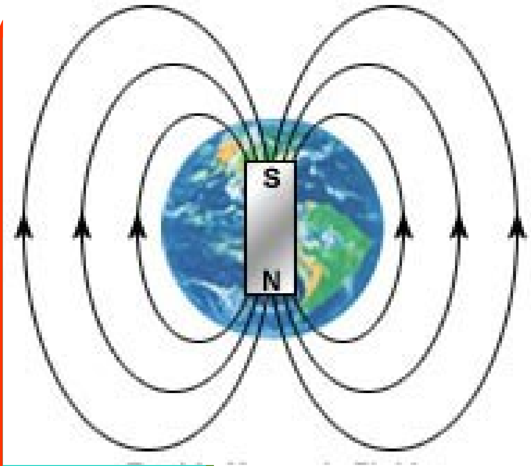
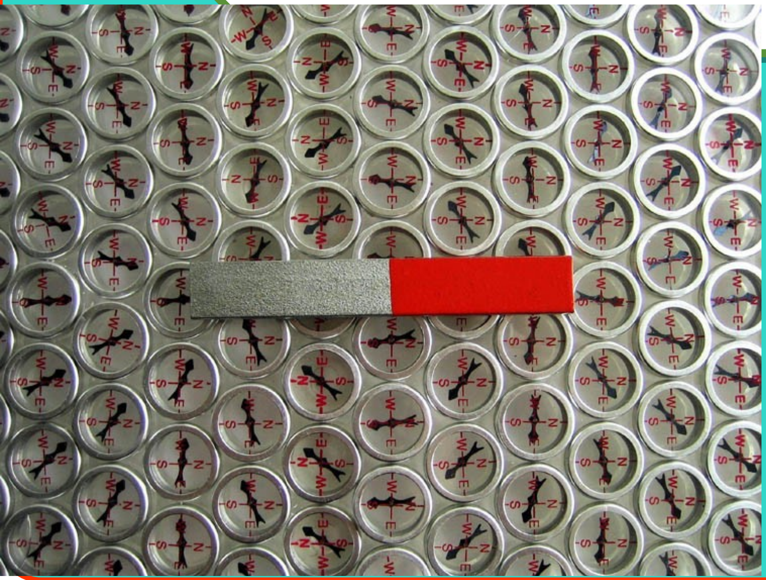


Chapter 28

Magnetic Fields



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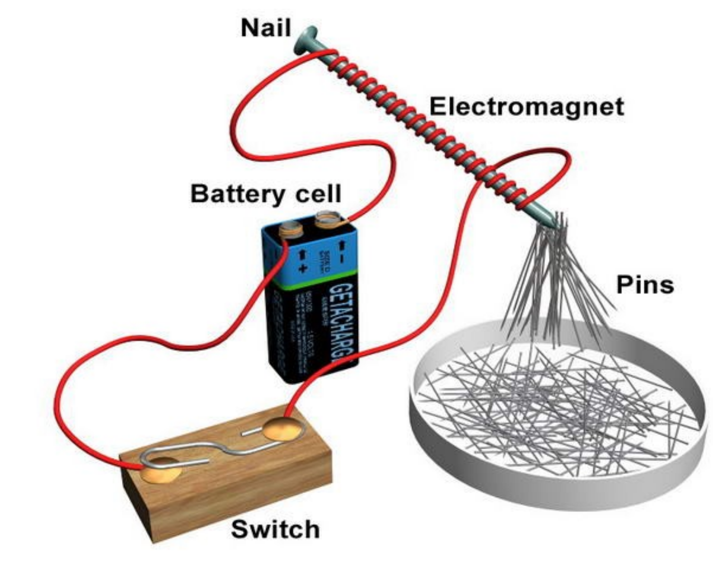


28 MAGNETIC FIELDS 735

28-2 What Produces a Magnetic Field?

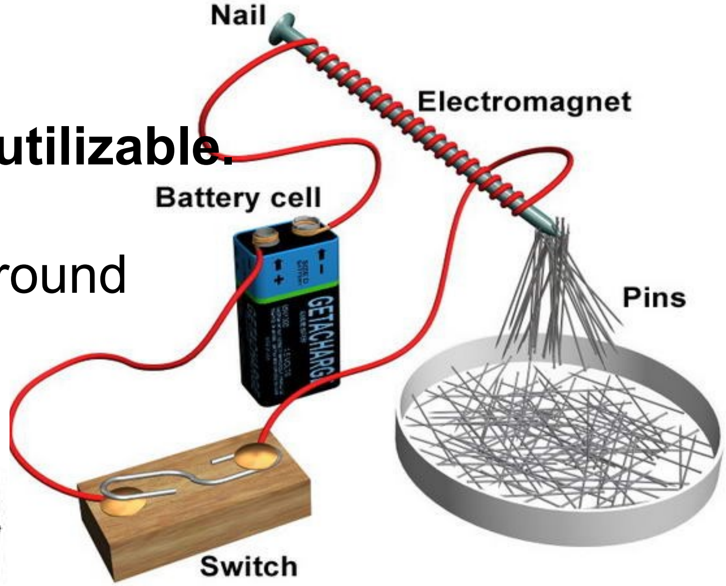
1. Moving electrically charged particles

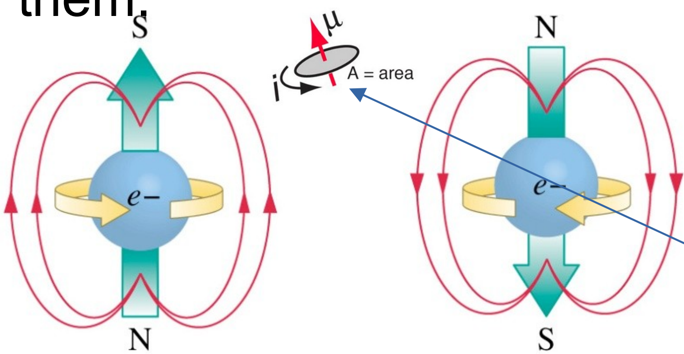
ex: current in a wire makes an **electromagnet.**

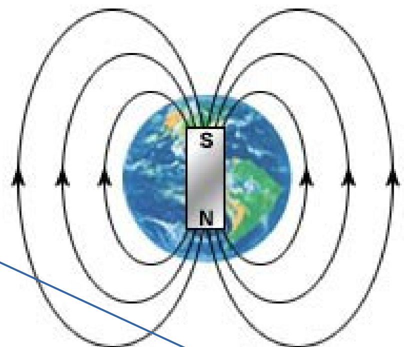
The current produces a magnetic field that is utilizable.

2. Spinning motion of the electrons.

These particles have an *intrinsic magnetic field* around them.



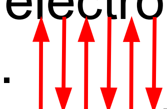




- The magnetic fields of the electrons in certain materials **add together** to give a **net magnetic field** around the material.

aligned in same direction

  - Such addition is the reason why a **permanent magnet**, has a permanent magnetic field.
- In other materials, the magnetic fields of the electrons **cancel out**, giving **no net magnetic field** surrounding the material.

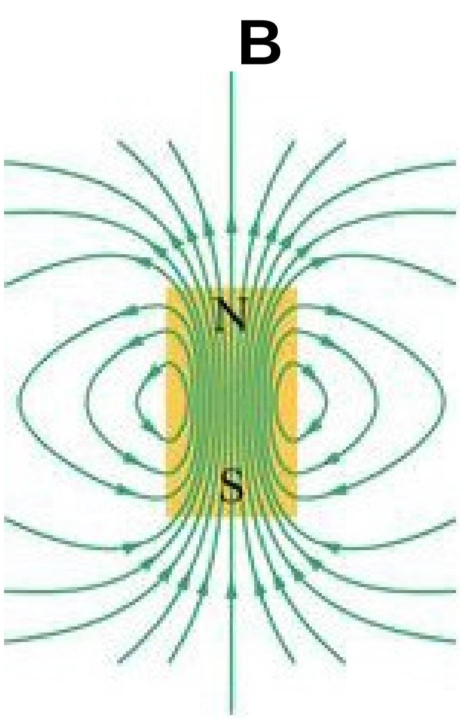


aligned in opposite directions

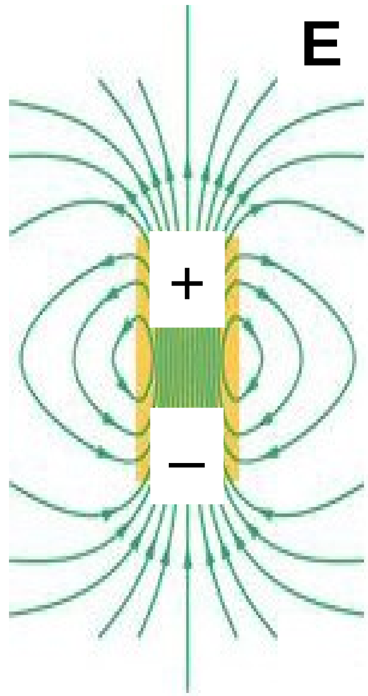
- Large objects, such as the earth, other planets, and stars, also produce magnetic fields.

28-2 What Produces a Magnetic Field?

Magnetic Field Direction



FROM North Poles
TO South Poles



Compare to Electric Field Directions

<u>Electric fields are created:</u>

•*microscopically*, by electric charges (fields) of elementary particles (electrons, protons)

•*macroscopically*, by adding the field of many elementary charges of the same sign

<u>Magnetic fields are created :</u>

•*microscopically*, by magnetic “moments” of elementary particles (electrons, protons, neutrons)

•*macroscopically*,

- by adding many microscopic magnetic moments (magnetic materials),
- by electric charges that move (electric currents).

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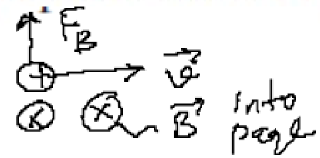
Magnetic Field $\vec{B}$ A **magnetic field** $\vec{B}$ is defined in terms of the force $\vec{F}_B$ acting on a test particle with charge q moving through the field with velocity $\vec{v}$:

RHR



$$\vec{F}_B = q\vec{v} \times \vec{B};$$

$$F_B = |q|vB \sin \phi,$$

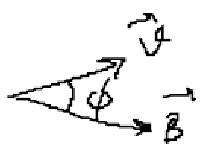


Table 28-1

Some Approximate Magnetic Fields

At surface of neutron star	10^8 T
Near big electromagnet	1.5 T
Near small bar magnet	10^{-2} T
At Earth's surface	10^{-4} T
In interstellar space	10^{-10} T
Smallest value in magnetically shielded room	10^{-14} T

ϕ is the angle between the directions of velocity $\vec{v}$ and magnetic field $\vec{B}$.

The SI unit for $\vec{B}$ is the **tesla** (T): $1\ \text{T} = 1\ \text{N/(A}\cdot\text{m)} = 10^4$ gauss.

How to Find the Magnetic Force on a Particle?

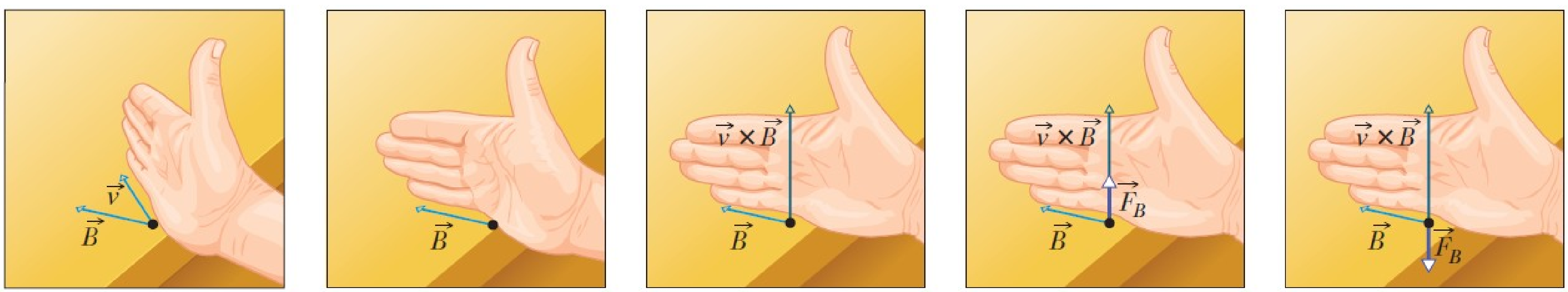
$$F_B = |q|vB \sin\phi,$$

$\vec{F}_B = \pm q\,\vec{v} \times \vec{B}$
q(−): direction is reversed!!

