$$q = \varepsilon_0 EA$$

  - in which A is the area of that part of the Gaussian surface through which there is a flux.
- the potential difference between the plates of a capacitor is related to the field E by

$$V_f - V_i = -\int_i^f \underbrace{\vec{E} \cdot d\vec{s}}_{\text{antiparallel}} \qquad V = \int_-^+ E\, ds = E \int_0^d ds = Ed.$$

- If V is the difference $V_f - V_i$ → $C = q/V = \varepsilon_0 EA/Ed$

We use Gauss' law to relate *q* and *E*. Then we integrate the *E* to get the potential difference.

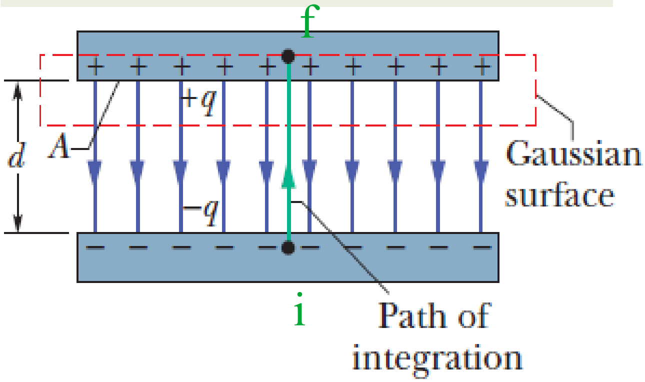


Fig. 25-5 A charged parallel-plate capacitor. A Gaussian surface encloses the charge on the positive plate. The integration of Eq. 25-6 is taken along a path extending directly from the negative plate to the positive plate.

only geometrical factors

$$\boxed{C = \frac{\varepsilon_0 A}{d} \quad \text{(parallel-plate capacitor).}}$$

$$\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}.$$

25-3 Calculating the Capacitance

A Cylindrical Capacitor :

- As a Gaussian surface, we choose a cylinder of length L and radius r, closed by end caps and placed as is shown.
- It is coaxial with the cylinders and encloses the central cylinder and thus also the charge q on that cylinder.

$$q = \varepsilon_0 E A = \varepsilon_0 E(\overbrace{2\pi r L}^{\text{surface area}}),$$

➡ $$E = \frac{q}{2\pi\varepsilon_0 L r}.$$

➡ $$V = \int_{-}^{+} \underbrace{E\,ds}_{\text{antiparallel}} = -\frac{q}{2\pi\varepsilon_0 L}\int_b^a \frac{dr}{r} = \frac{q}{2\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right),$$

From the relation *C= q/V*, we then have

➡ $$\boxed{C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)} \qquad \text{(cylindrical capacitor).}}$$

only geometrical factors

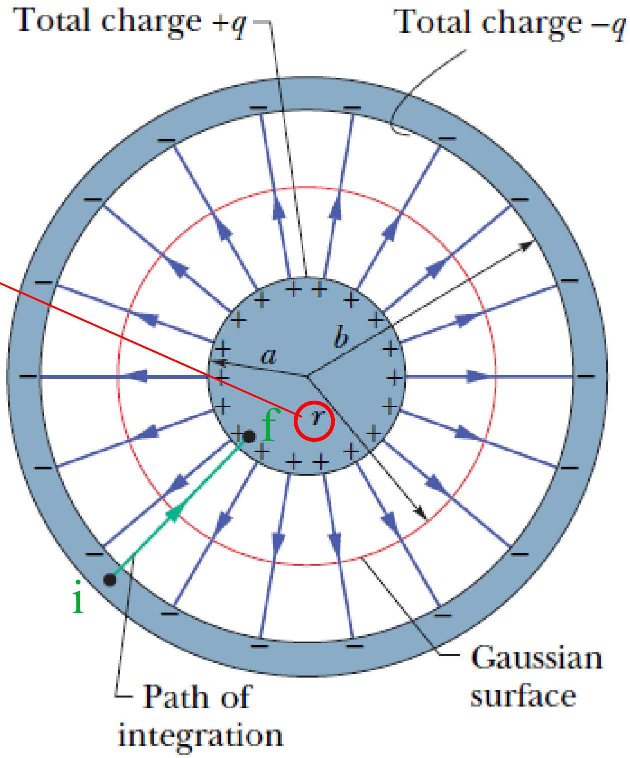


Fig. 25-6 A cross section of a long cylindrical capacitor, showing a cylindrical Gaussian surface of radius *r* (that encloses the positive plate) and the radial path of integration along which Eq. 25-6 is to be applied. This figure also serves to illustrate a spherical capacitor in a cross section through its center.

25-3 Calculating the Capacitance

A Spherical Capacitor:

- As a Gaussian surface, we choose a sphere of radius r and placed as is shown.
- It is cocentric with the spheres and encloses the central sphere and thus also the charge q on that sphere.

$$q = \varepsilon_0 EA = \varepsilon_0 E(\overbrace{4\pi r^2}^{\text{surface area}}),$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2},$$

$$V = \int_-^+ \underbrace{E\,ds}_{\text{antiparallel}} = -\frac{q}{4\pi\varepsilon_0}\int_b^a \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right) = \frac{q}{4\pi\varepsilon_0}\frac{b-a}{ab},$$

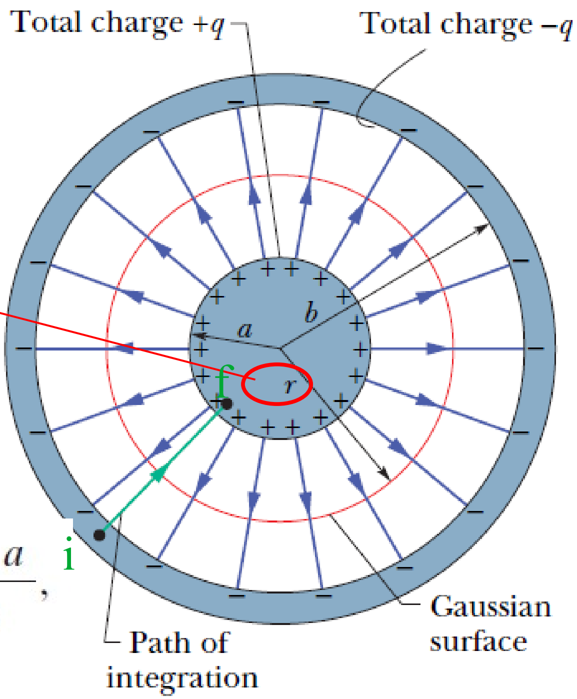


Fig. 25-6 A cross section of a long cylindrical capacitor, showing a cylindrical Gaussian surface of radius *r* (that encloses the positive plate) and the radial path of integration along which Eq. 25-6 is to be applied. This figure also serves to illustrate a spherical capacitor in a cross section through its center.

From the relation C= q/V, we then have

$$\boxed{C = 4\pi\varepsilon_0 \frac{ab}{b-a} \qquad \text{(spherical capacitor).}}$$

only geometrical factors

25-3 Calculating the Capacitance

An Isolated Sphere:

- We can assign a capacitance to a *single isolated spherical conductor* of radius R by assuming that the "missing plate" is a conducting sphere of infinite radius.
- The field lines that leave the surface of a positively charged isolated conductor must end somewhere;
  - the walls of the room in which the conductor is housed can serve effectively as our sphere of infinite radius.
- To find the capacitance of the conductor, we first rewrite the capacitance as:

$$C = 4\pi\varepsilon_0 \frac{a}{1 - a/b}.$$

- Now letting b→∞, and substituting R for a,

$$\boxed{C = 4\pi\varepsilon_0 R \qquad \text{(isolated sphere).}}$$

25-3 Calculating the Capacitance

Charging the Plates in a Parallel-Plate Capacitor:

In Fig. 25-7*a*, switch S is closed to connect the uncharged capacitor of capacitance $C = 0.25\ \mu\text{F}$ to the battery of potential difference $V = 12$ V. The lower capacitor plate has thickness $L = 0.50$ cm and face area $A = 2.0 \times 10^{-4}\ \text{m}^2$, and it consists of copper, in which the density of conduction electrons is $n = 8.49 \times 10^{28}$ electrons/m³. From what depth d within the plate (Fig. 25-7*b*) must electrons move to the plate face as the capacitor becomes charged?

n=# of charge carriers/volume

KEY IDEA

The charge collected on the plate is related to the capacitance and the potential difference across the capacitor by Eq. 25-1 ($q = CV$).

Calculations: Because the lower plate is connected to the negative terminal of the battery, conduction electrons move up to the face of the plate. From Eq. 25-1, the total charge magnitude that collects there is

$$q = CV = (0.25 \times 10^{-6}\ \text{F})(12\ \text{V}) = 3.0 \times 10^{-6}\ \text{C}.$$

Dividing this result by e gives us the number N of conduction electrons that come up to the face:

$$N = \frac{q}{e} = \frac{3.0 \times 10^{-6}\ \text{C}}{1.602 \times 10^{-19}\ \text{C}} = 1.873 \times 10^{13}\ \text{electrons}.$$

These electrons come from a volume that is the product of the face area A and the depth d we seek. Thus, from the density of conduction electrons (number per volume), we can write

$$n = \frac{N}{Ad},$$

or

$$d = \frac{N}{An} = \frac{1.873 \times 10^{13}\ \text{electrons}}{(2.0 \times 10^{-4}\ \text{m}^2)(8.49 \times 10^{28}\ \text{electrons/m}^3)} = 1.1 \times 10^{-12}\ \text{m} = 1.1\ \text{pm}. \qquad \text{(Answer)}$$

In common speech, we would say that the battery charges the capacitor by supplying the charged particles. But what the battery really does is set up an electric field in the wires and plate such that electrons very close to the plate face move up to the negative face.

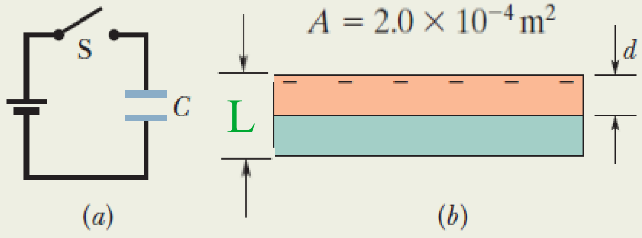


Fig. 25-7 (*a*) A battery and capacitor circuit. (*b*) The lower capacitor plate.

25-4 Capacitors in Parallel and Series

- When a potential difference V is applied across several capacitors connected **in parallel**, that potential difference V is applied across each capacitor.
- The total charge q stored on the capacitors is the sum of the charges stored on all the capacitors.
- Capacitors connected in parallel can be replaced with an equivalent capacitor that has the same total charge q and the same potential difference V as the actual capacitors.

$$q_1 = C_1 V, \quad q_2 = C_2 V, \quad \text{and} \quad q_3 = C_3 V.$$

$$\Downarrow$$

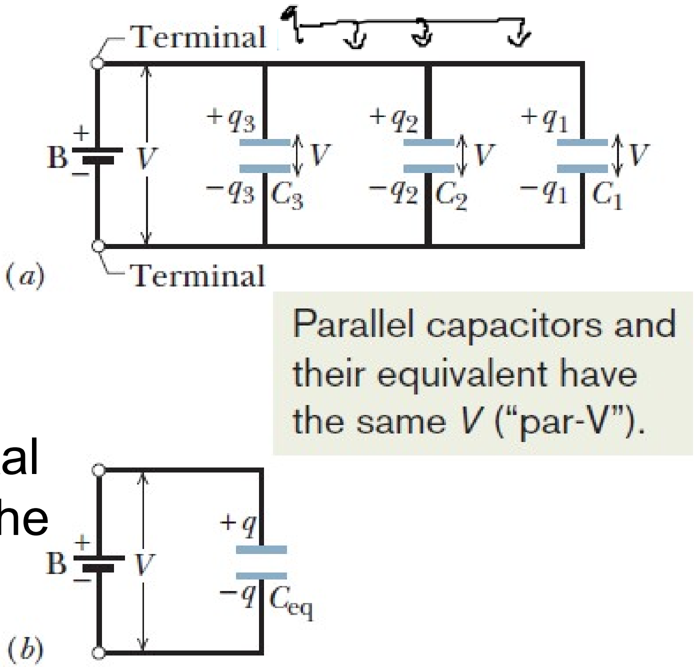
$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V.$$

$$\Downarrow$$

$$C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3,$$

$$\Downarrow$$

$$\boxed{C_{eq} = \sum_{j=1}^{n} C_j \qquad (n \text{ capacitors in parallel}).}$$



Parallel capacitors and their equivalent have the same *V* ("par-V").

Fig. 25-8 (*a*) Three capacitors connected in parallel to battery B. The battery maintains potential difference *V* across its terminals and thus across *each* capacitor. (*b*) The equivalent capacitor, with capacitance C_{eq}, replaces the parallel combination.

PARALLEL:

- *V* is same for all capacitors
- Total charge = sum of *Q*

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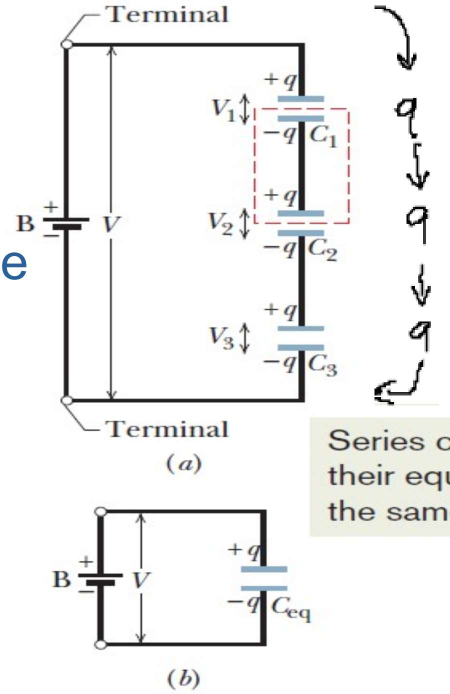
25-4 Capacitors in Parallel and Series

- When a potential difference V is applied across several capacitors connected **in series**, the capacitors have identical charge q.
- The sum of the potential differences across all the capacitors is equal to the applied potential difference V.
- Capacitors that are connected in series can be replaced with an equivalent capacitor that has the same charge q and the same total potential difference V as the actual series capacitors.