1. The magnitude of the force acting on a particle in a magnetic field is proportional to the charge q and speed v of the particle.

- if the charge is zero → the force is equal to zero
- if the particle is stationary → the force is equal to zero
- If $\vec{v}$ and $\vec{B}$ are parallel or antiparallel → the force is zero
- If $\vec{v}$ and $\vec{B}$ are perpendicular to each other → the force is at its Max.

2. The Direction of the force → use RIGHT HAND RULE



If q is positive

If q is negative

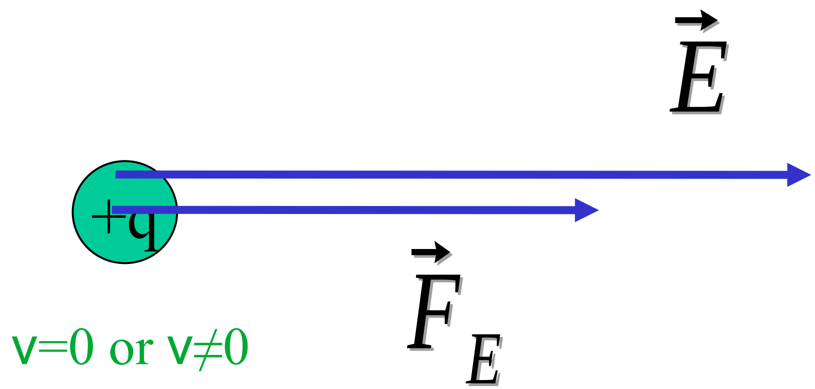
The force $\vec{F}_B$ acting on a charged particle moving with velocity $\vec{v}$ through a magnetic field $\vec{B}$ is *always* perpendicular to $\vec{v}$ and $\vec{B}$.

$|\vec{v}|$ does not change by applied B!!

Magnetic vs. Electric Forces

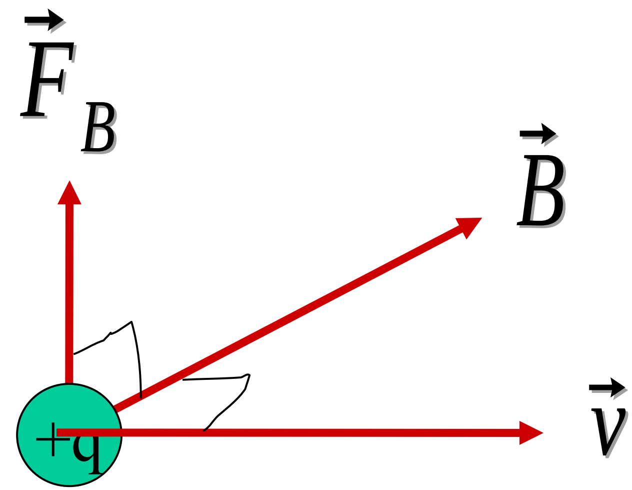
Electric Force on Charge Parallel to E:

$$\vec{F}_E = q\vec{E}$$



Magnetic Force on Charge Perpendicular to B and v.

$$\vec{F}_B = q\vec{v} \times \vec{B}$$



v≠0 new path is circular!

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Magnetic Field Lines (Similar to electric field lines)

1. the direction of the tangent to a magnetic field line at any point gives the direction of **B** at that point
2. the spacing of the lines represents the magnitude of **B** (stronger where the lines are closer together, and conversely).

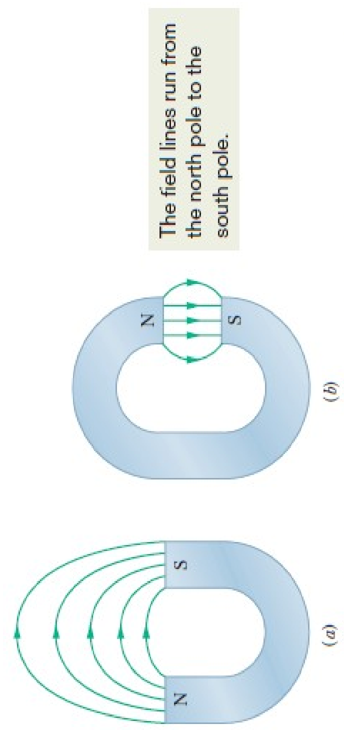
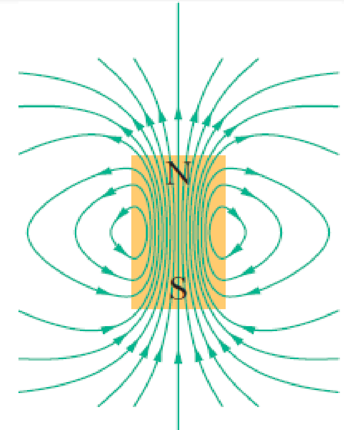
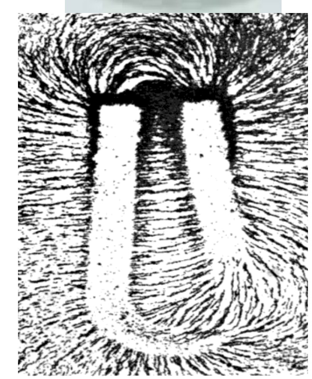


Fig. 28-5 (*a*) A horseshoe magnet and (*b*) a C-shaped magnet. (Only some of the external field lines are shown.)

- The end of a magnet from which the field lines emerge is called the *north pole* of the magnet; the other end, where field lines enter the magnet, is called the *south pole.*
- Because a magnet has two poles, it is said to be a **magnetic dipole**
- The external magnetic effects of a bar magnet are strongest near its ends, where the field lines are most closely spaced.
- Thus, the magnetic field of the bar magnet collects the iron filings mainly near the two ends of the magnet.



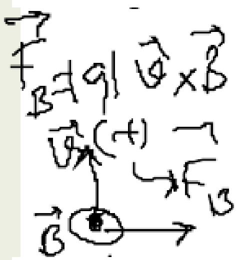


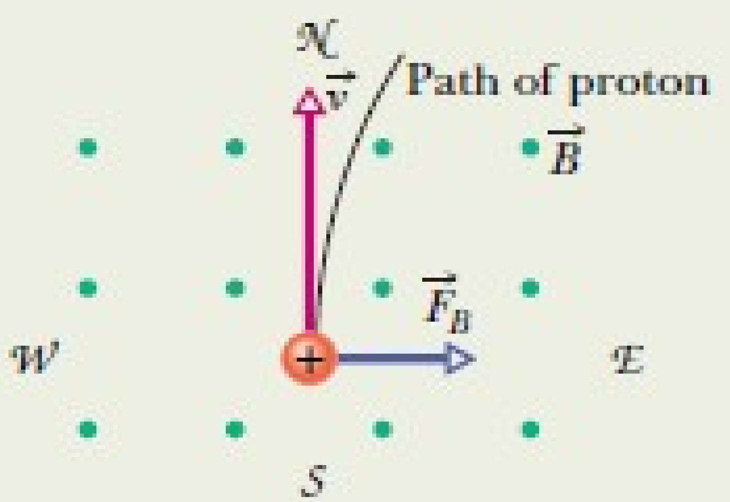
Opposite magnetic poles attract each other, and like magnetic poles repel each other.

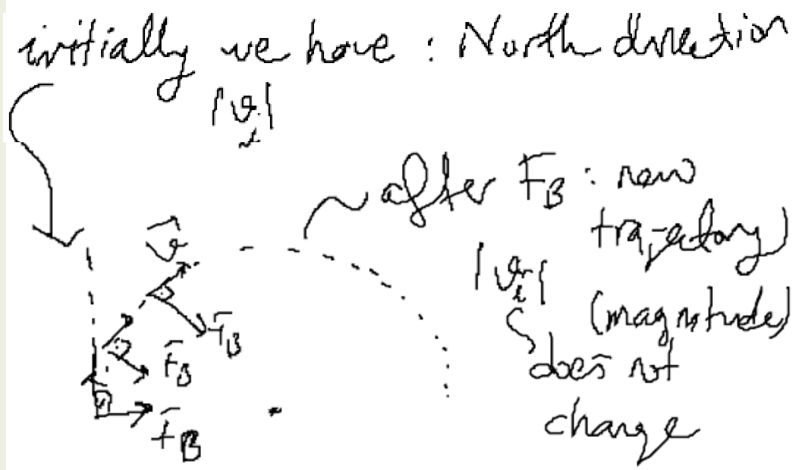
Sample Problem

Magnetic force on a moving charged particle

A uniform magnetic field $\vec{B}$, with magnitude 1.2 mT, is directed vertically upward throughout the volume of a laboratory chamber. A proton with kinetic energy 5.3 MeV enters the chamber, moving horizontally from south to north. What magnetic deflecting force acts on the proton as it enters the chamber? The proton mass is 1.67×10^{-27} kg. (Neglect Earth's magnetic field.)







A uniform magnetic field $\vec{B}$, with magnitude 1.2 mT, is directed vertically upward throughout the volume of a laboratory chamber. A proton with kinetic energy 5.3 MeV enters the chamber, moving horizontally from south to north. What magnetic deflecting force acts on the proton as it enters the chamber? The proton mass is 1.67×10^{-27} kg. (Neglect Earth's magnetic field.)

KEY IDEAS

Because the proton is charged and moving through a magnetic field, a magnetic force $\vec{F}_B$ can act on it. Because the initial direction of the proton's velocity is not along a magnetic field line, $\vec{F}_B$ is not simply zero.

Magnitude: To find the magnitude of $\vec{F}_B$, we can use Eq. 28-3 ($F_B = |q|vB \sin \phi$) provided we first find the proton's speed v. We can find v from the given kinetic energy because $K = \frac{1}{2}mv^2$. Solving for v, we obtain

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{(2)(5.3\text{ MeV})(1.60 \times 10^{-13}\text{ J/MeV})}{1.67 \times 10^{-27}\text{ kg}}} = 3.2 \times 10^7\text{ m/s}.$$

Equation 28-3 then yields

$$\begin{aligned} F_B &= |q|vB \sin \phi \\ &= (1.60 \times 10^{-19}\text{ C})(3.2 \times 10^7\text{ m/s}) \\ &\quad \times (1.2 \times 10^{-3}\text{ T})(\sin 90°) \\ &= 6.1 \times 10^{-15}\text{ N}. \end{aligned} \qquad \text{(Answer)}$$

This may seem like a small force, but it acts on a particle of small mass, producing a large acceleration; namely,

$$a = \frac{F_B}{m} = \frac{6.1 \times 10^{-15}\text{ N}}{1.67 \times 10^{-27}\text{ kg}} = 3.7 \times 10^{12}\text{ m/s}^2.$$

Direction: To find the direction of $\vec{F}_B$, we use the fact that $\vec{F}_B$ has the direction of the cross product $q\vec{v} \times \vec{B}$. Because the charge q is positive, $\vec{F}_B$ must have the same direction as $\vec{v} \times \vec{B}$, which can be determined with the right-hand rule for cross products (as in Fig. 28-2*d*). We know that $\vec{v}$ is directed horizontally from south to north and $\vec{B}$ is directed vertically up. The right-hand rule shows us that the deflecting force $\vec{F}_B$ must be directed horizontally from west to east, as Fig. 28-6 shows. (The array of dots in the figure represents a magnetic field directed out of the plane of the figure. An array of Xs would have represented a magnetic field directed into that plane.)

If the charge of the particle were negative, the magnetic deflecting force would be directed in the opposite direction—that is, horizontally from east to west. This is predicted automatically by Eq. 28-2 if we substitute a negative value for q.

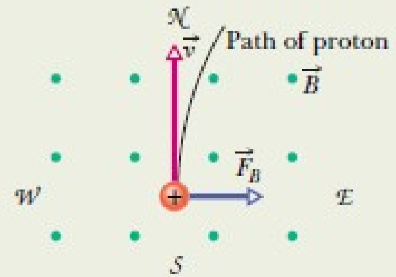
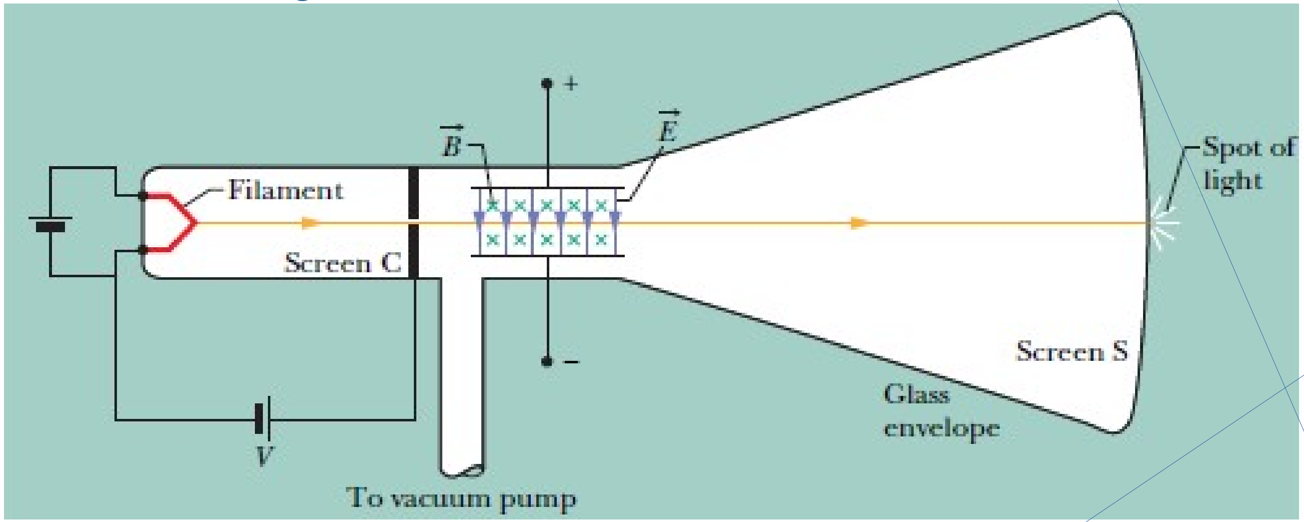


Fig. 28-6 An overhead view of a proton moving from south to north with velocity $\vec{v}$ in a chamber. A magnetic field is directed vertically upward in the chamber, as represented by the array of dots (which resemble the tips of arrows). The proton is deflected toward the east.

- Both a **E** and a **B** can produce a force on a charged particle.
- When the two fields are perpendicular to each other, they are said to be **crossed fields**. $B \perp E$
- What happens to charged particles as they move through crossed fields?

A modern version of apparatus for measuring the ratio of mass to charge for the electron.



- Thus, the crossed fields allow us to *measure the speed of the charged particles* passing through them.
- **Deflection** of the particle at the far end of the plates is

$$y = \frac{|q|EL^2}{2mv^2},$$

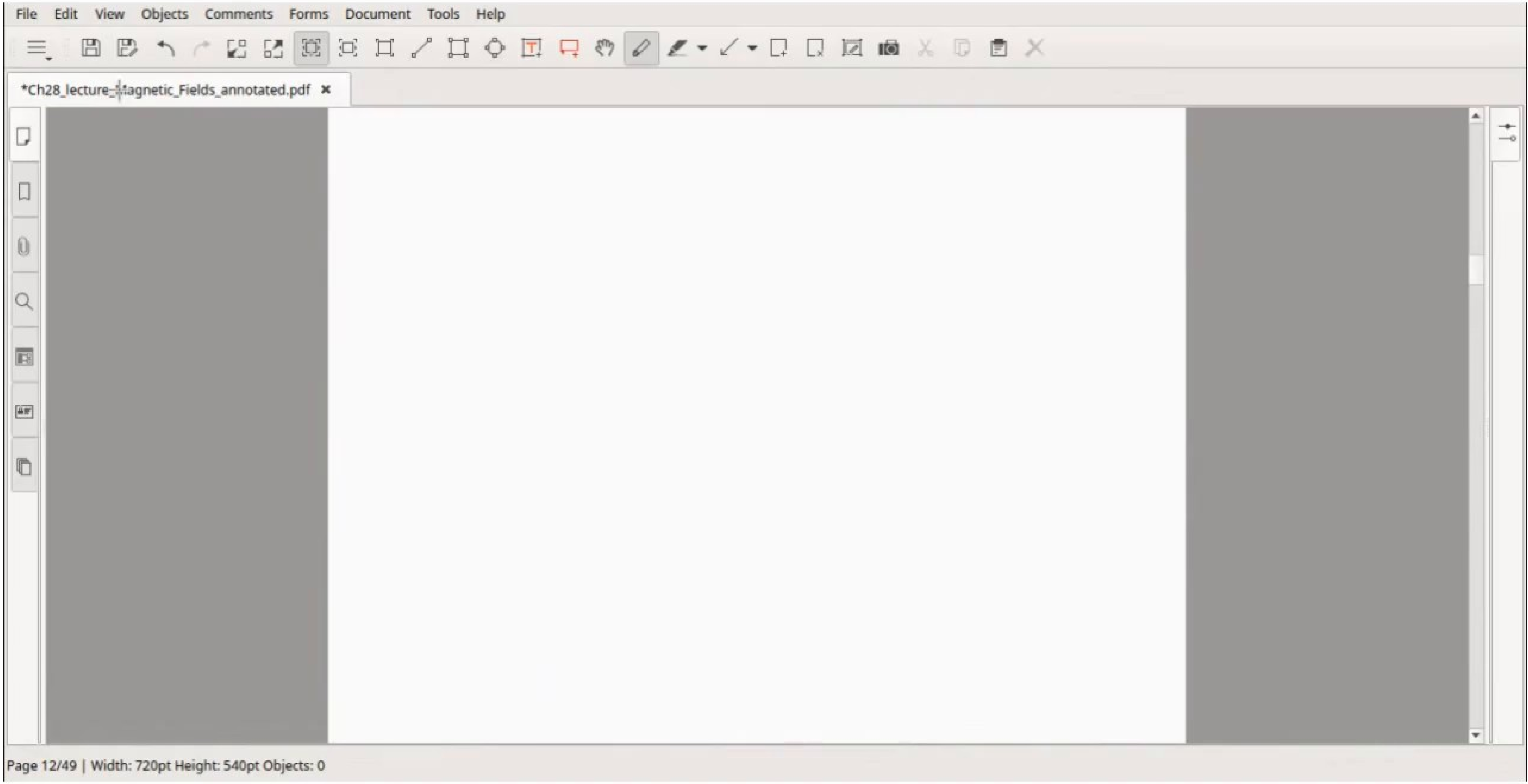
When the two fields in Fig. are adjusted so that the two deflecting forces cancel

$$|q|E = |q|vB\sin(90^\circ) = |q|vB$$

$$v = \frac{E}{B}.$$

$$\frac{m}{|q|} = \frac{B^2L^2}{2yE}$$

Video: Crossed Fields



- When a conducting strip carrying a current *i* is placed in a uniform magnetic field, some charge carriers (with charge *e*) **build up on one side** of the conductor, creating a potential difference *V* across the strip.
- *The polarities of the sides indicate the sign of the charge carriers.*

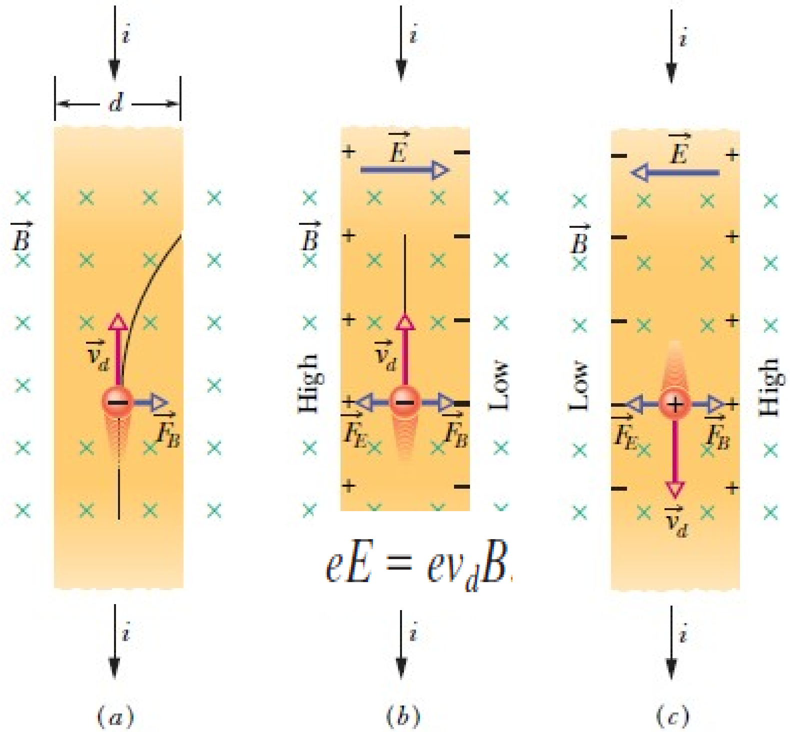


Fig. 28-8 A strip of copper carrying a current *i is immersed in a magnetic field . (a)The situation immediately after the* magnetic field is turned on. The curved path that will then be taken by an electron is shown. (*b*) *The situation at equilibrium.*

- A **Hall potential difference** *V* is associated with the electric field across strip width *d*, and the magnitude of that potential difference is $V = Ed$.
- When the electric and magnetic forces are in balance (Fig. 28-8*b*),
  - where v_d is the drift speed.

$$v_d = \frac{J}{ne} = \frac{i}{neA}$$

Where *J* is the current density, *A* the cross-sectional area, *e* the electronic charge, and *n* the number of charges per unit volume.

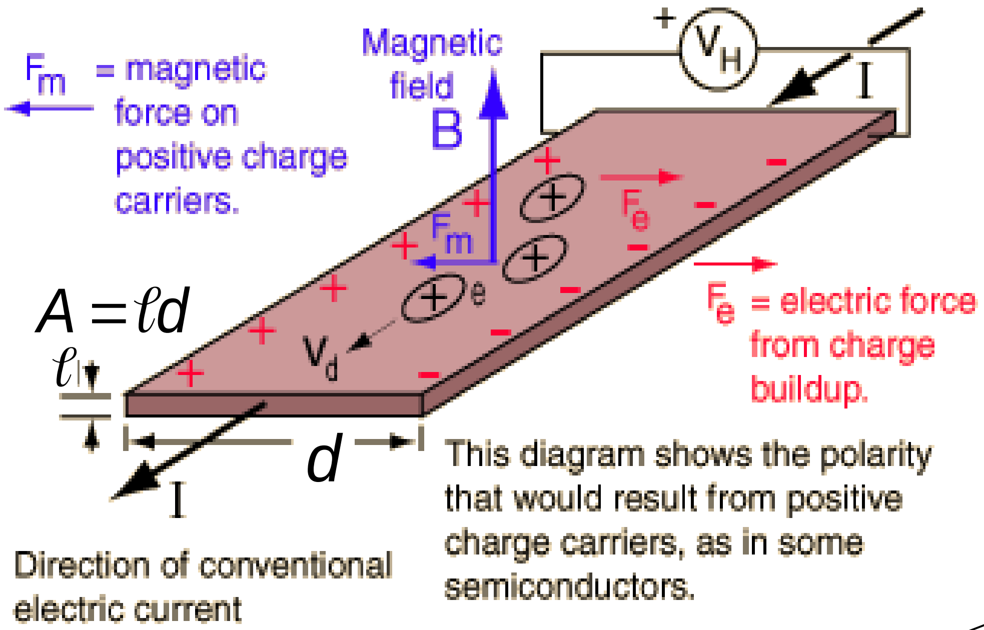
The **number density** of the charge carriers

$$n = \frac{Bi}{Vle},$$

Here, $l = (A/d)$, the thickness of the strip.