$V_1+V_2+V_3$
$= V$

Series capacitors and their equivalent have the same *q* ("seri-q").

Fig. 25-9 (*a*) Three capacitors connected in series to battery B. The battery maintains potential difference *V* between the top and bottom plates of the series combination. (*b*) The equivalent capacitor, with capacitance C_{eq}, replaces the series combination.

$$V_1 = \frac{q}{C_1}, \quad V_2 = \frac{q}{C_2}, \quad \text{and} \quad V_3 = \frac{q}{C_3}.$$

$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right).$$

$$C_{eq} = \frac{q}{V} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3},$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j} \quad (n \text{ capacitors in series}).$$

SERIES:

- *Q* is same for all capacitors
- Total potential difference = sum of *V*

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25-4 Capacitors in Parallel and Series

Example:

(a) Find the equivalent capacitance for the combination of capacitances shown in Fig. 25-10*a*, across which potential difference V is applied. Assume

$$C_1 = 12.0\ \mu\text{F}, \quad C_2 = 5.30\ \mu\text{F}, \quad \text{and} \quad C_3 = 4.50\ \mu\text{F}.$$

⇨

$$C_{12} = C_1 + C_2 = 12.0\ \mu\text{F} + 5.30\ \mu\text{F} = 17.3\ \mu\text{F}.$$

$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3} \qquad C_{123} = \frac{C_{12}C_3}{C_{12}+C_3}$$

$$= \frac{1}{17.3\ \mu\text{F}} + \frac{1}{4.50\ \mu\text{F}} = 0.280\ \mu\text{F}^{-1},$$

$$\boxed{C_{123} = \frac{1}{0.280\ \mu\text{F}^{-1}} = 3.57\ \mu\text{F}.} \qquad \text{(Answer)}$$

We first reduce the circuit to a single capacitor.

The equivalent of parallel capacitors is larger.

The equivalent of series capacitors is smaller.

Next, we work backwards to the desired capacitor.

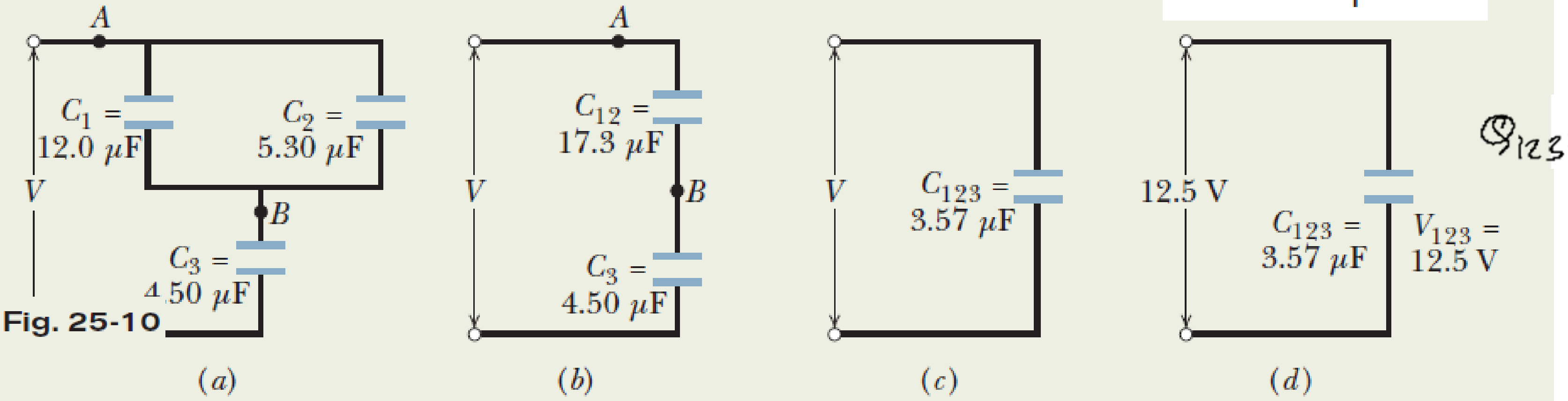


Fig. 25-10

$Q_{123} = ?$

25-4 Capacitors in Parallel and Series

(b) The potential difference applied to the input terminals in Fig. 25-10*a* is $V = 12.5$ V. What is the charge on C_1?

We first reduce the circuit to a single capacitor.

Next, we work backwards to the desired capacitor.

q_3, V_3 & q_1, V_1 & q_2, V_2

Applying $q = CV$ yields the charge.

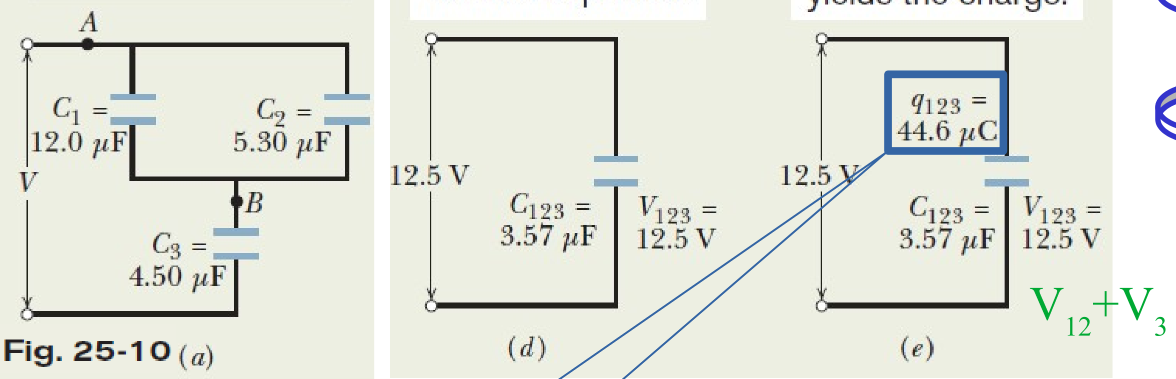


Fig. 25-10 (*a*)

$$q_{123} = C_{123}V = (3.57\ \mu\text{F})(12.5\ \text{V}) = 44.6\ \mu\text{C}.$$

$$q_{12} = q_{123} = 44.6\ \mu\text{C}.$$

$$V_{12} = \frac{q_{12}}{C_{12}} = \frac{44.6\ \mu\text{C}}{17.3\ \mu\text{F}} = 2.58\ \text{V}.$$

$$V_1 = V_{12} = 2.58\ \text{V},$$

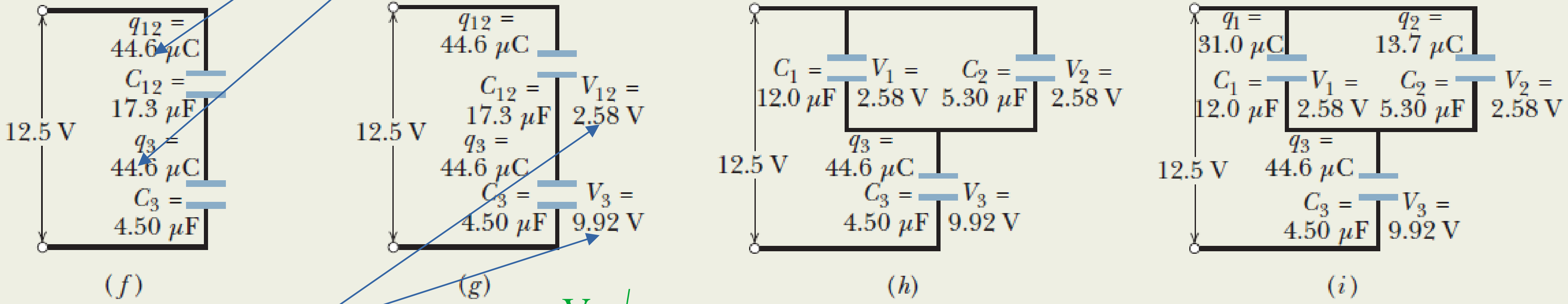
$$q_1 = C_1V_1 = (12.0\ \mu\text{F})(2.58\ \text{V}) = 31.0\ \mu\text{C}.$$

Series capacitors and their equivalent have the same q ("seri-q").

Applying $V = q/C$ yields the potential difference.

Parallel capacitors and their equivalent have the same V ("par-V").

Applying $q = CV$ yields the charge.



reverse operations

q_3, V_3 √

q_1, V_1 √ q_2, V_2 √

25-4 Capacitors in Parallel and Series

One Capacitor Charging up Another Capacitor:

Capacitor 1, with $C_1 = 3.55\ \mu\text{F}$, is charged to a potential difference $V_0 = 6.30$ V, using a 6.30 V battery. The battery is then removed, and the capacitor is connected as in Fig. 25-11 to an uncharged capacitor 2, with $C_2 = 8.95\ \mu\text{F}$. When switch S is closed, charge flows between the capacitors. Find the charge on each capacitor when equilibrium is reached.

$C_1 \rightarrow$ becomes a battery !

After the switch is closed, charge is transferred until the potential differences match.

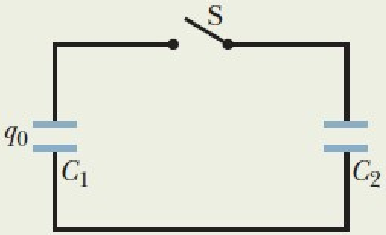


Fig. 25-11 A potential difference V_0 is applied to capacitor 1 and the charging battery is removed. Switch S is then closed so that the charge on capacitor 1 is shared with capacitor 2.

Calculations: Initially, when capacitor 1 is connected to the battery, the charge it acquires is, from Eq. 25-1,

$$q_0 = C_1V_0 = (3.55 \times 10^{-6}\ \text{F})\ (6.30\ \text{V})$$
$$= 22.365 \times 10^{-6}\ \text{C}.$$

initial charge → can flow through C_2 until balance

When switch S in Fig. 25-11 is closed and capacitor 1 begins to charge capacitor 2, the electric potential and charge on capacitor 1 decrease and those on capacitor 2 increase until

$$V_1 = V_2 \qquad \text{(equilibrium)}.$$

parallel connection

From Eq. 25-1, we can rewrite this as

$$\frac{q_1}{C_1} = \frac{q_2}{C_2} \qquad \text{(equilibrium)}.$$

Because the total charge cannot magically change, the total after the transfer must be

$$\underset{\text{final}}{q_1 + q_2} = \overset{\text{initial}}{q_0} \qquad \text{(charge conservation)};$$

thus

$$q_2 = q_0 - q_1.$$

We can now rewrite the second equilibrium equation as

$$\frac{q_1}{C_1} = \frac{q_0 - q_1}{C_2}.$$

Solving this for q_1 and substituting given data, we find

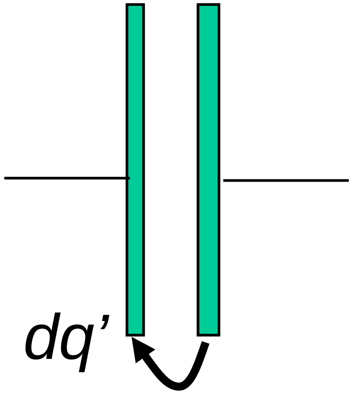
$$q_1 = 6.35\ \mu\text{C}. \qquad \text{(Answer)}$$

The rest of the initial charge ($q_0 = 22.365\ \mu\text{C}$) must be on capacitor 2:

$$q_2 = 16.0\ \mu\text{C}. \qquad \text{(Answer)}$$

25-5 Energy Stored in an Electric Field

The potential energy of a charged capacitor may be viewed as being stored in the electric field between its plates.



- Start out with uncharged capacitor
- **Transfer** small amount of charge dq **from one plate to the other** until charge on each plate has magnitude Q
- *How much work was needed*?
  - Suppose that, at a given instant, a charge dq’ has been transferred from one plate of a capacitor to the other. The increment of work required will be,

$$dW = V'\,dq' = \frac{q'}{C}\,dq'$$

  - The work required to bring the total capacitor charge up to a final value q is

$$W = \int dW = \frac{1}{C}\int_0^q q'\,dq' = \frac{q^2}{2C}.$$

  - **This work is stored as potential energy U in the capacitor**, so that,

$$U = \frac{q^2}{2C} \qquad \text{(potential energy).}$$

$$U = \tfrac{1}{2}CV^2 \qquad \text{(potential energy).}$$

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25-5 Energy Stored in an Electric Field

The potential energy of a charged capacitor may be viewed as being stored in the electric field between its plates.