Fig. 28-8 (c) For the same current direction, if the charge carriers were positively charged, they would pile up on the right side, and the right side would be at the higher potential.

Hall Voltage For Positive Charge Carriers



The current expressed in terms of the drift velocity is:

$$I = neAv_d$$

$$F_{\text{mag}} = ev_d B$$

$$F_{\text{mag}} = \frac{eIB}{neA} = \frac{IB}{nA}$$

$$F_{\text{elec}} = eE = e\frac{V}{d} = e\frac{V\ell}{A}$$

At equilibrium:

$$F_{\text{mag}} = F_{\text{elec}}$$

$$\frac{IB}{nA} = e\frac{V\ell}{A}$$

$$V_{\text{Hall}} = \frac{IB}{ne\ell}$$

$V_{\text{Hall}} = +$ then carriers +

$V_{\text{Hall}} = -$ then carriers –

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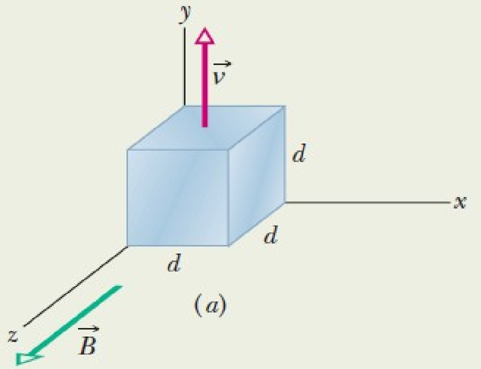
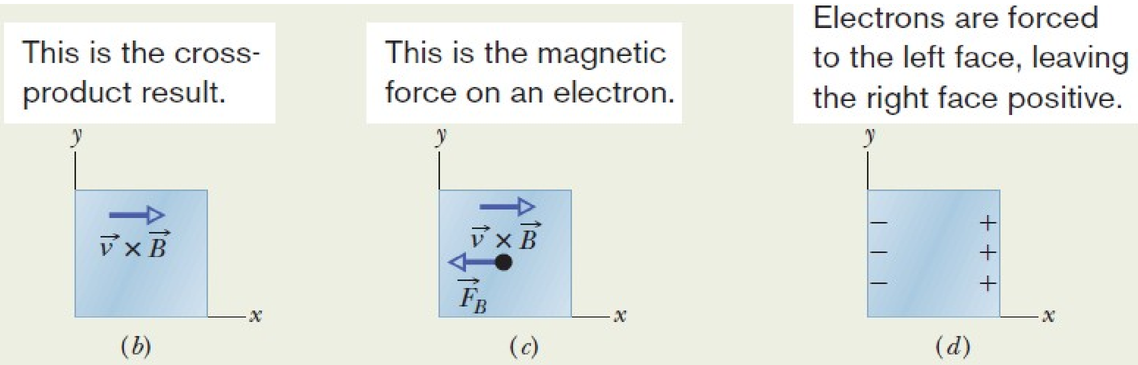
Example, Potential Difference Setup Across a Moving Conductor:





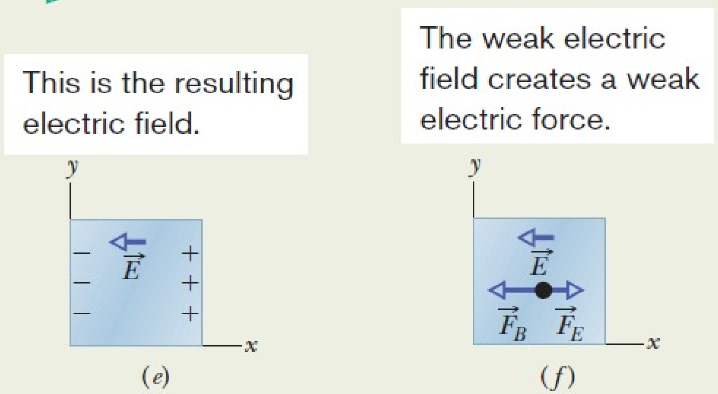


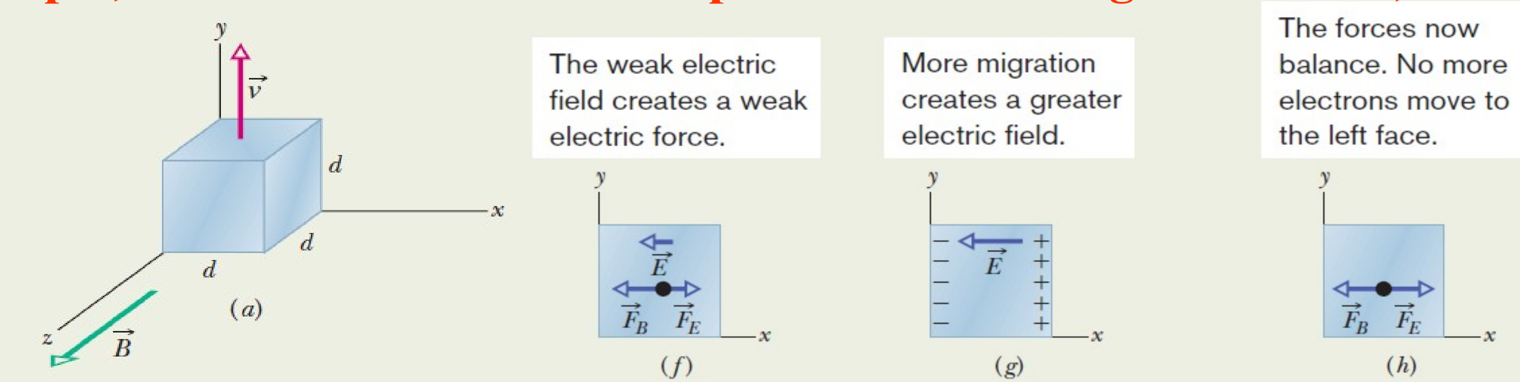
Figure 28-9*a* shows a solid metal cube, of edge length $d = 1.5$ cm, moving in the positive y direction at a constant velocity $\vec{v}$ of magnitude 4.0 m/s. The cube moves through a uniform magnetic field $\vec{B}$ of magnitude 0.050 T in the positive z direction.

(a) Which cube face is at a lower electric potential and which is at a higher electric potential because of the motion through the field?

Reasoning: When the cube first begins to move through the magnetic field, its electrons do also. Because each electron has charge q and is moving through a magnetic field with velocity $\vec{v}$, the magnetic force $\vec{F}_B$ acting on the electron is given by Eq. 28-2. Because q is negative, the direction of $\vec{F}_B$ is opposite the cross product $\vec{v} \times \vec{B}$, which is in the positive direction of the x axis (Fig. 28-9*b*). Thus, $\vec{F}_B$ acts in the negative direction of the x axis, toward the left face of the cube (Fig. 28-9*c*).

Most of the electrons are fixed in place in the atoms of the cube. However, because the cube is a metal, it contains conduction electrons that are free to move. Some of those conduction electrons are deflected by $\vec{F}_B$ to the left cube face, making that face negatively charged and leaving the right face positively charged (Fig. 28-9*d*). This charge separation produces an electric field $\vec{E}$ directed from the positively charged right face to the negatively charged left face (Fig. 28-9*e*). Thus, the left face is at a lower electric potential, and the right face is at a higher electric potential.

Example, Potential Difference Setup Across a Moving Conductor, cont.:



(b) What is the potential difference between the faces of higher and lower electric potential?

1. The electric field $\vec{E}$ created by the charge separation produces an electric force $\vec{F}_E = q\vec{E}$ on each electron (Fig. 28-9*f*). Because q is negative, this force is directed opposite the field $\vec{E}$—that is, rightward. Thus on each electron, $\vec{F}_E$ acts toward the right and $\vec{F}_B$ acts toward the left.

2. When the cube had just begun to move through the magnetic field and the charge separation had just begun, the magnitude of $\vec{E}$ began to increase from zero. Thus, the magnitude of $\vec{F}_E$ also began to increase from zero and was initially smaller than the magnitude $\vec{F}_B$. During this early stage, the net force on any electron was dominated by $\vec{F}_B$, which continuously moved additional electrons to the left cube face, increasing the charge separation (Fig. 28-9*g*).

3. However, as the charge separation increased, eventually magnitude F_E became equal to magnitude F_B (Fig. 28-9*h*). The net force on any electron was then zero, and no additional electrons were moved to the left cube face. Thus, the magnitude of $\vec{F}_E$ could not increase further, and the electrons were then in equilibrium.

Calculations: We seek the potential difference V between the left and right cube faces after equilibrium was reached (which occurred quickly). We can obtain V with Eq. 28-9 ($V = Ed$) provided we first find the magnitude E of the electric field at equilibrium. We can do so with the equation for the balance of forces ($F_E = F_B$).

For F_E, we substitute $|q|E$, and then for F_B, we substitute $|q|vB \sin\phi$ from Eq. 28-3. From Fig. 28-9*a*, we see that the angle ϕ between velocity vector $\vec{v}$ and magnetic field vector $\vec{B}$ is 90°; thus $\sin\phi = 1$ and $F_E = F_B$ yields

$$|q|E = |q|vB \sin 90° = |q|vB.$$

This gives us $E = vB$; so $V = Ed$ becomes

$$V = vBd. \qquad (28\text{-}13)$$

Substituting known values gives us

$$V = (4.0\ \text{m/s})(0.050\ \text{T})(0.015\ \text{m})$$
$$= 0.0030\ \text{V} = 3.0\ \text{mV}. \qquad \text{(Answer)}$$

28-6 A Circulating Charged Particle

- A charged particle with mass *m* and charge magnitude |*q*| moving with velocity perpendicular to a uniform magnetic field will travel in a circle.
- The magnetic force continuously deflects the particle, and since ***B*** and ***v*** are always <u>perpendicular</u> to each other, this deflection causes the particle to follow a circular path.
- Applying Newton's 2nd law to the circular motion yields

$$|q|vB = \frac{mv^2}{r}$$

$F_B = F_c$

$T = \frac{x}{v} \rightarrow \frac{1}{T} = \frac{v}{2\pi R}$

from which we find the radius *r* of the circle to be

$$r = \frac{mv}{|q|B}$$

- The frequency of revolution *f*, the angular frequency, and the period of the motion *T* are given by

$$f = \frac{\omega}{2\pi} = \frac{1}{T} = \frac{|q|B}{2\pi m}$$

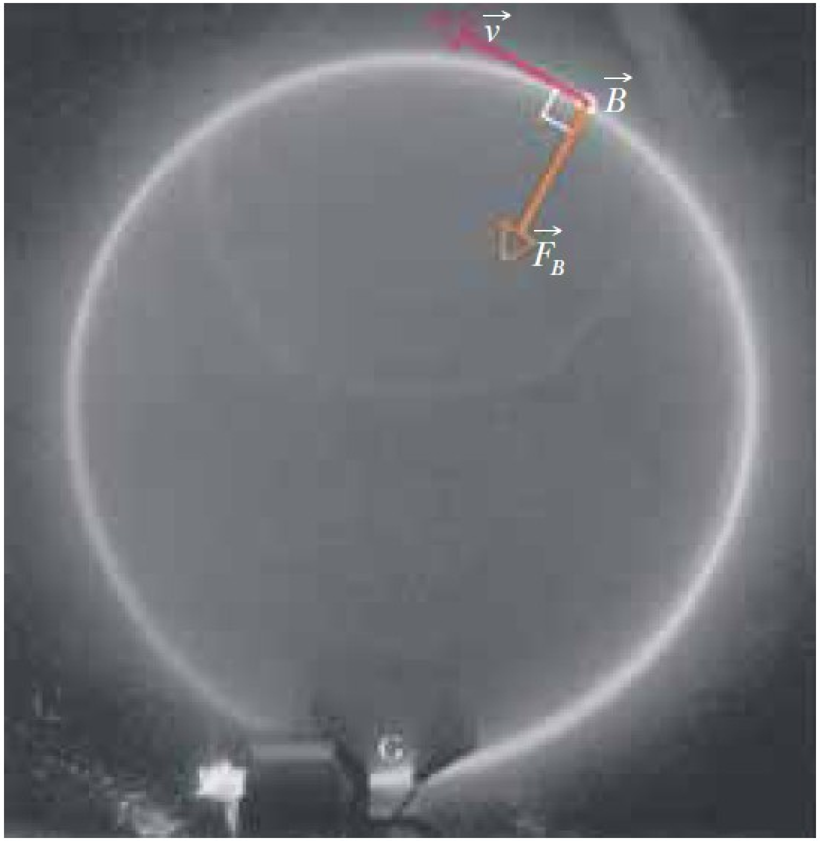


Fig. 28-10 Electrons circulating in a chamber containing gas at low pressure (their path is the glowing circle). A uniform magnetic field, B, pointing directly out of the plane of the page, fills the chamber. Note the radially directed magnetic force F_B ; for circular motion to occur, F_B must point toward the center of the circle, *(Courtesy John Le P. Webb, Sussex University, England)*

Example, Uniform Circular Motion of a Charged Particle in a Magnetic Field:

Figure 28-12 shows the essentials of a *mass spectrometer,* which can be used to measure the mass of an ion; an ion of mass m (to be measured) and charge q is produced in source S. The initially stationary ion is accelerated by the electric field due to a potential difference V. The ion leaves S and enters a separator chamber in which a uniform magnetic field $\vec{B}$ is perpendicular to the path of the ion. A wide detector lines the bottom wall of the chamber, and the $\vec{B}$ causes the ion to move in a semicircle and thus strike the detector. Suppose that $B = 80.000$ mT, $V = 1000.0$ V, and ions of charge $q = +1.6022 \times 10^{-19}$ C strike the detector at a point that lies at $x = 1.6254$ m. What is the mass m of the individual ions, in atomic mass units (Eq. 1-7: 1 u = 1.6605×10^{-27} kg)?