- In a parallel-plate capacitor, the energy density **u** -that is, the potential energy per unit volume between the plates- should be *uniform*.
- We can find **u** by dividing the total potential energy by the volume **Ad** of the space between the plates

$$u = \frac{U}{Ad} = \frac{CV^2}{2Ad}.$$

- But since($C = \varepsilon_0 A/d$), this result becomes

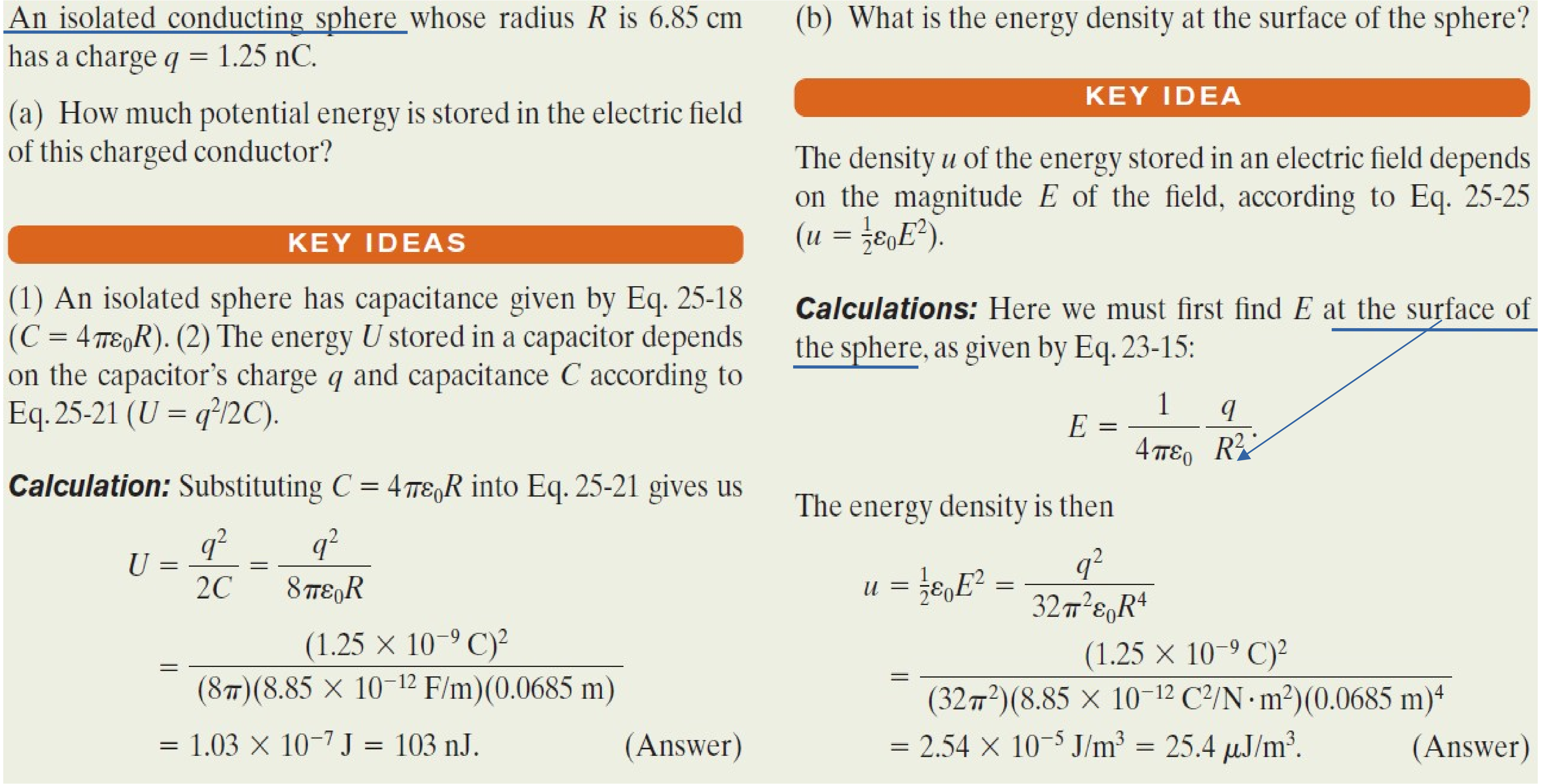
$$u = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2.$$

- However, ($E=-\Delta V/\Delta s$), V/d equals the electric field magnitude E. Therefore;

$$\boxed{u = \frac{1}{2}\varepsilon_0 E^2 \qquad \text{(energy density).}}$$

25-5 Energy Stored in an Electric Field

Potential Energy and Energy Density of an Electric Field:



An isolated conducting sphere whose radius R is 6.85 cm has a charge $q = 1.25$ nC.

(a) How much potential energy is stored in the electric field of this charged conductor?

KEY IDEAS

(1) An isolated sphere has capacitance given by Eq. 25-18 ($C = 4\pi\varepsilon_0 R$). (2) The energy U stored in a capacitor depends on the capacitor's charge q and capacitance C according to Eq. 25-21 ($U = q^2/2C$).

Calculation: Substituting $C = 4\pi\varepsilon_0 R$ into Eq. 25-21 gives us

$$U = \frac{q^2}{2C} = \frac{q^2}{8\pi\varepsilon_0 R}$$

$$= \frac{(1.25 \times 10^{-9}\ \mathrm{C})^2}{(8\pi)(8.85 \times 10^{-12}\ \mathrm{F/m})(0.0685\ \mathrm{m})}$$

$$= 1.03 \times 10^{-7}\ \mathrm{J} = 103\ \mathrm{nJ}. \qquad \text{(Answer)}$$

(b) What is the energy density at the surface of the sphere?

KEY IDEA

The density u of the energy stored in an electric field depends on the magnitude E of the field, according to Eq. 25-25 ($u = \frac{1}{2}\varepsilon_0 E^2$).

Calculations: Here we must first find E at the surface of the sphere, as given by Eq. 23-15:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}.$$

The energy density is then

$$u = \tfrac{1}{2}\varepsilon_0 E^2 = \frac{q^2}{32\pi^2\varepsilon_0 R^4}$$

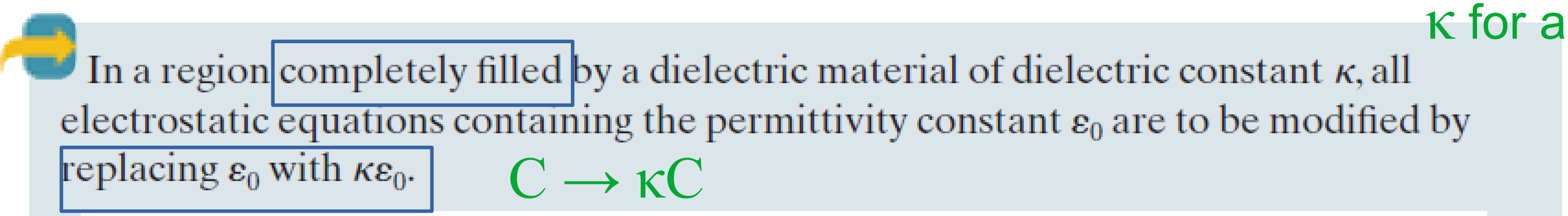
$$= \frac{(1.25 \times 10^{-9}\ \mathrm{C})^2}{(32\pi^2)(8.85 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2})(0.0685\ \mathrm{m})^4}$$

$$= 2.54 \times 10^{-5}\ \mathrm{J/m^3} = 25.4\ \mu\mathrm{J/m^3}. \qquad \text{(Answer)}$$

25-6 Capacitor with a Dielectric

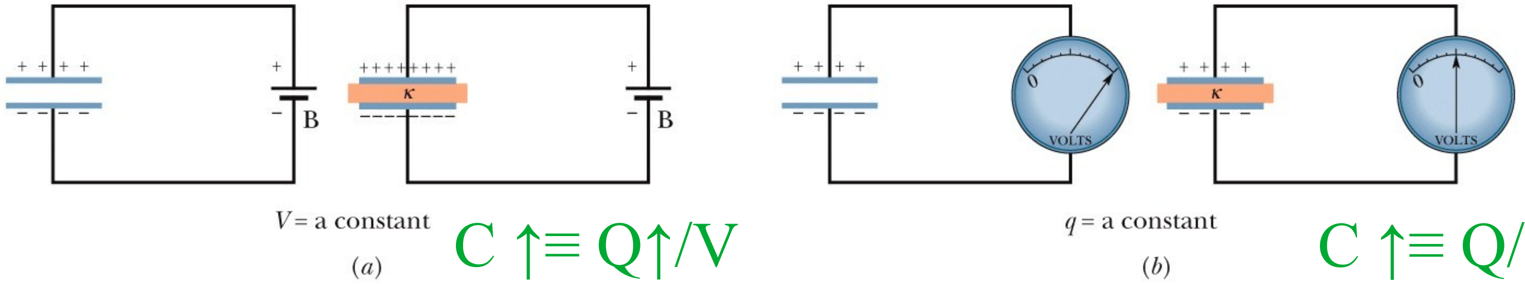
- If the **space between** the plates of a capacitor is **completely filled** with a **dielectric material**, the *capacitance C* in vacuum (or, effectively, in air) is *multiplied by the material's dielectric constant κ*, (Greek kappa) which is a number greater than 1.

κ for air is 1



In a region completely filled by a dielectric material of dielectric constant κ, all electrostatic equations containing the permittivity constant ε_0 are to be modified by replacing ε_0 with $\kappa\varepsilon_0$.

$C \rightarrow \kappa C$



V = a constant

(*a*)

$C \uparrow \equiv Q\uparrow/V$

q = a constant

(*b*)

$C \uparrow \equiv Q/V\downarrow$

(a)**If the potential difference between the plates of a capacitor is maintained**, as by the presence of battery B, the effect of a dielectric is to increase the charge on the plates.

(b) **If the charge on the capacitor plates is maintained**, as in this case by isolating the capacitor, the effect of a dielectric is to reduce the potential difference between the plates. The scale shown is that of a potentiometer, a device used to measure potential difference (here, between the plates). A capacitor cannot discharge through a potentiometer.

25-6 Capacitor with a Dielectric

- A **dielectric**, is an insulating material such as mineral oil or plastic, and is characterized by a numerical factor κ, called the *dielectric constant of the material*.
- Some dielectrics, such as strontium titanate, can increase the capacitance by more than two orders of magnitude.
- The introduction of a dielectric also limits the potential difference that can be applied between the plates to a certain value V_{max}, called the **breakdown potential**.
- ***Every dielectric material has a characteristic dielectric strength, which is the maximum value of the electric field that it can tolerate without breakdown.***

$$C=\kappa\varepsilon_0 A/d \text{ (parallel plate capacitor)}$$

$$C=\kappa C_0$$

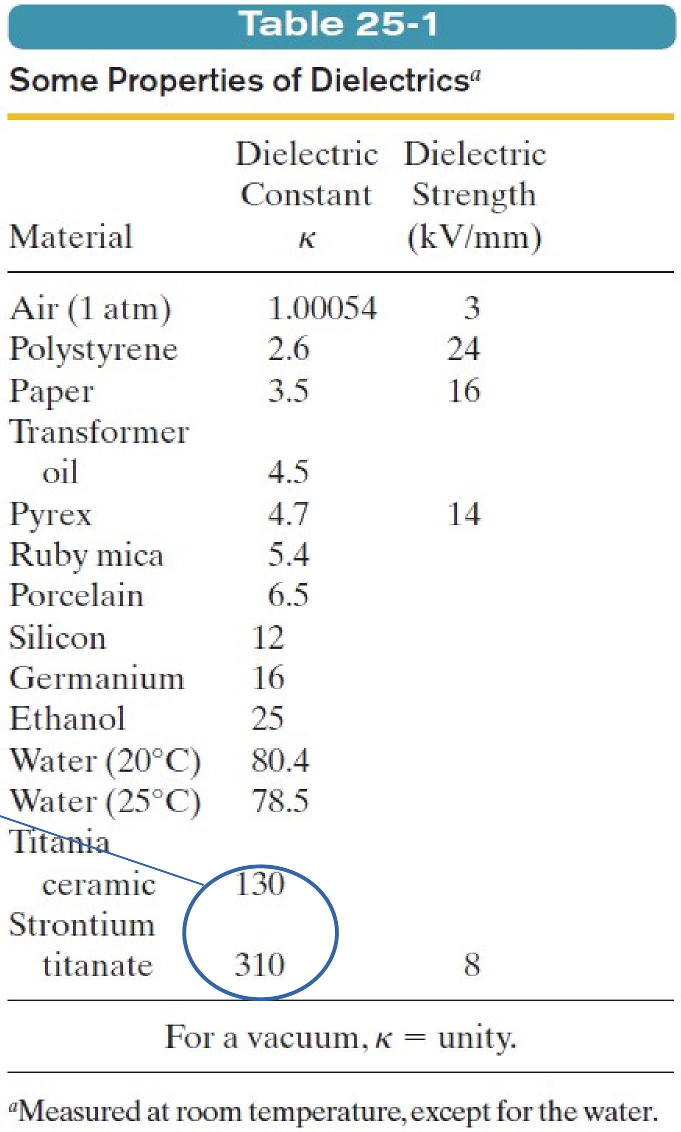


Table 25-1

Some Properties of Dielectrics[a]

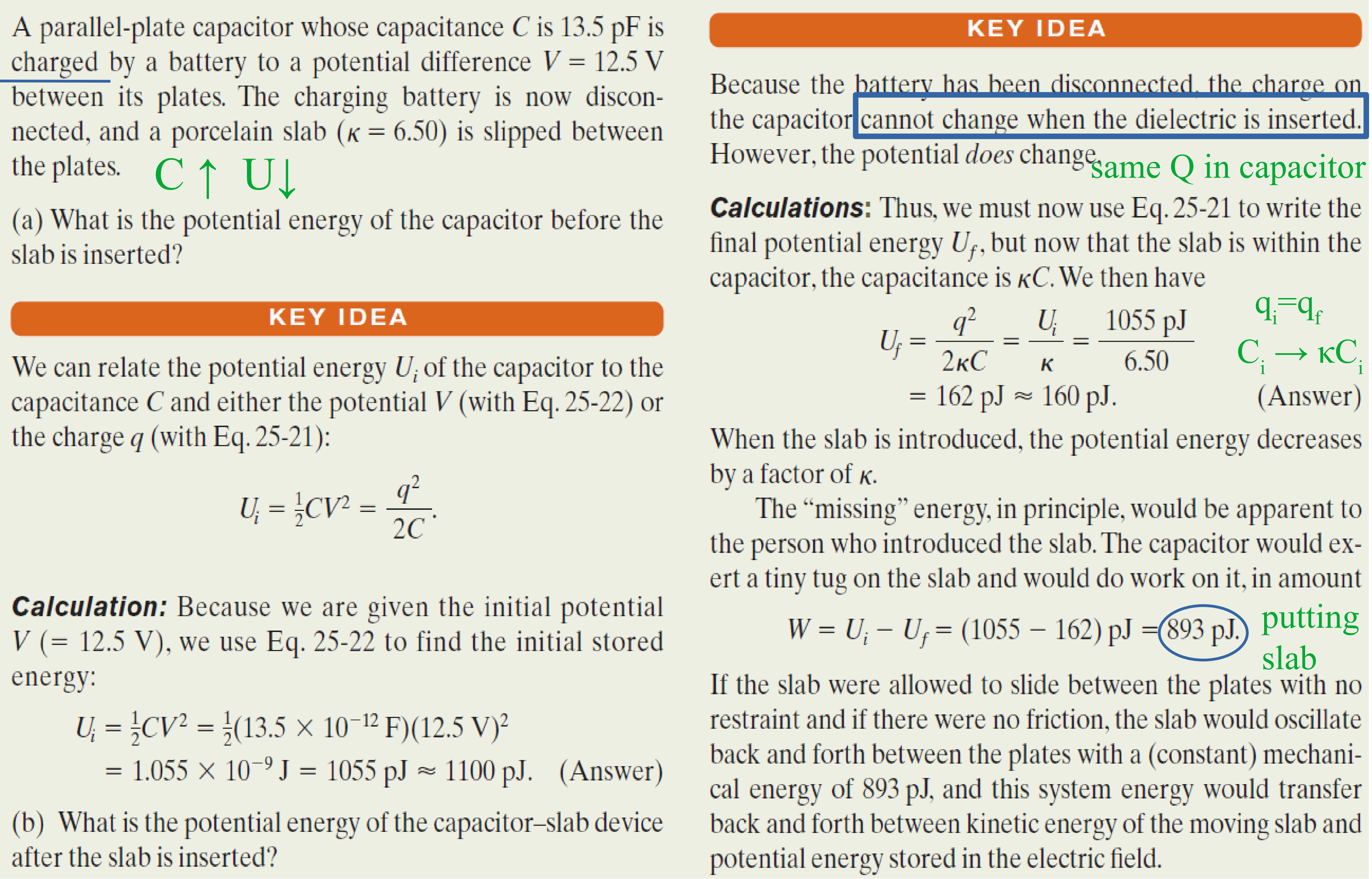
Material	Dielectric Constant κ	Dielectric Strength (kV/mm)
Air (1 atm)	1.00054	3
Polystyrene	2.6	24
Paper	3.5	16
Transformer oil	4.5	
Pyrex	4.7	14
Ruby mica	5.4	
Porcelain	6.5	
Silicon	12	
Germanium	16	
Ethanol	25	
Water (20°C)	80.4	
Water (25°C)	78.5	
Titania ceramic	130	
Strontium titanate	310	8

For a vacuum, κ = unity.

[a] Measured at room temperature, except for the water.

25-6 Capacitor with a Dielectric

Work and Energy when a Dielectric is inserted inside a Capacitor:



A parallel-plate capacitor whose capacitance C is 13.5 pF is charged by a battery to a potential difference $V = 12.5$ V between its plates. The charging battery is now disconnected, and a porcelain slab ($\kappa = 6.50$) is slipped between the plates. $C\uparrow\ U\downarrow$

(a) What is the potential energy of the capacitor before the slab is inserted?

KEY IDEA

We can relate the potential energy U_i of the capacitor to the capacitance C and either the potential V (with Eq. 25-22) or the charge q (with Eq. 25-21):

$$U_i = \tfrac{1}{2}CV^2 = \frac{q^2}{2C}.$$

Calculation: Because we are given the initial potential V (= 12.5 V), we use Eq. 25-22 to find the initial stored energy:

$$U_i = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(13.5\times10^{-12}\text{ F})(12.5\text{ V})^2$$
$$= 1.055\times10^{-9}\text{ J} = 1055\text{ pJ} \approx 1100\text{ pJ}. \quad \text{(Answer)}$$

(b) What is the potential energy of the capacitor–slab device after the slab is inserted?

KEY IDEA

Because the battery has been disconnected, the charge on the capacitor cannot change when the dielectric is inserted. However, the potential *does* change. same Q in capacitor

Calculations: Thus, we must now use Eq. 25-21 to write the final potential energy U_f, but now that the slab is within the capacitor, the capacitance is κC. We then have

$$U_f = \frac{q^2}{2\kappa C} = \frac{U_i}{\kappa} = \frac{1055\text{ pJ}}{6.50}$$
$$= 162\text{ pJ} \approx 160\text{ pJ}. \quad \text{(Answer)}$$

$q_i=q_f$

$C_i \rightarrow \kappa C_i$

When the slab is introduced, the potential energy decreases by a factor of κ.

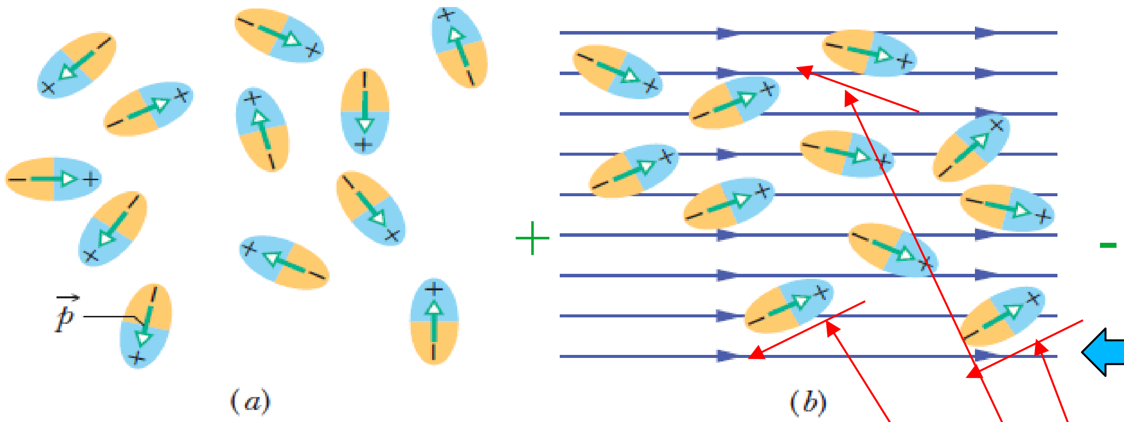
The "missing" energy, in principle, would be apparent to the person who introduced the slab. The capacitor would exert a tiny tug on the slab and would do work on it, in amount

$$W = U_i - U_f = (1055 - 162)\text{ pJ} = 893\text{ pJ}.$$

putting slab

If the slab were allowed to slide between the plates with no restraint and if there were no friction, the slab would oscillate back and forth between the plates with a (constant) mechanical energy of 893 pJ, and this system energy would transfer back and forth between kinetic energy of the moving slab and potential energy stored in the electric field.

25-6 Capacitor with a Dielectric FYI



Atomic View

Polar Dielectrics

(a) Molecules with a permanent electric dipole moment, showing their random orientation in the absence of an external electric field.

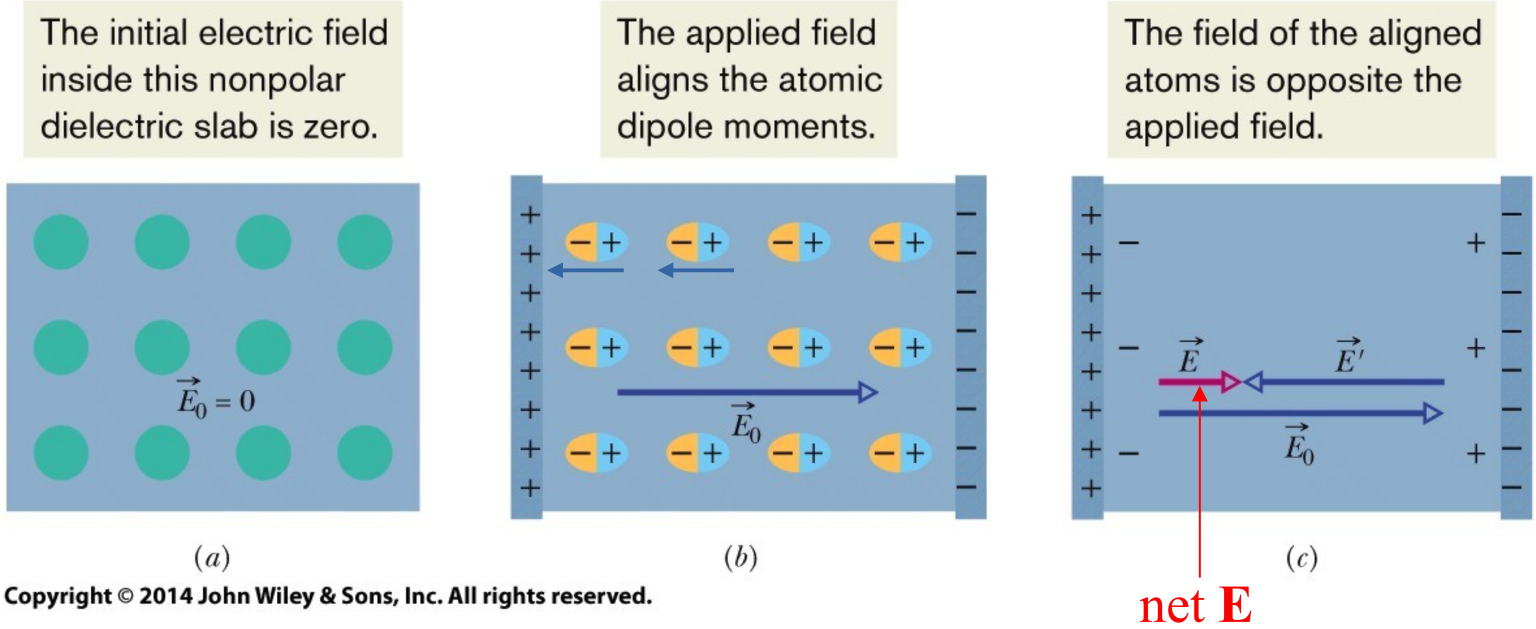
(b) An electric field is applied, producing partial alignment of the dipoles. Thermal agitation prevents complete alignment.

1. **Polar dielectrics**. The molecules of some dielectrics, like water, have *permanent electric dipole moments*.
   - In such materials (called polar dielectrics), the electric dipoles tend to line up with an external electric field as in Figure.
   - Since the molecules are continuously push&pull each other as a result of their random thermal motion, this alignment is not complete, but it becomes more complete as the magnitude of the applied field is increased.
   - **The alignment of the electric dipoles produces an electric field that is directed opposite the applied field and is smaller in magnitude.**

25-6 Capacitor with a Dielectric FYI

Atomic View

Nonpolar Dielectrics


The initial electric field inside this nonpolar dielectric slab is zero.

The applied field aligns the atomic dipole moments.

The field of the aligned atoms is opposite the applied field.

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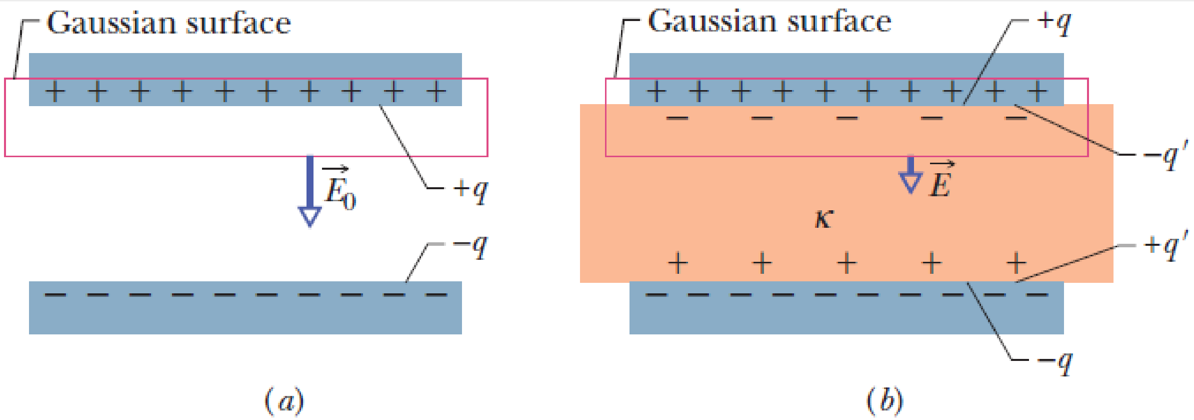
2. **Nonpolar dielectrics**. Regardless of whether they have permanent electric dipole moments, molecules *acquire dipole moments by induction* when placed in an external electric field.
   - This occurs because the external field tends to “stretch” the molecules, slightly separating the centers of negative and positive charge.

25-7 Dielectrics and Gauss' Law Capacitor with a Dielectric

Fig. 25-16 A parallel-plate capacitor (*a*) without and (*b*) with a dielectric slab inserted. The charge q on the plates is assumed to be the same in both cases.



- In Fig. 25-16a, without a dielectric. We enclose the charge q on the top plate with a Gaussian surface and then apply Gauss' law. If E_0 represents the magnitude of the field, we have $$\varepsilon_0 \oint \vec{E}\cdot d\vec{A} = \varepsilon_0 EA = q, \implies E_0 = \frac{q}{\varepsilon_0 A}. \quad \sigma/\varepsilon_0$$

- In Fig. 25-16b, with the dielectric in place. Now the surface encloses **two types of charge**: *It still encloses charge +q on the top plate, but it now also encloses the induced charge –q' on the top face of the dielectric*.
  1. The **charge on the conducting plate (q)** is said to be **free charge** because it can move if we change the electric potential of the plate;
  2. The **induced charge (q')** on the surface of the dielectric is **not free charge** because it cannot move from that surface.

C↑ U↓ E↓

$$\varepsilon_0 \oint \vec{E}\cdot d\vec{A} = \varepsilon_0 EA = q - q', \implies \boxed{E = \frac{q-q'}{\varepsilon_0 A}.}$$

25-7 Dielectrics and Gauss' Law Capacitor with a Dielectric

- The <u>effect of the dielectric</u> is **to weaken** the original field E_0 by a factor of κ and

$$E = \frac{E_0}{\kappa} = \frac{q}{\kappa\varepsilon_0 A}. \qquad E = \frac{q - q'}{\varepsilon_0 A}. \Longrightarrow q - q' = \frac{q}{\kappa}.$$

- When a dielectric is present, Gauss' law may be generalized to

$$\varepsilon_0 \oint \kappa\vec{E}\cdot d\vec{A} = q \qquad \text{(Gauss' law with dielectric).}$$

where q is the <u>free charge</u>. *Any induced surface charge is accounted for by including the dielectric constant κ inside the integral*.

no consideraton of induced charge! only free charge since $\varepsilon_0 \rightarrow \varepsilon_0\kappa$

Inserting a dielectric into a capacitor causes induced charge to appear on the faces of the dielectric and weakens the electric field between the plates.