Finding speed: When the ion emerges from the source, its kinetic energy is approximately zero. At the end of the acceleration, its kinetic energy is $\frac{1}{2}mv^2$. Also, during the acceleration, the positive ion moves through a change in potential of $-V$. Thus, because the ion has positive charge q, its potential energy changes by $-qV$. If we now write the conservation of mechanical energy as

$$\Delta K + \Delta U = 0,$$

we get

$$\tfrac{1}{2}mv^2 - qV = 0$$

or

$$v = \sqrt{\frac{2qV}{m}}. \qquad (28\text{-}22)$$

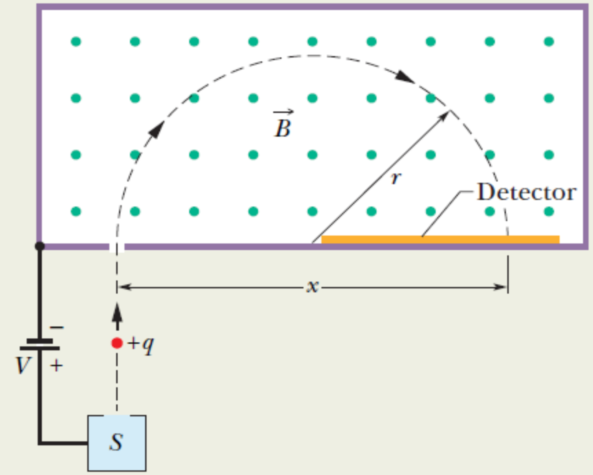


Fig. 28-12 Essentials of a mass spectrometer. A positive ion, after being accelerated from its source S by a potential difference V, enters a chamber of uniform magnetic field $\vec{B}$. There it travels through a semicircle of radius r and strikes a detector at a distance x from where it entered the chamber.

Finding mass: Substituting this value for v into Eq. 28-16 gives us

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \frac{1}{B}\sqrt{\frac{2mV}{q}}.$$

Thus,

$$x = 2r = \frac{2}{B}\sqrt{\frac{2mV}{q}}.$$

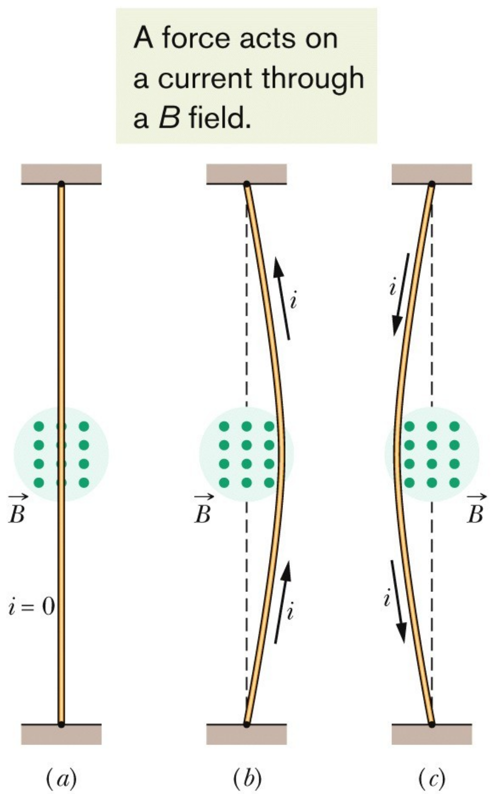
Solving this for m and substituting the given data yield

$$m = \frac{B^2qx^2}{8V}$$

$$= \frac{(0.080000\ \text{T})^2(1.6022 \times 10^{-19}\ \text{C})(1.6254\ \text{m})^2}{8(1000.0\ \text{V})}$$

$$= 3.3863 \times 10^{-25}\ \text{kg} = 203.93\ \text{u}. \qquad \text{(Answer)}$$

28-8 Magnetic Force on a Current-Carrying Wire



A flexible wire passes between the pole faces of a magnet (only the farther pole face is shown). (a) Without current in the wire, the wire is **straight**. (b) With upward current, the wire is deflected **rightward**. (c) With downward current, the deflection is **leftward**.

A straight wire carrying a current *i* in a uniform magnetic field experiences a sideways force

$$\vec{F}_B = i\vec{L} \times \vec{B}.$$

The force acting on a current element in a magnetic field is

$$d\vec{F}_B = i\, d\vec{L} \times \vec{B}.$$

The direction of the length vector **L** or **dL** is that of the current **i**

$$\vec{F}_B = q\vec{v} \times \vec{B} = i t v B = i x B$$

$$\vec{F}_B = i\vec{l} \times \vec{B}$$

$$i = \frac{\Delta q}{\Delta t} = \frac{q}{t}$$

$$v = \frac{\Delta x}{\Delta t} = \frac{x}{t}$$

Consider a length L of the wire in the figure.

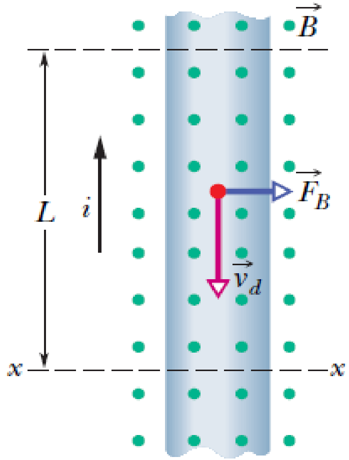


Fig. 28-15 A close-up view of a section of the wire of Fig. 28-14*b*. The current direction is upward, which means that electrons drift downward. A magnetic field that emerges from the plane of the page causes the electrons and the wire to be deflected to the right.

- All the conduction electrons in this section of wire will drift past plane *xx* in a time $t = L/v_d$.
- Thus, in that time a charge will pass through that plane that is given by

$$q = it = i\frac{L}{v_d}$$

$$F_B = qv_dB\sin\phi = \frac{iL}{v_d}v_dB\sin 90^\circ$$

$$F_B = iLB.$$

$$\vec{F}_B = i\vec{L} \times \vec{B} \qquad \text{(force on a current).}$$

- Here ***L*** is a length vector that has magnitude *L* and is directed along the wire segment in the direction of the (conventional) current.
- If a wire is not straight or the field is not uniform, we can imagine the wire broken up into small straight segments.
  - The force on the wire as a whole is then the vector sum of all the forces on the segments that make it up.
  - In the differential limit, we can write $d\vec{F}_B = i\,d\vec{L} \times \vec{B},$ and we can find the resultant force on any given arrangement of currents by integrating Eq. 28-28 over that arrangement.

28-8 Magnetic Force on a Current-Carrying Wire

Sample Problem

Magnetic force on a wire carrying current

A straight, horizontal length of copper wire has a current $i = 28$ A through it. What are the magnitude and direction of the minimum magnetic field $\vec{B}$ needed to suspend the wire—that is, to balance the gravitational force on it? The linear density (mass per unit length) of the wire is 46.6 g/m. m/l

$F_B = F_g$

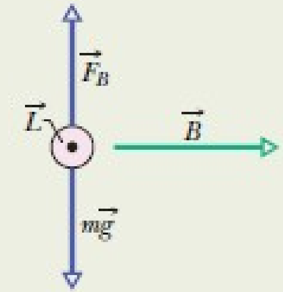


Fig. 28-17 A wire (shown in cross section) carrying current out of the page.

KEY IDEAS

(1) Because the wire carries a current, a magnetic force $\vec{F}_B$ can act on the wire if we place it in a magnetic field $\vec{B}$. To balance the downward gravitational force $\vec{F}_g$ on the wire, we want $\vec{F}_B$ to be directed upward (Fig. 28-17). (2) The direction of $\vec{F}_B$ is related to the directions of $\vec{B}$ and the wire's length vector $\vec{L}$ by Eq. 28-26 ($\vec{F}_B = i\vec{L} \times \vec{B}$).

Calculations: Because $\vec{L}$ is directed horizontally (and the current is taken to be positive), Eq. 28-26 and the right-hand rule for cross products tell us that $\vec{B}$ must be horizontal and rightward (in Fig. 28-17) to give the required upward $\vec{F}_B$.

The magnitude of $\vec{F}_B$ is $F_B = iLB \sin\phi$ (Eq. 28-27). Because we want $\vec{F}_B$ to balance $\vec{F}_g$, we want

$$iLB \sin\phi = mg, \qquad (28\text{-}29)$$

where mg is the magnitude of $\vec{F}_g$ and m is the mass of the wire.

We also want the minimal field magnitude B for $\vec{F}_B$ to balance $\vec{F}_g$. Thus, we need to maximize $\sin\phi$ in Eq. 28-29. To do so, we set $\phi = 90°$, thereby arranging for $\vec{B}$ to be perpendicular to the wire. We then have $\sin\phi = 1$, so Eq. 28-29 yields

$$B = \frac{mg}{iL\sin\phi} = \frac{(m/L)g}{i}. \qquad (28\text{-}30)$$

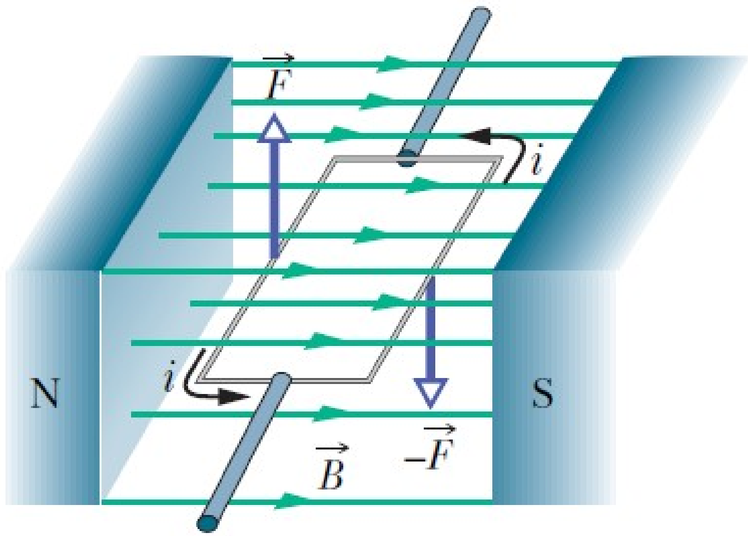
We write the result this way because we know m/L, the linear density of the wire. Substituting known data then gives us

$$B = \frac{(46.6 \times 10^{-3}\ \text{kg/m})(9.8\ \text{m/s}^2)}{28\ \text{A}}$$

$$= 1.6 \times 10^{-2}\ \text{T}. \qquad \text{(Answer)}$$

This is about 160 times the strength of Earth's magnetic field.