$\boxed{C\uparrow\ U\downarrow\ E\downarrow}$

25-7 Dielectrics and Gauss' Law Capacitor with a Dielectric

Dielectric Partially Filling a Gap in a Capacitor:

Figure 25-17 shows a parallel-plate capacitor of plate area A and plate separation d. A potential difference V_0 is applied between the plates by connecting a battery between them. The battery is then disconnected, and a dielectric slab of thickness b and dielectric constant κ is placed between the plates as shown. Assume $A = 115\ \text{cm}^2$, $d = 1.24$ cm, $V_0 = 85.5$ V, $b = 0.780$ cm, and $\kappa = 2.61$.

no fully filling $C\ \kappa C_0$

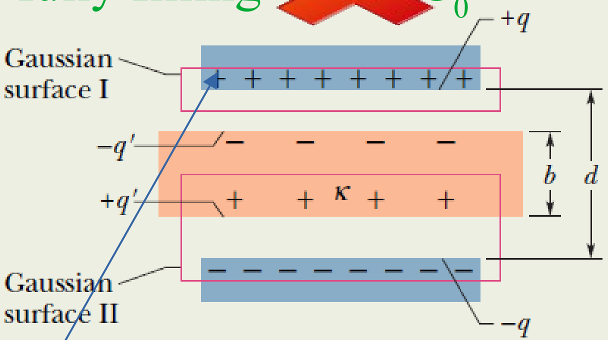


Fig. 25-17 A parallel-plate capacitor containing a dielectric slab that only partially fills the space between the plates.

(a) What is the capacitance C_0 before the dielectric slab is inserted?

Calculation: From Eq. 25-9 we have

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\ \text{F/m})(115 \times 10^{-4}\ \text{m}^2)}{1.24 \times 10^{-2}\ \text{m}}$$

without dielectric

$$= 8.21 \times 10^{-12}\ \text{F} = 8.21\ \text{pF}. \qquad \text{(Answer)}$$

(b) What free charge appears on the plates?

Calculation: From Eq. 25-1,

$$q = C_0 V_0 = (8.21 \times 10^{-12}\ \text{F})(85.5\ \text{V})$$
$$= 7.02 \times 10^{-10}\ \text{C} = 702\ \text{pC}. \qquad \text{(Answer)}$$

stored charge

(c) What is the electric field E_0 in the gaps between the plates and the dielectric slab?

Calculations: That surface passes through the gap, and so it encloses *only* the free charge on the upper capacitor plate. Electric field pierces only the bottom of the Gaussian surface. Because there the area vector $d\vec{A}$ and the field vector $\vec{E}_0$ are both directed downward, the dot product in Eq. 25-36 becomes

$$\vec{E}_0 \cdot d\vec{A} = E_0\, dA \cos 0° = E_0\, dA.$$

Equation 25-36 then becomes

$$\varepsilon_0 \kappa E_0 \oint dA = q.$$

The integration now simply gives the surface area A of the plate. Thus, we obtain

$$\varepsilon_0 \kappa E_0 A = q,$$

general formula

or

$$E_0 = \frac{q}{\varepsilon_0 \kappa A}.$$

$\varepsilon_0 \rightarrow \varepsilon_0 \kappa$

We must put $\kappa = 1$ here because Gaussian surface I does not pass through the dielectric. Thus, we have

$$E_0 = \frac{q}{\varepsilon_0 \kappa A} = \frac{7.02 \times 10^{-10}\ \text{C}}{(8.85 \times 10^{-12}\ \text{F/m})(1)(115 \times 10^{-4}\ \text{m}^2)}$$
$$= 6900\ \text{V/m} = 6.90\ \text{kV/m}. \qquad \text{(Answer)}$$

Note that the value of E_0 does not change when the slab is introduced because the amount of charge enclosed by Gaussian surface I in Fig. 25-17 does not change.

25-7 Dielectrics and Gauss' Law Capacitor with a Dielectric

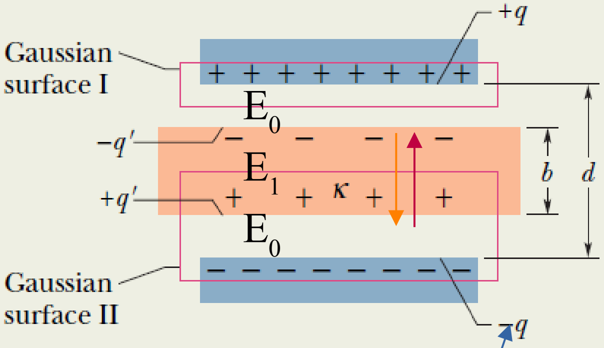


Fig. 25-17 A parallel-plate capacitor containing a dielectric slab that only partially fills the space between the plates.

(d) What is the electric field E_1 in the dielectric slab?

Calculations: That surface encloses free charge $-q$ and induced charge $+q'$, but we ignore the latter when we use Eq. 25-36. We find

$$\varepsilon_0 \oint \kappa \vec{E}_1 \cdot d\vec{A} = -\varepsilon_0 \kappa E_1 A = -q. \qquad (25\text{-}37)$$

The first minus sign in this equation comes from the dot product $\vec{E}_1 \cdot d\vec{A}$ along the top of the Gaussian surface because now the field vector $\vec{E}_1$ is directed downward and the area vector $d\vec{A}$ (which, as always, points outward from the interior of a closed Gaussian surface) is directed upward. With 180° between the vectors, the dot product is negative.

Now $\kappa = 2.61$. Thus, Eq. 25-37 gives us

$$E_1 = \frac{q}{\varepsilon_0 \kappa A} = \frac{E_0}{\kappa} = \frac{6.90\ \text{kV/m}}{2.61}$$
$$= \boxed{2.64\ \text{kV/m}}. \qquad \text{(Answer)}$$

(e) What is the potential difference V between the plates after the slab has been introduced?

Calculation: Within the dielectric, the path length is b and the electric field is E_1. Within the two gaps above and below the dielectric, the total path length is $d - b$ and the electric field is E_0. Equation 25-6 then yields

$$V = \int_-^+ E\, ds = E_0(d - b) + E_1 b$$
$$= (6900\ \text{V/m})(0.0124\ \text{m} - 0.00780\ \text{m})$$
$$+ (2640\ \text{V/m})(0.00780\ \text{m})$$
$$= 52.3\ \text{V}. \qquad \text{(Answer)}$$

This is less than the original potential difference of 85.5 V.

(f) What is the capacitance with the slab in place between the plates of the capacitor? not κC_0 since not fully insert

Calculation: Taking q from (b) and V from (e), we have

same without slab

$$C = \frac{q}{V} = \frac{7.02 \times 10^{-10}\ \text{C}}{52.3\ \text{V}}$$
$$= 1.34 \times 10^{-11}\ \text{F} = 13.4\ \text{pF}. \qquad \text{(Answer)}$$

This is greater than the original capacitance of 8.21 pF.

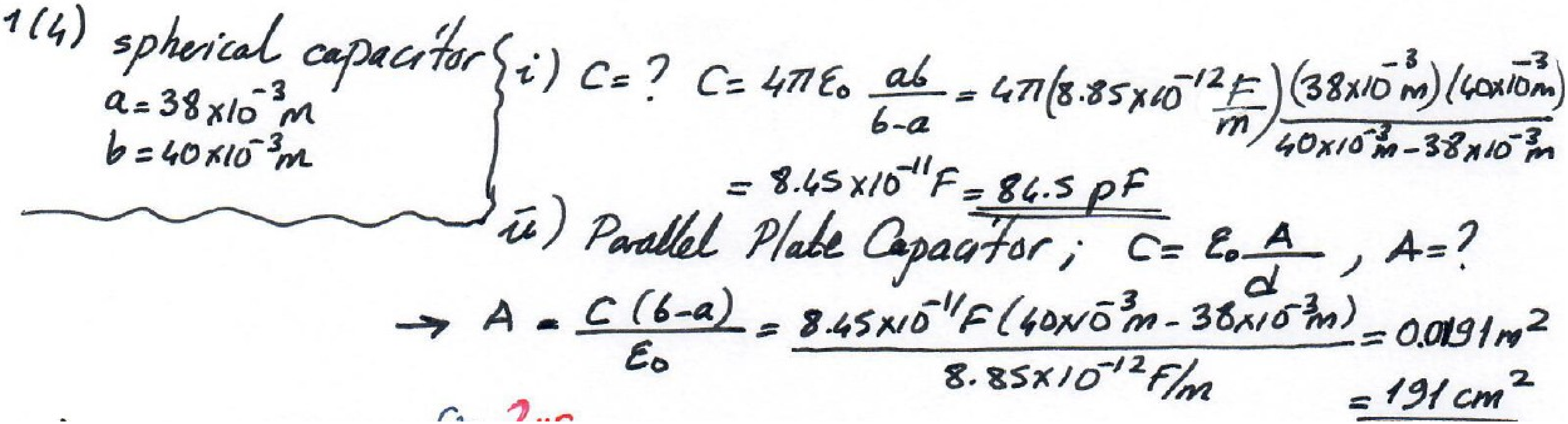
$$\boxed{C\uparrow\ E\downarrow\ V\downarrow}$$

$13.4 > 8.21$ √

but it is not 2.62*8.21 (κC_0)

25 Solved Problems

1. The plates of a spherical capacitor have radii 38.0 mm and 40.0 mm. (a) Calculate the capacitance. (b) What must be the plate area of a parallel-plate capacitor with the same plate separation and capacitance?



1(a) spherical capacitor

$a = 38\times10^{-3}\,m$

$b = 40\times10^{-3}\,m$

i) $C = ?$ $C = 4\pi\varepsilon_0 \frac{ab}{b-a} = 4\pi\left(8.85\times10^{-12}\frac{F}{m}\right)\frac{(38\times10^{-3}\,m)(40\times10^{-3}\,m)}{40\times10^{-3}\,m - 38\times10^{-3}\,m}$

$= 8.45\times10^{-11}\,F = \underline{\underline{84.5\,pF}}$

ii) Parallel Plate Capacitor; $C = \varepsilon_0 \frac{A}{d}$, $A = ?$

$\rightarrow A = \frac{C\,(b-a)}{\varepsilon_0} = \frac{8.45\times10^{-11}\,F\,(40\times10^{-3}\,m - 38\times10^{-3}\,m)}{8.85\times10^{-12}\,F/m} = 0.0191\,m^2$

$= \underline{191\,cm^2}$

25 Solved Problems

2. In Figure, a 20.0 V battery is connected across capacitors of capacitances C_1=C_6=3.00 μF and C_3=C_5=2.00C_2=2.00C_4= 4.00 μF. What are (a) the equivalent capacitance C_{eq} of the capacitors and (b) the charge stored by C_{eq}? What are (c) V_1 and (d) q_1 of capacitor 1, (e)V_2 and (f)q_2 of capacitor 2, and (g)V_3 and (h) q_3 of capacitor 3?

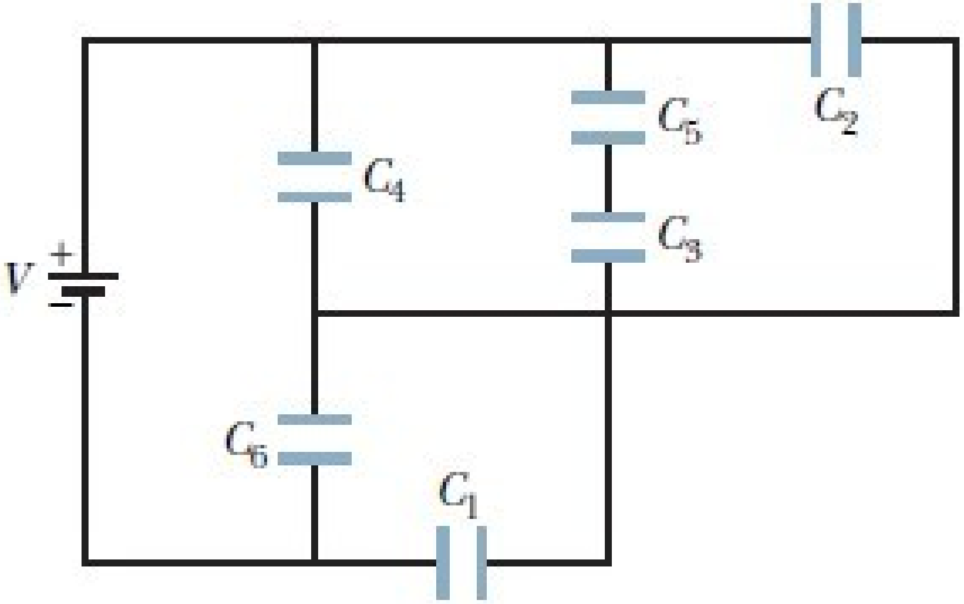
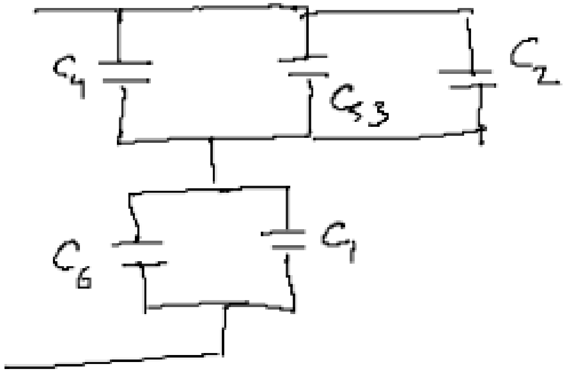


Fig. 25-31 Problem 15.

25 Solved Problems



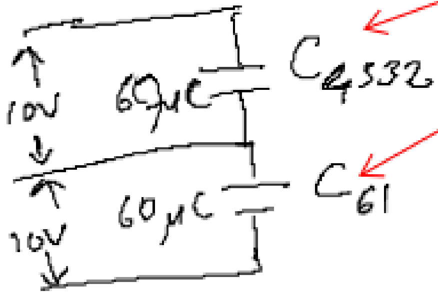
$C_{53} \rightarrow \frac{1}{C_{53}} = \frac{1}{C_5} + \frac{1}{C_3} \rightarrow C_{53} = \frac{C_5 C_3}{C_5 + C_3} = \frac{4\mu F\, 4\mu F}{4\mu F + 4\mu F}$

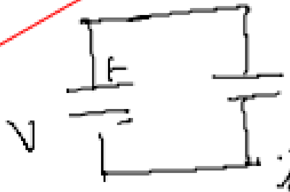
$= 2\mu F$

$C_{532} = \cancel{\frac{C_{53} C_2}{C_{53} + C_2}} = C_2 + C_{53} = 2\mu F + 2\mu F = 4\mu F$

$C_{4532} = C_4 + C_{532} = 2\mu F + 4\mu F = 6\mu F$

$C_{61} = C_6 + C_1 = 3\mu F + 3\mu F = 6\mu F$



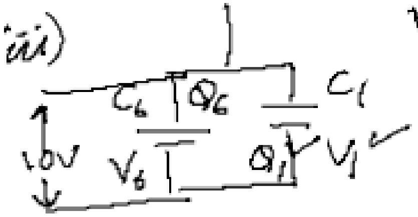


$C_{eqv} \left(= C_{614532}\right) = \frac{C_{61} \times C_{4532}}{C_{61} + C_{4532}} = 3\mu F$ (i)

ii) $C_{eqv} = \frac{Q}{V} \rightarrow Q = 3\mu F \times 20V = 60\mu C$

iv) $Q_1 = ?$ $Q_1 = C_1 V_1 = 3\mu F\ 10V = 30\mu C$

iii)



$\underline{V_1 = 10V}$

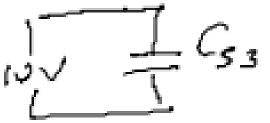
v)

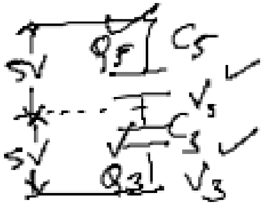
$V_2 = 10V$ vi) $Q_2 = C_2 V_2$

$= 2\mu F\ 10V$

$= 20\mu C$

vii)





$C_5 = 4\mu F = C_3$ $\Big\{ Q_{53} = C_{53} V = 2\mu F\ 10V = 20\mu C$

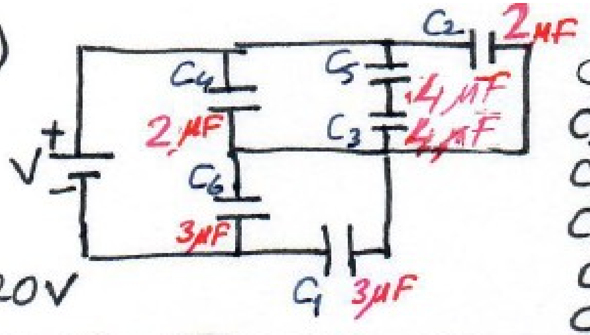
$Q_{53} = Q_3 = Q_5 = 20\mu C$

viii)

$C_3 = \frac{Q_3}{V} \rightarrow V_3 = \frac{20\mu C}{6\mu F} = 5V$

25 Solved Problems

2(15)



V=20V

$C_1 = 3\mu F$
$C_2 = 2\mu F$
$C_3 = 4\mu F$
$C_4 = 2\mu F$
$C_5 = 4\mu F$
$C_6 = 3\mu F$

i) Equivalent capacitance=?

$$\frac{1}{C_{35}} = \frac{1}{C_3} + \frac{1}{C_5} \rightsquigarrow C_{35} = \frac{(4\mu F)(4\mu F)}{4\mu F + 4\mu F} = 2\mu F$$

$$C_{235} = C_2 + C_{35} = 4\ \mu F$$

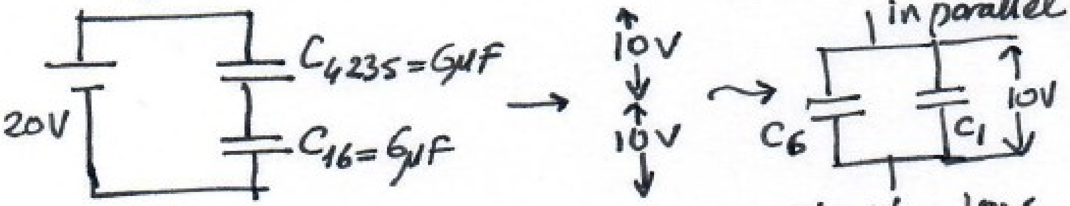
$$C_{4235} = C_4 + C_{235} = 6\ \mu F$$

$$C_{16} = C_1 + C_6 = 6\mu F$$

$$\frac{1}{C_{164235}} = \frac{1}{C_{16}} + \frac{1}{C_{4235}} \rightsquigarrow C_{eqv} = \frac{(6\mu F)(6\mu F)}{6\mu F + 6\mu F} = \underline{\underline{3\ \mu F}}$$

ii) $C = \frac{Q}{V} \rightsquigarrow q_{stored} = C_{eqv} V = (3\times10^{-6} F)(20V) = \underline{\underline{60\mu C}}$

iii) $V_1 = ?$



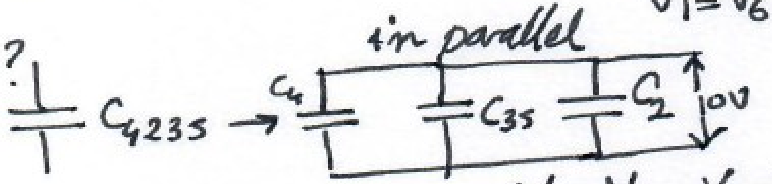
$V_1 = V_6 = \underline{\underline{10V}}$

iv) $q_1 = ?$ $q = CV$

$q_1 = (3\mu F)(10V)$

$= \underline{\underline{30\mu C}}$

v) $V_2 = ?$

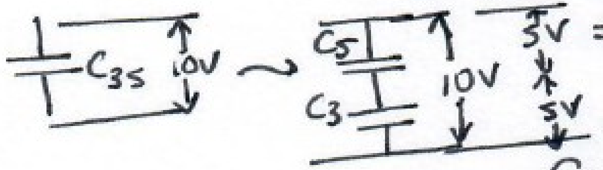


$V_4 = V_{35} = V_2 = \underline{10V}$

vi) $q_2 = ?$ $q_2 = (2\mu F)(10V)$

$= \underline{\underline{20\mu C}}$

vii) $V_3 = ?$



$\Rightarrow V_3 = V_5 = \underline{5V}$

since $C_3 = C_5$

viii) $q_3 = ?$ $q_3 = (4\mu F)(5V)$

$= \underline{\underline{20\mu C}}$

25 Solved Problems

3. In Fig. 25-37, V = 10 V, C_1 = 10 μF, and $C_2 = C_3$ = 20 μF. Switch S is first thrown to the left side until capacitor 1 reaches equilibrium. Then the switch is thrown to the right. When equilibrium is again reached, how much charge is on capacitor 1?

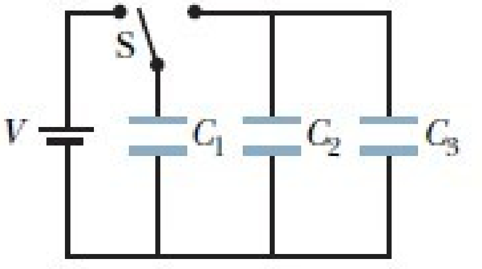
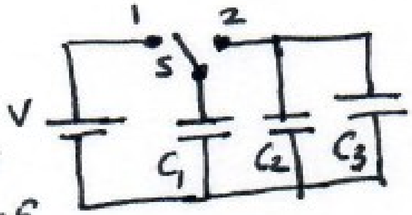


Fig. 25-37 Problem 22.

4(22)



$V = 10\,V$

$C_1 = 10\,\mu F$

$C_2 = C_3 = 20\,\mu F$

$q_1 = ?$ at case 2

<u>case 1:</u> $C = \frac{Q}{V} \sim Q_1 = C_1 V = (10\,\mu F)(10\,v) = 100\,\mu C$

<u>case 2:</u> Equilibrium is reached. $\underbrace{Q_1}_{\text{case 1}} = \underbrace{q_1 + q_2 + q_3}_{\text{Case 2}}$

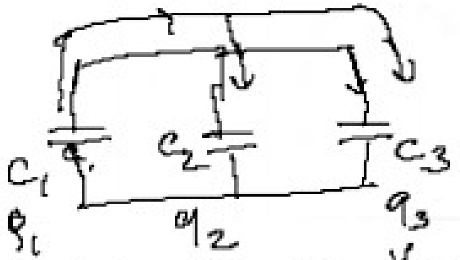
they are in parallel and $C_2 = C_3 \rightarrow q_2 = q_3$

→ same potential, different charges

$\rightarrow C_1 = \frac{q_1}{V}$ & $C_2 = \frac{q_2}{V} \Rightarrow \frac{q_1}{C_1} = \frac{q_2}{C_2} \sim \frac{q_1}{10\,\mu f} = \frac{q_2}{20\,\mu f}$

$\Rightarrow q_2 = 2q_1 \sim Q_1 = \frac{q_2}{2} + q_2 + q_2 = 5/2\, q_2 = 100\,\mu C$

$\rightarrow q_2 = 40\,\mu C = q_3 \sim \underline{q_1 = 20\,\mu C}$



$V_1 = V_2 = V_3 = V$

$V_1 = V_2 \Rightarrow \frac{q_1}{C_1} = \frac{q_2}{C_2}$

$\downarrow$

$\rightarrow q_2 = 2q_1$

$100\,\mu C = 20\,\mu C + 40\,\mu C + 40\,\mu C$

$Q_{\text{initial}} = Q_{\text{final}}$

25 Solved Problems

4. Figure 25-42 shows a 12.0 V battery and four uncharged capacitors of capacitances $C_1 = 1.00$ μF, $C_2 = 2.00$ μF, $C_3 = 3.00$ μF, and $C_4 = 4.00$ μF. If only switch S_1 is closed, what is the charge on (a) capacitor 1, (b) capacitor 2, (c) capacitor 3, and (d) capacitor 4? If both switches are closed, what is the charge on (e) capacitor 1, (f) capacitor 2, (g) capacitor 3, and (h) capacitor 4?

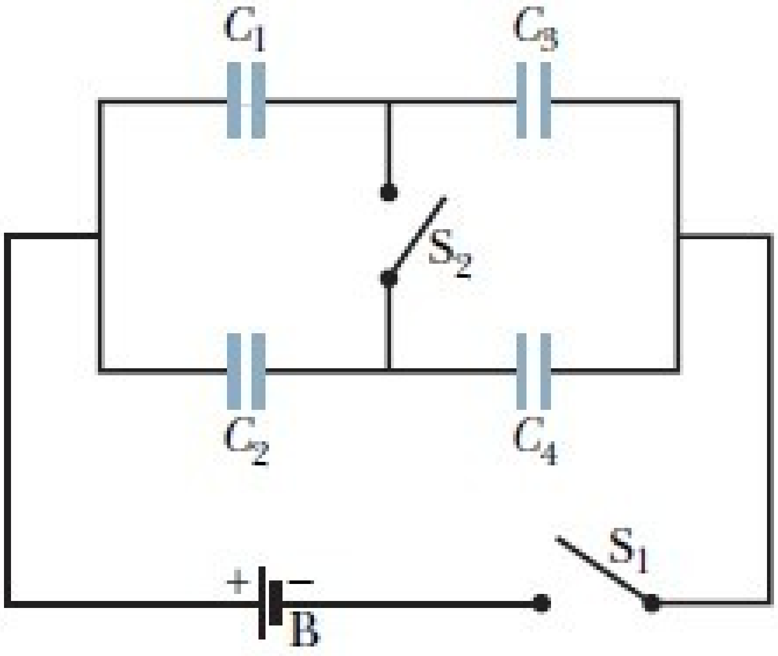
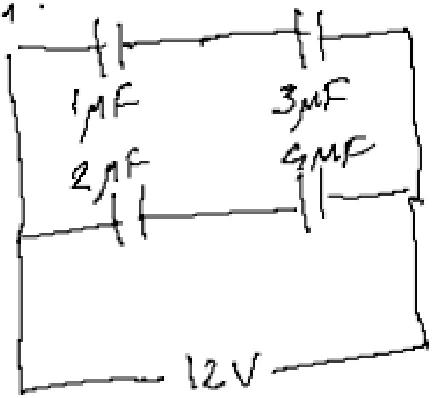


Fig. 25-42 Problem 27.

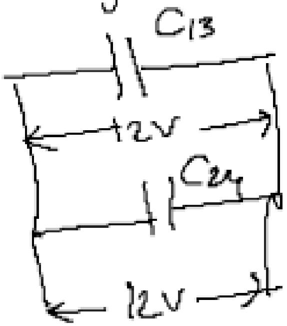
25 Solved Problems

S_1:



q_1, q_2, q_3, q_4 ?

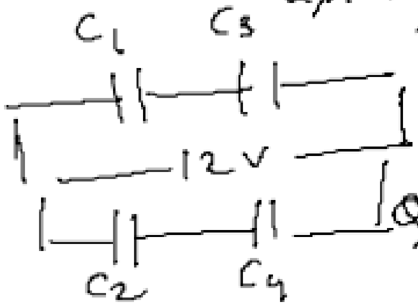
First find the Ceq!