The elements of an electric motor

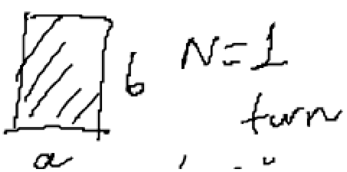


The elements of an electric motor:

- A rectangular loop of wire, carrying a current and free to rotate about a fixed axis, is placed in a magnetic field.
- Magnetic forces on the wire produce a torque that rotates it.
  - The two magnetic forces ***F*** and –***F*** produce a torque on the loop, tending to rotate it about its central axis.
- A commutator (not shown) reverses the direction of the current every half-revolution so that the torque always acts in the same direction (FYI).

- **Magnitude of the total torque on the coil (of area A and N turns):**

$$\tau = N\tau' = NiabB\sin\theta = (NiA)B\sin\theta,$$



$F_B = ibB\sin\varphi$, $\varphi = 90$ $\tau = rF_B = aibB\sin\theta$ $\tau = (iA)B = \mu B$

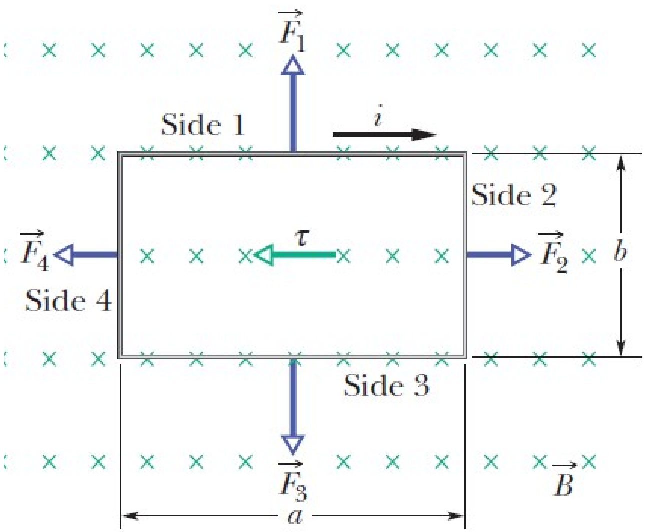
μ:magnetic moment

28-9 Torque on a Current Loop

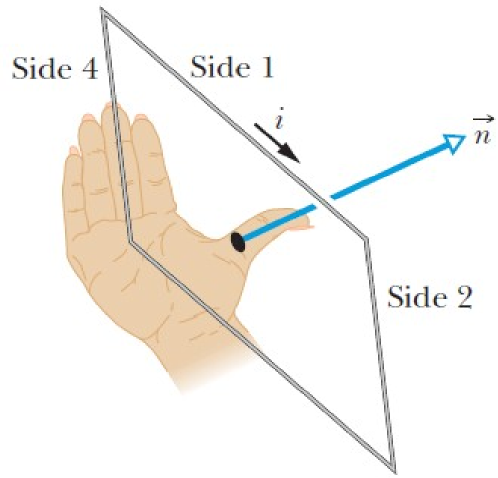
- A rectangular loop, of length *a* and width *b* and carrying a current *i*, is located in a uniform magnetic field. A torque acts to align the normal vector with the direction of the field.
- For side 2 the magnitude of the force acting on this side is $F_2 = ibB \sin(90° - \theta) = ibB \cos\theta = F_4$. → F_2 and F_4 cancel out exactly.
- Forces F_1 and F_3 have the common magnitude *iaB.* As Fig. 28-19*c* shows, these two forces do not share the same line of action; so they produce a net torque.

$$\tau' = \left(iaB\frac{b}{2}\sin\theta\right) + \left(iaB\frac{b}{2}\sin\theta\right) = \boxed{iabB\sin\theta.}$$

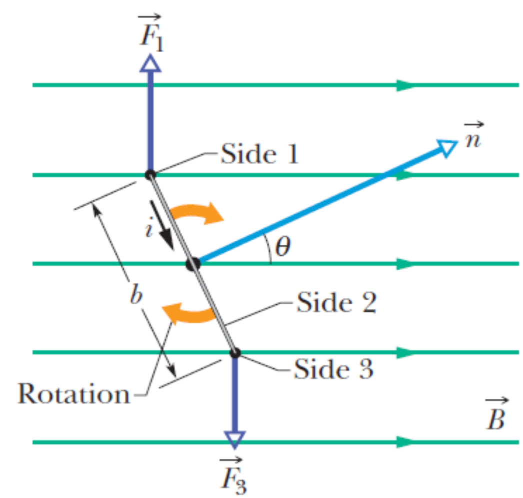
FYI



The loop as seen by looking in the direction of the magnetic field



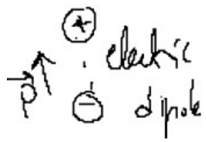
A perspective of the loop showing how the right-hand rule gives the direction of , which is perpendicular to the plane of the loop



A side view of the loop, from side 2. The loop rotates as indicated.

28-10 The Magnetic Dipole Moment: μ

- The coil behaves like a bar magnet placed in the magnetic field.
- Thus, like a bar magnet, a current-carrying coil is said to be a *magnetic dipole*.
- Assign a **magnetic dipole moment**



The magnetic moment vector attempts to align with the magnetic field.

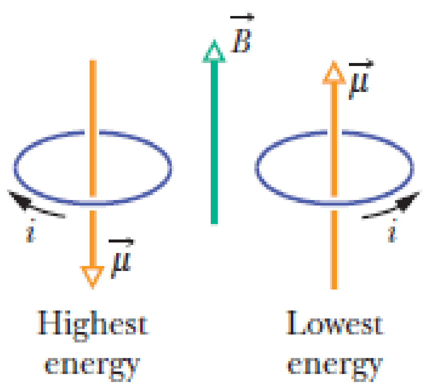


Fig. 28-20 The orientations of highest and lowest energy of a magnetic dipole (here a coil carrying current) in an external magnetic field $\vec{B}$. The direction of the current i gives the direction of the magnetic dipole moment $\vec{\mu}$ via the right-hand rule shown for $\vec{n}$ in Fig. 28-19*b*.

MAGNETIC DIPOLE MOMENT

- The direction: grasp the coil with the fingers of your right hand in the direction of current *i*; the outstretched thumb of that hand gives the direction (RHR).
- The magnitude of is given by

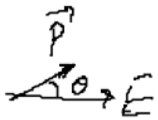
$$\mu = NiA \qquad \text{(magnetic moment).}$$

N: number of turns in the coil
i: current through the coil
A: area enclosed by each turn of the coil

- Then the torque on the coil due to a magnetic field can be written as:

$$\vec{\tau} = \vec{\mu} \times \vec{B},$$

$$\vec{p} \times \vec{E} = \vec{\tau}_E$$
$$-\vec{p} \cdot \vec{E} = U_E$$



- A magnetic dipole in an external magnetic field has an energy that depends on the dipole's orientation in the field.

U(θ)

28-10 The Magnetic Dipole Moment: μ

- The orientation energy of a magnetic dipole in a magnetic field is:

$$U(\theta) = -\vec{\mu} \cdot \vec{B}.$$

- If an external agent rotates a magnetic dipole from an initial orientation U_i to some other orientation U_f and the dipole is stationary both initially and finally, the work W_a done on the dipole by the agent is

$$W_a = \Delta U = U_f - U_i.$$

$$\mu = NiA \qquad \text{(magnetic moment)},$$

$$U(\theta) = -\vec{\mu} \cdot \vec{B}.$$

From the above equations, one can see that the unit of $\boldsymbol{\mu}$ can be the joule per tesla (J/T), or the ampere-square meter.

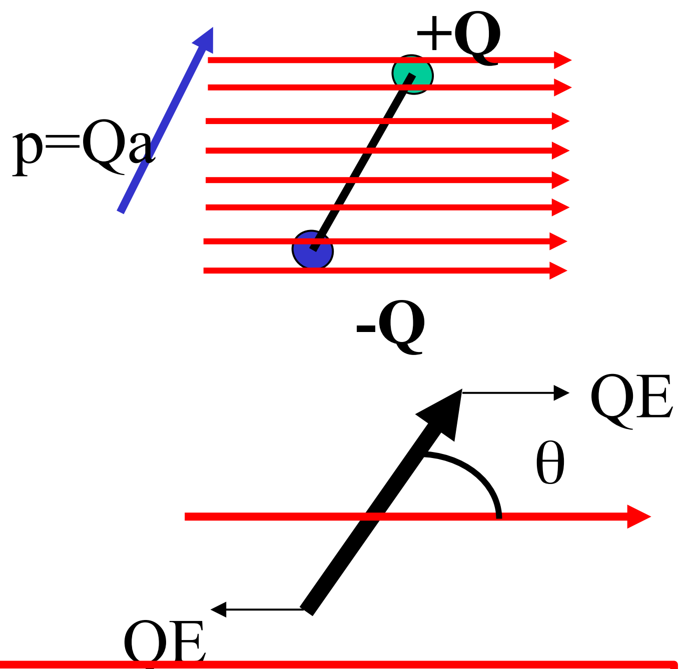
Table 28-2

Some Magnetic Dipole Moments

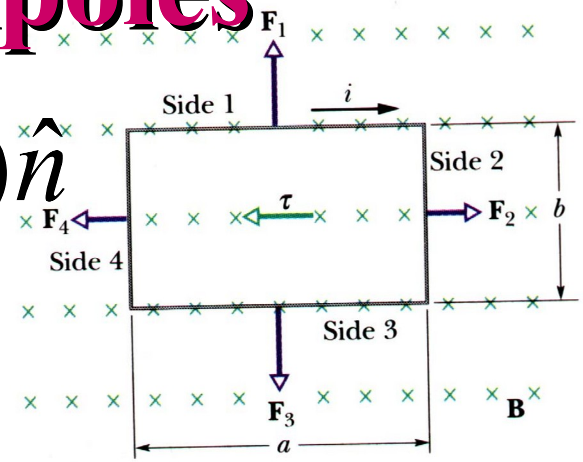
Small bar magnet	5 J/T
Earth	8.0×10^{22} J/T
Proton	1.4×10^{-26} J/T
Electron	9.3×10^{-24} J/T

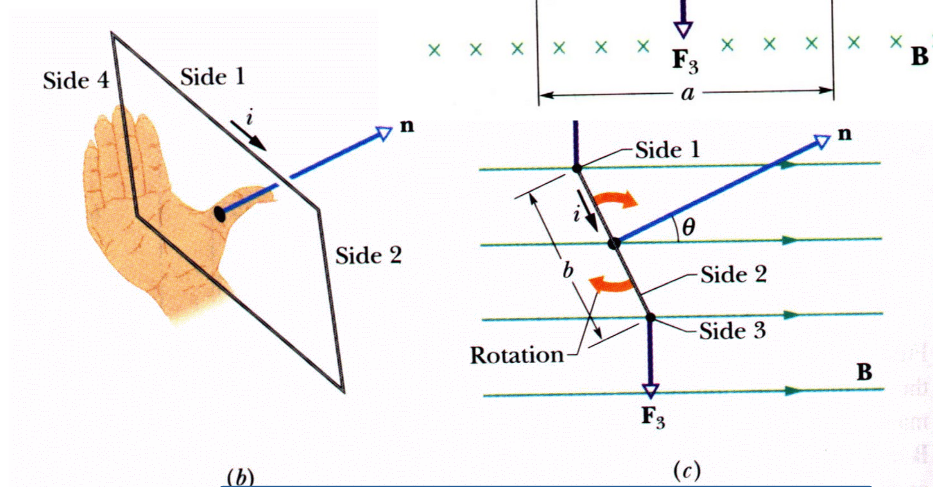
μ ↑↑ ↑↑ ⇝ magnet: Fe, Ni

Electric vs. Magnetic Dipoles



$$\vec{\mu} = (NiA)\hat{n}$$





$$\vec{\tau}_E = \vec{p} \times \vec{E}$$

$$U_E = -\vec{p} \cdot \vec{E}$$

$$\vec{\tau}_B = \vec{\mu} \times B$$

$$U_B = -\vec{\mu} \cdot \vec{B}$$

http://www.phys.lsu.edu/~jdowling/PHYS21132-SP15/lectures/index.html

Sample Problem

Rotating a magnetic dipole in a magnetic field

Figure 28-21 shows a circular coil with 250 turns, an area A of 2.52×10^{-4} m^2, and a current of 100 μA. The coil is at rest in a uniform magnetic field of magnitude $B = 0.85$ T, with its magnetic dipole moment $\vec{\mu}$ initially aligned with $\vec{B}$.