$C = \frac{Q}{V}$

$C_{13} = \frac{1\mu F\, 3\mu F}{1\mu F + 3\mu F} = \frac{3}{4}\mu F$

$C_{24} = \frac{2\mu F\, 4\mu F}{2\mu F + 4\mu F} = \frac{8}{6}\mu F$



$\rightarrow q_1 = q_3$

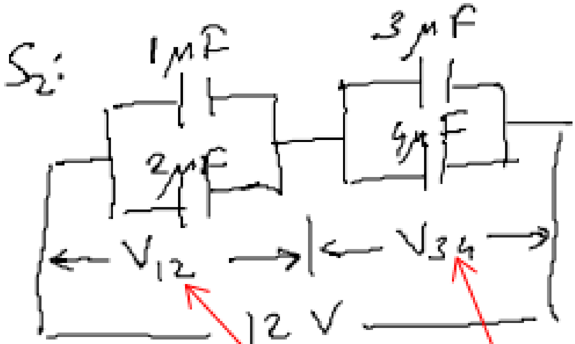
$Q_{13} = ? = q_1 = q_3$

$Q_{13} = C_{13} V = \frac{3}{4}\, 12$

$= 9\mu C$

$Q_{24} = C_{24} V = \left(\frac{8}{6}\mu F\right) 12V = 16\mu C$

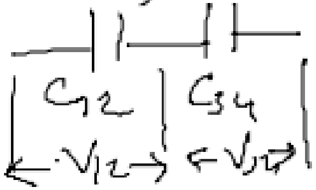
S_2:



$V_{12} + V_{34} = 12V$

$C_{12} = 3\mu F$

$C_{34} = 7\mu F$



$\rightarrow C_{12} V_{12} = C_{34} V_{34}$

$3\mu F\, V_{12} = 7\mu F (12V - V_{12})$

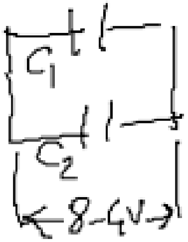
$10\mu F\, V_{12} = 7\mu F\; 12V$

$V_{12} = \left(\frac{7}{10}\right) 12V = 8.40\, V$

$\Rightarrow V_{34} = 12V - 8.4V = 3.6V$

$q_{12} = q_{34}$

~~$Q_{12} = C_{12} V_{12} = (3\mu F)(8.40V)$~~ = not sequential



$Q_1 = C_1\; 8.4V = 8.4\mu C$

$Q_2 = C_2\; 8.4V = 16.8\mu C$

$Q_3 = C_3\; 3.6V = 10.8\mu C$

$Q_4 = C_4\; 3.6V = 14.4\mu C$

25 Solved Problems

7 (27) $V = 12.0\ \text{V}$

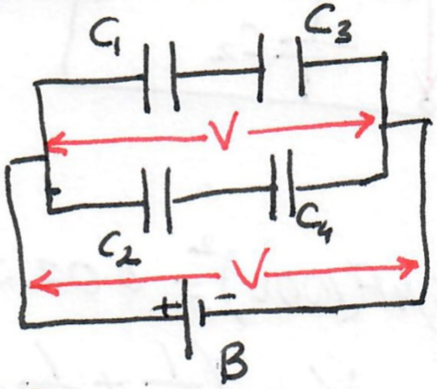
$C_1 = 1.00\ \mu F$

$C_2 = 2.00\ \mu F$

$C_3 = 3.00\ \mu F$

$C_4 = 4.00\ \mu F$

Case: S_1 Closed



C_1 & C_3 in series

C_2 & C_4 in series

i) $\Rightarrow q_1 = q_3$ & $q_2 = q_4$

$$q_1 = q_3 = C_{eqv} V = \left(\frac{1}{C_1} + \frac{1}{C_3}\right) V$$

$$= \frac{C_1 C_3}{C_1 + C_3} V = \frac{(1\mu F)(3\mu F)}{\mu F + 3\mu F} 12\text{V}$$

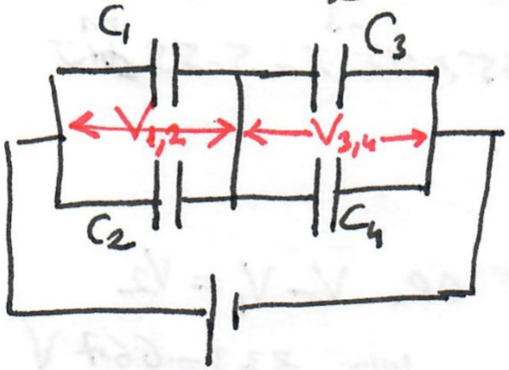
$$\boxed{q_1 = q_3 = 9\ \mu F}$$

ii)

$$q_2 = q_4 = C_{eqv} V = \frac{C_2 C_4}{C_2 + C_4} V = 16\ \mu F$$

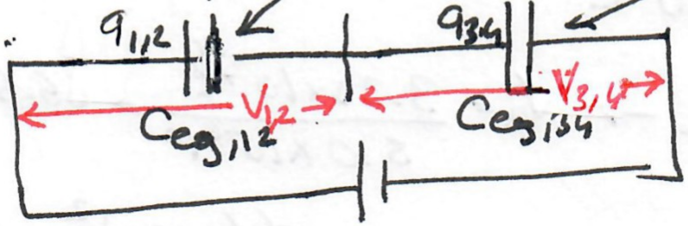
iii) $9\ \mu F$ iv) $16\ \mu F$

Case: S_1 & S_2 Closed



$C_{eq,12} = C_1 + C_2$ $C_{eq,34} = C_3 + C_4$

$q_{1,2}$; $V_{1,2}$ $q_{3,4}$; $V_{3,4}$



$q_{1,2} = q_{3,4}$

$(C_1 + C_2)V_{1,2} = (C_3 + C_4)V_{3,4}$

$V_{1,2} + V_{3,4} = V$

25 Solved Problems

$$\rightarrow (C_1+C_2)V_{12}=(C_3+C_4)(V-V_{12})$$

$$(C_1+C_2)V_{12}+(C_3+C_4)V_{12}=(C_3+C_4)V$$

$$V_{12}(C_1+C_2+C_3+C_4)=(C_3+C_4)V$$

$$V_{12}=\frac{C_3+C_4}{(C_1+C_2+C_3+C_4)}V=\frac{3\mu F+4\mu F}{(1\mu F+2\mu F+3\mu F+4\mu F)}12V=8.40V$$

v) $q_1=C_1V_{12}=1\mu F * 8.40V=8.40\mu C$

vi) $q_2=C_2V_{12}=2\mu F\times 8.40V=16.8\mu C$

vii) $q_3=C_3(V-V_{12})=3\mu F * 3.60V=10.8\mu C$

viii) $q_4=C_4(V-V_{12})=4\mu F\times 3.60=14.4\mu C$

25 Solved Problems

5. In Fig. 25-28, a potential difference V=100 V is applied across a capacitor arrangement with capacitances C_1=10.0 µF, C_2=5.00 µF, and C_3=4.00 µF. What are (a) charge q_3, (b) potential difference V_3, and (c) stored energy U_3 for capacitor 3, (d) q_1, (e) V_1, and (f) U_1 for capacitor 1, and (g) q_2, (h) V_2, and (i) U_2 for capacitor 2?

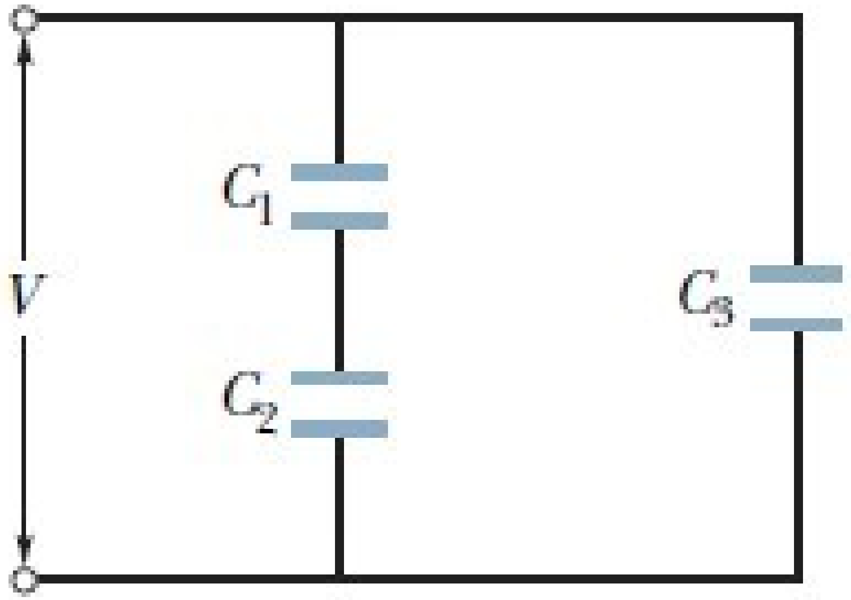


Fig. 25-28 Problems 10 and 34.

q_i & V_i & u_i

$C = \frac{q}{V}$

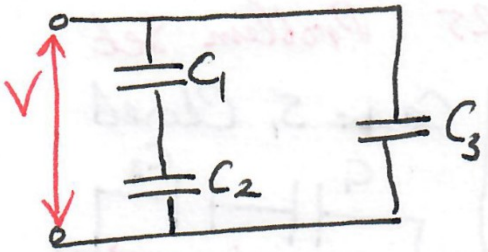
25 Solved Problems

8 (34) $V = 100\,\mathrm{V}$

$C_1 = 10.0\,\mu\mathrm{F}$

$C_2 = 5.0\,\mu\mathrm{F}$

$C_3 = 4.0\,\mu\mathrm{F}$



in series $\rightsquigarrow q_1 = q_2$

i) $q_3 = ?$

$q_3 = C_3 V = (4.0\,\mu\mathrm{F}\ 100\,\mathrm{V})$

$= 400\,\mu\mathrm{F} = 4 \times 10^{-4}$

ii) $V_3 = V = 100\,\mathrm{V}$

iii) $U_3 = \frac{1}{2} C_3 V_3^2 = \frac{1}{2}\, 4\,\mu\mathrm{F} (100\,\mathrm{V})^2 = 0.02\,\mathrm{J}$

iv) $q_1 = ?$ $\quad q_1 = q_2 = C_{eq,12} V = \left(\frac{1}{C_1} + \frac{1}{C_2}\right) V = \left(\frac{C_2 C_1}{C_1 + C_2}\right) V = \left(\frac{10\,\mu\mathrm{F}\ 5\,\mu\mathrm{F}}{10\,\mu\mathrm{F} + 5\,\mu\mathrm{F}}\right) 100\,\mathrm{V}$

$q_1 = 333.3\,\mu\mathrm{C} = 3.33 \times 10^{-4}\,\mathrm{C}$

v) $V_1 = ?$ $\quad \frac{q_1}{C_1} = V_1 \rightarrow V_1 = \frac{3.33 \times 10^{-4}\,\mathrm{C}}{10.0\,\mu\mathrm{F}} = \frac{3.33 \times 10^{-4}\,\mathrm{C}}{10 \times 10^{-6}\,\mathrm{F}} = 33.3\,\mathrm{V}$

vi) $U_1 = ?$ $\quad U_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2}\, 10.0\,\mu\mathrm{F} (33.3\,\mathrm{V})^2 = 5.55 \times 10^{-3}\,\mathrm{J} = 5.55\,\overset{m}{\cancel{\mu}}\mathrm{J}$

vii) $q_2 = q_1 = 3.33 \times 10^{-4}\,\mathrm{C}$

viii) $V_2 = ?$ $\quad \frac{q_2}{C_2} = V_2 \rightarrow V_2 = \frac{3.33 \times 10^{-4}\,\mathrm{C}}{5.0 \times 10^{-6}\,\mathrm{F}} = 66.66\,\mathrm{V}$ $\quad$ OR $\quad V - V_1 = V_2$

$100 - 33.3 = 66.7\,\mathrm{V}$

ix) $U_2 = ?$ $\quad U_1 = \frac{1}{2} C_2 V_2^2 = \frac{1}{2}\, 5.0 \times 10^{-6} (66.7\,\mathrm{V})^2 = 1.11 \times 10^{-2}\,\mathrm{J} = 11.1\,\mathrm{mJ}$

25 Solved Problems

6. Figure shows a parallel plate capacitor with a plate area A=5.56 cm^2 and separation d=5.56 mm. The left half of the gap is filled with material of dielectric constant κ_1=7.00; the right half is filled with material of dielectric constant κ_2=12.0. What is the capacitance?

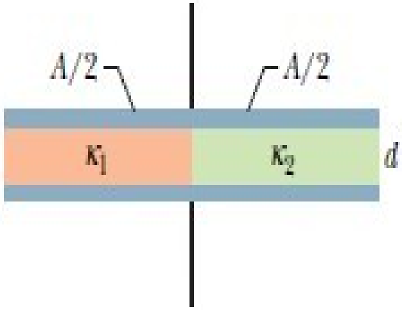
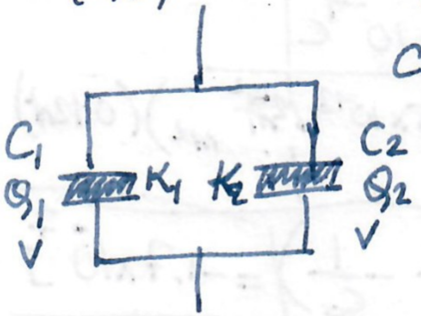


Fig. 25-47

11 (48)



$Q = Q_1 + Q_2$

$CV = C_1V + C_2V$

$\Rightarrow C = C_1 + C_2$

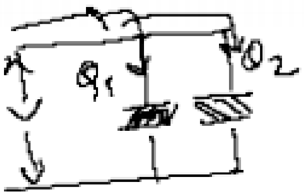
$= K_1 C_{01} + K_2 C_{02}$

$$C = K_1 \varepsilon_0 \frac{A/2}{d} + K_2 \varepsilon_0 \frac{A/2}{d}$$

$$= 19 \varepsilon_0 \frac{A/2}{d} = 19 \times \left(8.85\times10^{-12} \frac{F}{m}\right)\left(\frac{5.56\times10^{-4} m^2/2}{5.56\times10^{-3} m}\right)$$

$$= \underline{\underline{8.4\ pf}}$$

$$\frac{C}{V} = \frac{C}{\frac{N}{C} m} = \frac{C^2}{Nm} \equiv \text{farad}$$



$C = K\varepsilon_0 \frac{A}{d}$

$Q = Q_1 + Q_2$

$C = C_1 + C_2 = 7\varepsilon_0 \frac{A/2}{d} + 12\varepsilon_0 \frac{A/2}{d}$

$= K_1 \underset{C_0}{C_{01}} + K_2 \underset{C_0}{C_{02}}$

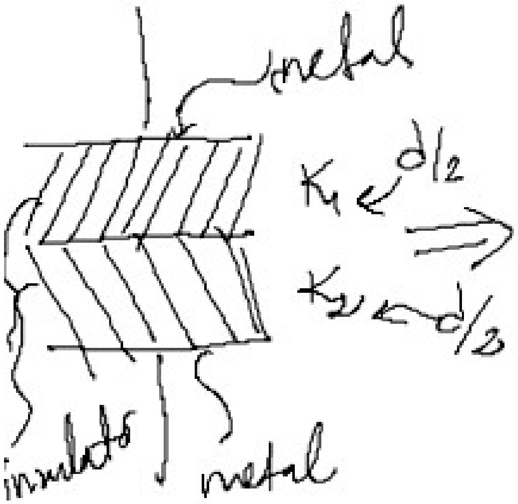
$C_0 = \varepsilon_0 \frac{A/2}{d}$

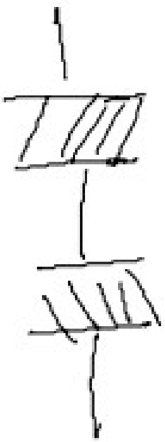
$\underline{\underline{C = 19 C_0}}$

$C > C_0$

$C\uparrow$ by insertion of dielectric

25 Solved Problems





$C_1 = K C_{01}$

$C_2 = K C_{02}$

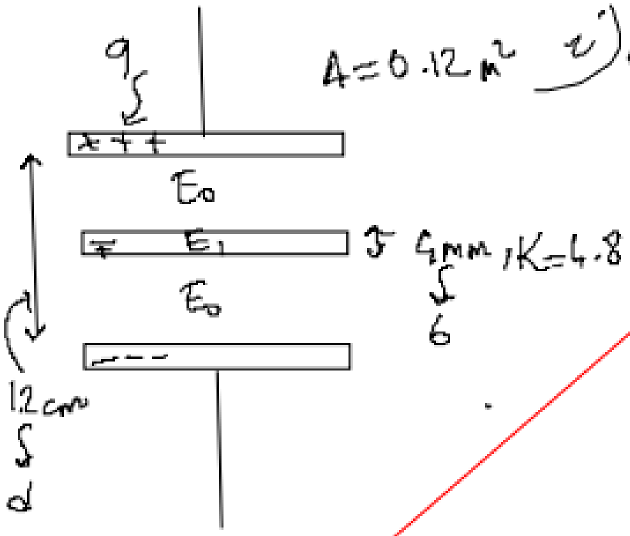
d is halved

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

7. A parallel-plate capacitor has plates of area 0.12 m^2 and a separation of 1.2 cm. A battery charges the plates to a potential difference of 120 V and is then disconnected. A dielectric slab of thickness 4.0 mm and dielectric constant 4.8 is then placed symmetrically between the plates.