(a) In Fig. 28-21, what is the direction of the current in the coil?

Right-hand rule: Imagine cupping the coil with your right hand so that your right thumb is outstretched in the direction of $\vec{\mu}$. The direction in which your fingers curl around the coil is the direction of the current in the coil. Thus, in the wires on the near side of the coil—those we see in Fig. 28-21—the current is from top to bottom.

(b) How much work would the torque applied by an external agent have to do on the coil to rotate it 90° from its initial orientation, so that $\vec{\mu}$ is perpendicular to $\vec{B}$ and the coil is again at rest?

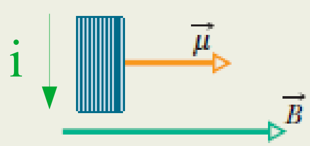


Fig. 28-21 A side view of a circular coil carrying a current and oriented so that its magnetic dipole moment is aligned with magnetic field $\vec{B}$.

KEY IDEA

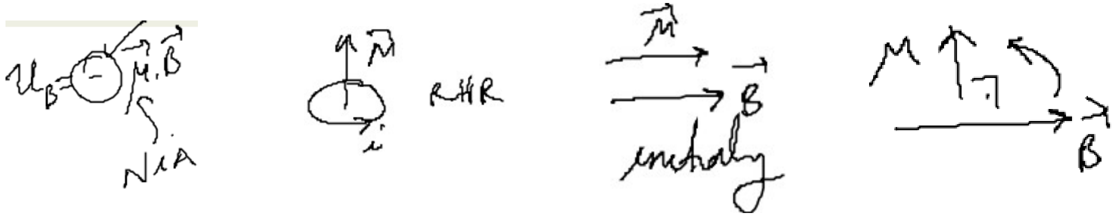
The work W_a done by the applied torque would be equal to the change in the coil's orientation energy due to its change in orientation.

Calculations: From Eq. 28-39 ($W_a = U_f - U_i$), we find

$$W_a = U(90°) - U(0°)$$
$$= -\mu B \cos 90° - (-\mu B \cos 0°) = 0 + \mu B$$
$$= \mu B.$$

Substituting for μ from Eq. 28-35 ($\mu = NiA$), we find that

$$W_a = (NiA)B$$
$$= (250)(100 \times 10^{-6}\ \text{A})(2.52 \times 10^{-4}\ \text{m}^2)(0.85\ \text{T})$$
$$= 5.355 \times 10^{-6}\ \text{J} \approx 5.4\ \mu\text{J}. \qquad \text{(Answer)}$$



(+) should be given into the system to have a rotation

1. A proton moves through a uniform magnetic field given by **B**=(10**i**-20**j**+30**k**)mT. At time t_1, the proton has a velocity given by **v**=v_x**i**+v_y**j**+(2.0 km/s)**k** and the magnetic force on the proton is $\mathbf{F}_B$=(4.0x10^{-17} N)**i**+(2.0x10^{-17} N)**j**. At that instant, what are (a) v_x and (b) v_y? no B(r) or B(t) dependence

1(6) proton

$\vec{B}=(10\hat{i}-20\hat{j}+30\hat{k})\,mT$: uniform

at t_1: $\vec{v}_p = v_x\hat{i}+v_y\hat{j}+2\times10^{3}\,m/s\,\hat{k}$

$\vec{F}_p = 4\times10^{-17}N\hat{i}+2\times10^{-17}N\hat{j}$

$1.6\times10^{-19}C$ $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & 2\times10^{3}m/s \\ 10mT & -20mT & 30mT \end{vmatrix}$

$\vec{F}=q\vec{v}\times\vec{B}$ $\rightarrow 1.6\times10^{-19}c\left[\hat{i}(30\times10^{-3}v_y+(20\times10^{-3})(2\times10^{3}))-\hat{j}(30\times10^{-3}v_x-10\times10^{-3}\times2\times10^{3})\right.$ $\left.+\hat{k}(-20\times10^{-3}v_x-10\times10^{-3}v_y)\right]=4\times10^{-17}N\hat{i}+2\times10^{-17}N\hat{j}$

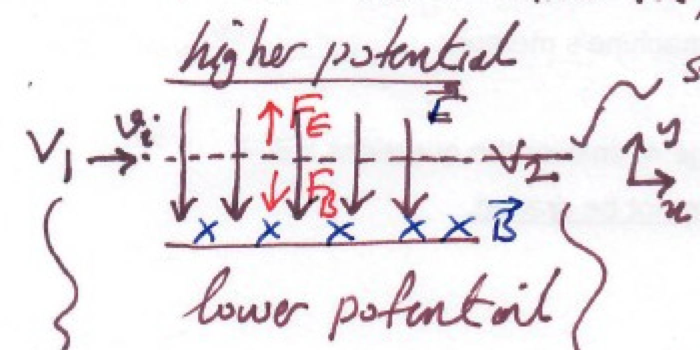
$-20\times10^{-3}v_x-10\times10^{-3}v_y = 0 \rightarrow v_x=-v_y/2$

$\rightarrow 30\times10^{-3}v_y+40=250$

$v_y=7\times10^{3}\,m/s$ | $v_x=-3.5\times10^{3}\,m/s$

2. In Figure, an electron accelerated from rest through potential difference $V_1 = 1.00$ kV enters the gap between two parallel plates having separation $d = 20.0$ mm and potential difference $V_2 = 100$ V. The lower plate is at the lower potential. Neglect fringing and assume that the electron's velocity vector is perpendicular to the electric field vector between the plates. In unit-vector notation, what uniform magnetic field allows the electron to travel in a straight line in the gap? B → for keeping horizontal motion → NO DEFLECTION

2(g) $V_1 = 1$ kV & $d = 20\times10^{-3}$ m, $V_2 = 100$ V, $m_e = 9.11\times10^{-31}$ kg



straight line $\Rightarrow |\vec{F}_B| = |\vec{F}_E|$

$|q| v_i B = |q| E$

$\sqrt{\frac{2qV_1}{m_e}}\, B = \frac{V_2}{d}$

$\Delta U = qV_1 - 0$

$= (1.6\times10^{-19}\,C)\, 1\times10^{3}\,V$

$\Delta U = \Delta K = \frac{1}{2} m_e v_i^2$

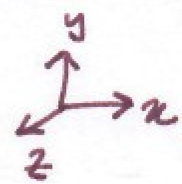
$V_2 = Ed$

$\Rightarrow E = \frac{V_2}{d}$

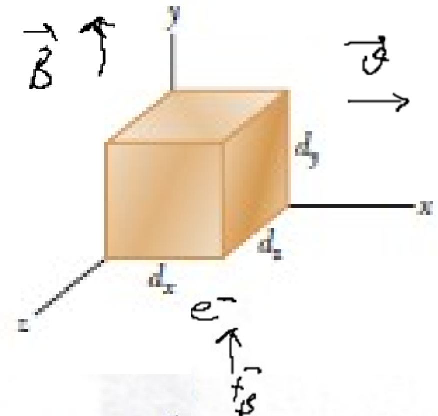
$\rightarrow B = \frac{100\,V}{20\times10^{-3}\,m} \sqrt{\frac{9.11\times10^{-31}\,kg}{2\times1.6\times10^{-19}\,C\; 1\times10^{3}\,V}}$

$B = 2.67\times10^{-4}\,T$

$\vec{B} = 2.67\times10^{-4}\,T\,(-\hat{k})$



3. In Figure, a conducting rectangular solid of dimensions d_x=5.00 m, d_y=3.00 m, and d_z=2.00 m moves at constant velocity **v**=(20.0 m/s)**i** through a uniform magnetic field **B**=(30.0 mT)**j.** What are the resulting (a) electric field within the solid, in unit-vector notation, and (b) potential difference across the solid?



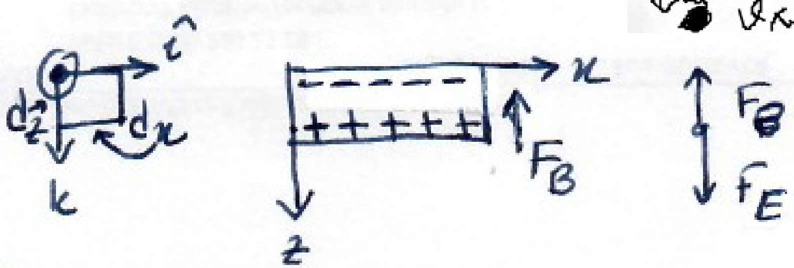
for e^- ~ f_B? $\vec{v} \times \vec{B}$

conducting solid

$d_x = 5\,m$
$d_y = 3\,m$
$d_z = 2\,m$

$\vec{v} = 20\,m/s\,\hat{i}$
$\vec{B} = 30\times10^{-3}\,T\,\hat{j}$

i)



when they are equal

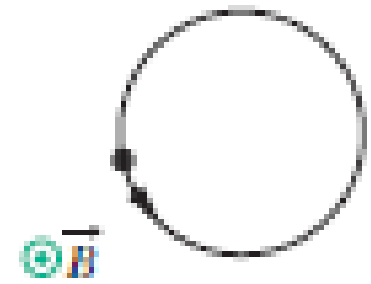
$qE = qvB$

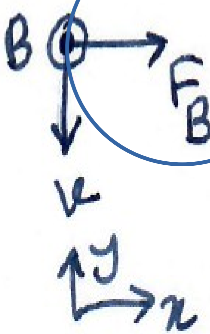
$E = vB$

$\rightsquigarrow |\vec{E}| = v|\vec{B}| = (20\,m/s)(30\times10^{-3}\,T) = 0.6\,V/m \rightsquigarrow \vec{E} = (0.6\,V/m)(-\hat{k})$

ii) $V = Ed \rightsquigarrow V = (0.6\,V/m)\,2m = 1.2\,V$

4. In Figure, a particle moves along a circle in a region of uniform magnetic field of magnitude B=4.00 mT. The particle is either a proton or an electron (you must decide which). It experiences a magnetic force of magnitude 3.20×10^{-15} N. What are (a) the particle's speed, (b) the radius of the circle, and (c) the period of the motion?





It should be electron!