(a) What is the capacitance before the slab is inserted?
(b) What is the capacitance with the slab in place?
(c) What is the free charge q before and
(d) after the slab is inserted?
(e) What is the magnitude of the electric field in the space between the plates and dielectric and
(f) in the dielectric itself?
(g) With the slab in place, what is the potential difference across the plates?
(h) How much external work is involved in inserting the slab?

25 Solved Problems



A=0.12 m²

i) $C_0 = \varepsilon_0 \frac{A}{d}$ = 89 pF initially capacitance $\}\ E_0, V_0$

gap is not fully filled ⇒ $C \cancel{= K} C_0$

$V = \int_{-}^{+} E\,ds = E_0(d-b) + E_1 b$ $\left\{ \begin{array}{l} \varepsilon_0 E_0 A = q \\ K\varepsilon_0 E_1 A = q \end{array} \right.$

$= \frac{q}{\varepsilon_0 A}(d-b) + \frac{q}{K\varepsilon_0 A} b$; so what is q?

⇒ free charge or initially stored charge

$C_0 = \frac{q}{V} \rightarrow q = C_0 V = (89\,pF)(120\,V) = 11\,nC$

(iii) (before slab)

$C = \frac{q}{V}$ (iv) charge remains the same after insertion of the slab

$$C = \frac{\cancel{q}}{\frac{\cancel{q}}{\varepsilon_0 A}(d-b) + \frac{\cancel{q}}{K\varepsilon_0 A} b} = \frac{\varepsilon_0 A K}{K(d-b)+b} = 1.2\times10^{2}\,pF = \underset{iv)}{0.12\,nF} \Rightarrow C\uparrow ✓$$

$E\downarrow$ ✓

$V\downarrow$ ✓

v) $E_0 = \frac{q}{\varepsilon_0 A} = 10$ kV/m

vi) $E_1 = \frac{E_0}{K} = \frac{10\,kV/m}{4.8} = 2.1$ kV/m

vii) V=? 120 V ✓

viii) $V = E_0(d-b) + E_1 b = 88$ V

ix) $W = -\Delta U = -\frac{q^2}{2}\Big(\underbrace{\frac{1}{C}}_{U_f} - \underbrace{\frac{1}{C_0}}_{U_i}\Big)$

$\boxed{W = -1.7\times10^{-7}\,J}$ negative work!

25 Solved Problems

12 (53) $A = 0.12\ m^2$

$d = 1.2cm = 0.012m$

$V = 120V$

then disconencted

slab with thickness $0.004\ m$

$\kappa = 4.8$

$\downarrow$ capacitance should be increased

i) Initially $C = ?$ $C = \frac{\varepsilon_0 A}{d} =$

$$C_0 = \frac{(8.55\times10^{-2}\ C^2N/m^2)(0.12m)}{1.2\times10^{-3}m} = 89\ pF$$

ii) After slab ;

See sample problem : Dielectric partially filling the gap in a capacitor

$$V = \int_{-}^{+} E\,ds = E_0(d-b) + E_1 b$$

$$V = E_0(d-b) + \frac{E_0}{K} b = \frac{q}{\varepsilon_0 A K}(K(d-b)+b)$$

$$\frac{V}{q} = \frac{1}{C} = \frac{1}{\varepsilon_0 A K}(K(d-b)+b)$$

$$\Rightarrow C = \frac{\varepsilon_0 A K}{K(d-b)+b} = 1.2\times10^{2}\ pF = 0.12\ nF$$

iii) before $q = C_0 V = (89pF)(120V)$

$= 11\ nC$

iv) after. since battery is disconnected q will ~~the~~ remain the same after the insertion of the slab $q = 11\ nC$

$$\text{v)}\ E_0 = \frac{q}{\varepsilon_0 A} = \frac{11\times10^{-9}C}{(8.85\times10^{-12}\ C^2/Nm^2)(0.12m^2)}$$

$= 10\ kV/m$

$$\text{vi)}\ E_1 = \frac{E_0}{K} = \frac{10\ kV/m}{4.8} = 2.1\ kV/m$$

$$\text{vii)}\ V = E_0(d-b) + E_1 b = 88\ V$$

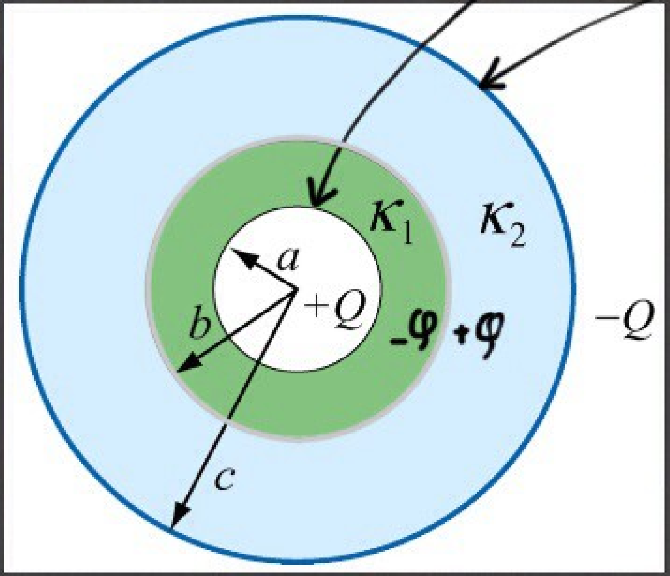
$(10\ kV/m)(0.012\ m - 0.004\ m) + (2.1\ kV/m)(0.40\times10^{-3}m)$

$$\text{viii)}\ W_{ext} = -\Delta U = \frac{q^2}{2}\left[\frac{1}{C} - \frac{1}{C_0}\right] = -1.7\times10^{-7}\ J$$

25 Solved Problems

By Aziz Kolkıran

Ex: Capacitor with two dielectrics-3



The system runs as two serially connected spherical capacitors

$$\Rightarrow \frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}$$

capacitance of a spherical cap.

$$C= 4\pi\epsilon_0\left(\frac{r_1 r_2}{r_2-r_1}\right)$$

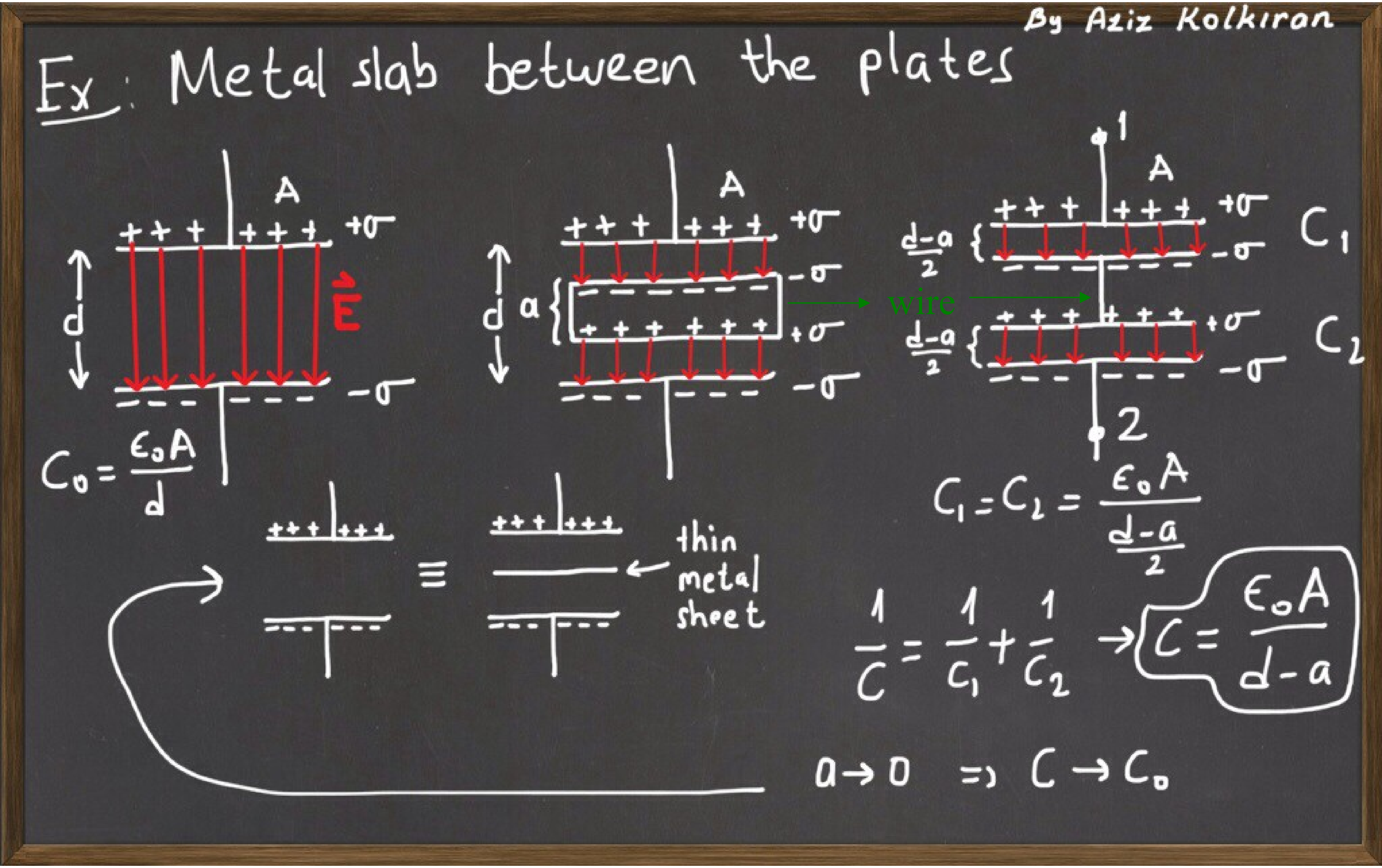
$$C_1=K_1\,4\pi\epsilon_0\left(\frac{ab}{b-a}\right)$$

$$C_2=K_2\,4\pi\epsilon_0\left(\frac{bc}{c-b}\right) \Rightarrow C=\frac{C_1 C_2}{C_1+C_2}$$

$$\boxed{C=\frac{4\pi\epsilon_0\,abc}{K_1 a(c-b)+K_2 c(b-a)}}$$

check $K_1, K_2\rightarrow 1$ $\quad C\rightarrow 4\pi\epsilon_0\,\frac{ac}{c-a}$

25 Solved Problems



By Aziz Kolkıran

Ex: Metal slab between the plates

$$C_0 = \frac{\epsilon_0 A}{d}$$

$$C_1 = C_2 = \frac{\epsilon_0 A}{\frac{d-a}{2}}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \rightarrow \boxed{C = \frac{\epsilon_0 A}{d-a}}$$

$$a \to 0 \Rightarrow C \to C_0$$

25 Summary

Capacitor and Capacitance

- The capacitance of a capacitor is defined as:

$$q = CV$$ **Eq. 25-1**

Determining Capacitance

- Parallel-plate capacitor: $C = \frac{\varepsilon_0 A}{d}$. **Eq. 25-9**
- Cylindrical Capacitor: $C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}$. **Eq. 25-14**
- Spherical Capacitor: $C = 4\pi\varepsilon_0 \frac{ab}{b-a}$. **Eq. 25-17**
- Isolated sphere: $C = 4\pi\varepsilon_0 R$. **Eq. 25-18**

Capacitor in parallel and series

- In parallel: $C_{eq} = \sum_{j=1}^{n} C_j$ **Eq. 25-19**
- In series $\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$ **Eq. 25-20**

Potential Energy and Energy Density

- Electric Potential Energy (U):

$$U = \frac{q^2}{2C} = \tfrac{1}{2}CV^2$$ **Eq. 25-21&22**

- Energy density (*u*)

$$u = \tfrac{1}{2}\varepsilon_0 E^2.$$ **Eq. 25-25**

Capacitance with a Dielectric

- If the space between the plates of a capacitor is completely filled with a dielectric material, the capacitance *C* is increased by a factor *κ*, called the dielectric constant, which is characteristic of the material.

Gauss' Law with a Dielectric

- When a dielectric is present, Gauss' law may be generalized to

$$\varepsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q.$$ **Eq. 25-36**