$B = 4\times10^{-3}\,T$

$F_B = 3.2\times10^{-15}\,N$

i) $\vec{F} = q\,\vec{v}\times\vec{B} \Rightarrow 3.2\times10^{-15}\,N = 1.6\times10^{-19}\,C\,|v|\,4\times10^{-3}\,T$

$\rightarrow |v| = 5\times10^{6}\,m/s$

ii) $\frac{mv^2}{R} = F \rightarrow \frac{(9.1\times10^{-31}\,kg)(5\times10^{6}\,m/s)^2}{3.2\times10^{-15}\,N} = R = 7.12\times10^{-3}\,m$

$F_c = F_B$

iii) $T = \frac{2\pi R}{v} = \frac{2\pi(7.12\times10^{-3}\,m)}{5\times10^{6}\,m/s} = 8.9\times10^{-9}\,s$

5. Mass Ratio

Mass ratio

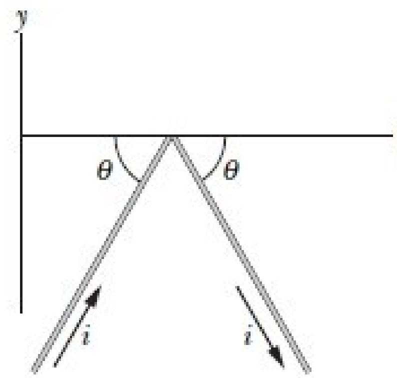
Particle A with charge q and mass m_A and particle B with charge $2q$ and mass m_B, are accelerated from rest by a potential difference ΔV, and subsequently deflected by a uniform magnetic field into semicircular paths. The radii of the trajectories by particle A and B are R and $2R$, respectively. The direction of the magnetic field is perpendicular to the velocity of the particle. What is their mass ratio?

$$KE = \Delta U \rightarrow \frac{1}{2} m v^2 = q \Delta V \Rightarrow v = \sqrt{\frac{2q\Delta V}{m}}$$

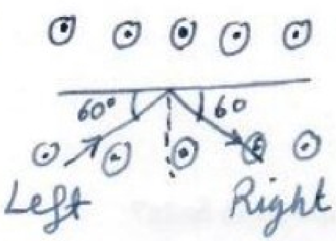
$$\frac{mv^2}{r} = qvB \rightarrow r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2q\Delta V}{m}} = \frac{1}{B}\sqrt{\frac{2m\Delta V}{q}} \rightarrow r \propto \sqrt{\frac{m}{q}}$$

$$\frac{r_A}{r_B} = \sqrt{\frac{m_A/q_A}{m_B/q_B}} \Rightarrow \frac{R}{2R} = \sqrt{\frac{m_A/q}{m_B/2q}} \Rightarrow \boxed{\frac{m_A}{m_B} = \frac{1}{8}}$$

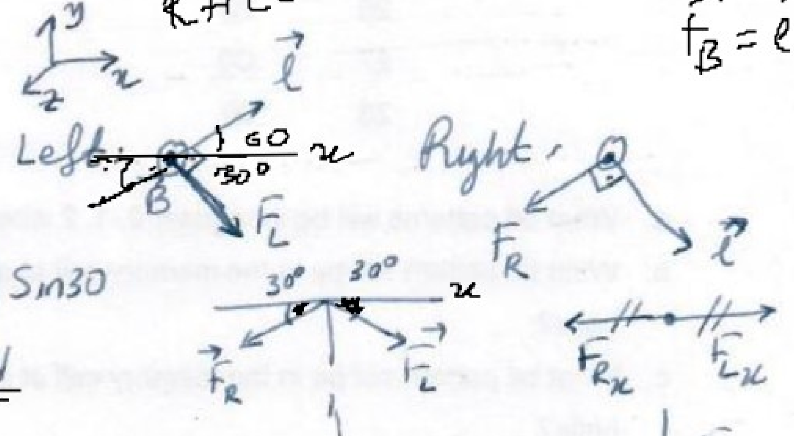
6. The bent wire shown in Figure lies in a uniform magnetic field. Each straight section is 2.0 m long and makes an angle of $\theta = 60°$ with the x axis, and the wire carries a current of 2.0 A. What is the net magnetic force on the wire in unit-vector notation if the magnetic field is given by (a) 4.0**k** T and (b) 4.0**i** T



6 (43) i) $\vec{B} = 4\hat{k}$



RHCS $\quad \vec{F}_B = i\,\vec{\ell} \times \vec{B}$



$F_{R_y} = |\vec{F}_R| \sin 30 = (i\,L|\vec{B}|\sin 90) \sin 30$

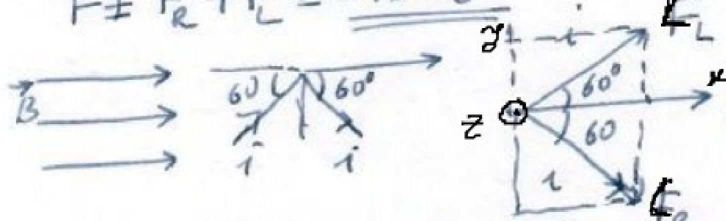
$= (2A)(2m)(4T)\,1\,\frac{1}{2} = 8\,N$

$F_{L_y} = 8N \rightarrow F_y = 16\,N$

$\vec{F} = \vec{F}_R + \vec{F}_L = -16N\,\hat{j}$

ℓ: length

ii) $\vec{B} = 4\hat{i}$



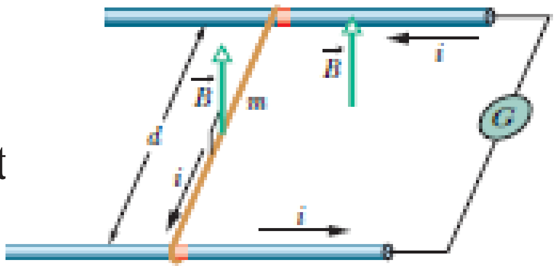
$F_{Lx} \parallel \vec{B}$, $F_{Rx} \parallel \vec{B}$ } 0 $\quad F_{Ly} \perp \vec{B}$, $F_{Ry} \perp \vec{B}$

$\vec{F} = \vec{F}_L + \vec{F}_R$

$= i\,\ell |B| \sin 60 (-\hat{k}) + i\,\ell |B| \sin 60 (\hat{k}) = 0$

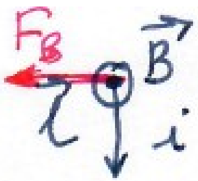
no magnetic force for $B \rightarrow B_x$

7. In Figure, a metal wire of mass m=24.1mg can slide with negligible friction on two horizontal parallel rails separated by distance d=2.56 cm. The track lies in a vertical uniform magnetic field of magnitude 56.3 mT. At time t=0, device G is connected to the rails, producing a constant current i=9.13 mA in the wire and rails (even as the wire moves). At t=61.1 ms, what are the wire's (a) speed and (b) direction of motion (left or right)?



$\times 10$ m/s

$m = 24.1\times10^{-6}$ kg
$d = 2.56\times10^{-2}$ m
$B = 56.3\times10^{-3}$ T
$i = 9.13\times10^{-3}$ A
$t = 61.1\times10^{-3}$ s



i) $\vec{F}_B = i\vec{l}\times\vec{B} \sim |F_B| = i\,l\,B\sin 90^\circ$

$= (9.13\times10^{-3}\,A)(2.56\times10^{-2}\,m)(56.3\times10^{-3}\,T)$

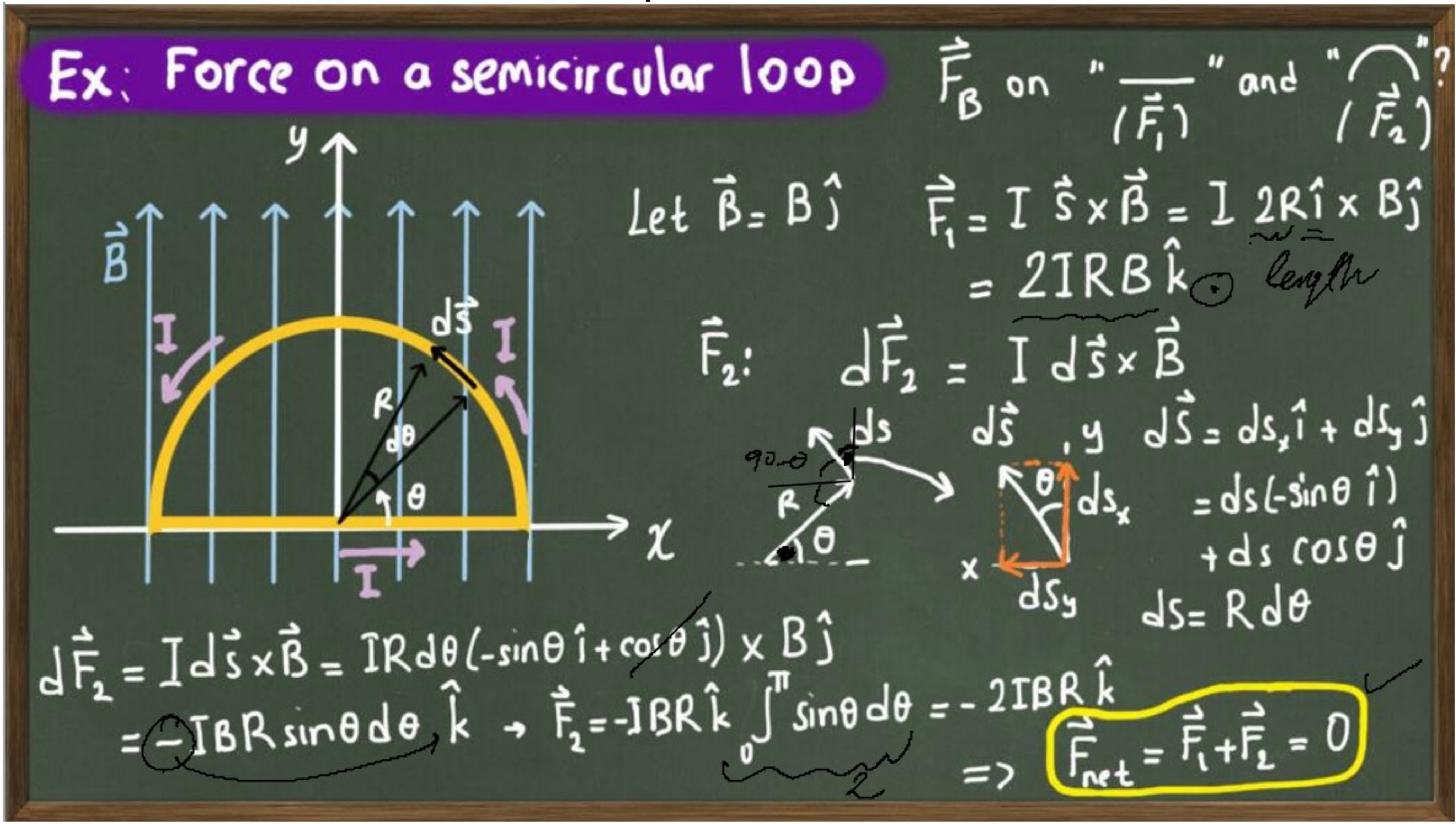
$= 1.32\times10^{-5}\,N$

$F_B = ma \sim a = \dfrac{1.32\times10^{-5}\,N}{24.1\times10^{-6}\,kg} = 5.46\times10^{-1}\,m/s^2$

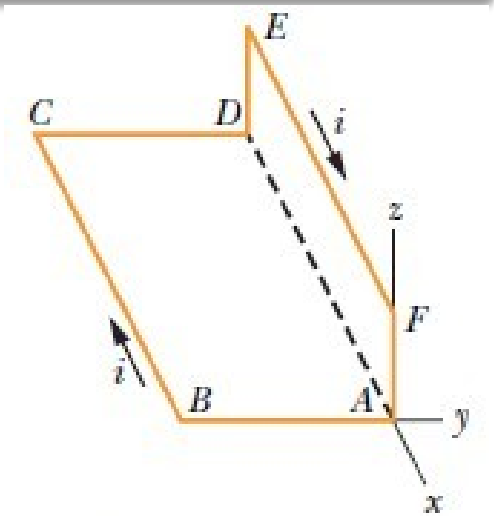
$\sim v = v_0^{\,0} + at = (5.46\times10^{-1}\,m/s^2)(61.1\times10^{-3}\,s) = 33.5\times10^{-3}\,m/s$

ii) Left.

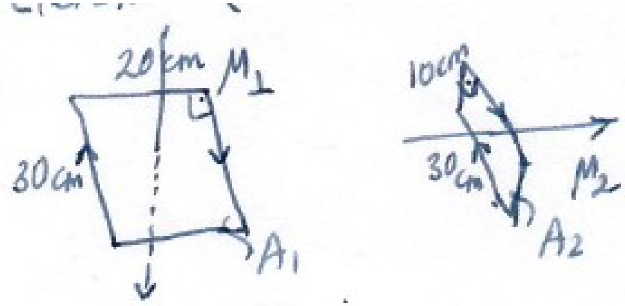
8. Force on a semicircular loop



9. Figure shows a current loop ABCDEFA carrying a current i=5.00 A. The sides of the loop are parallel to the coordinate axes shown, with AB=20.0 cm, BC=30.0 cm, and FA=10.0 cm. In unit- vector notation, what is the magnetic dipole moment of this loop? (Hint: Imagine equal and opposite currents i in the line segment AD; then treat the two rectangular loops ABCDA and ADEFA.)

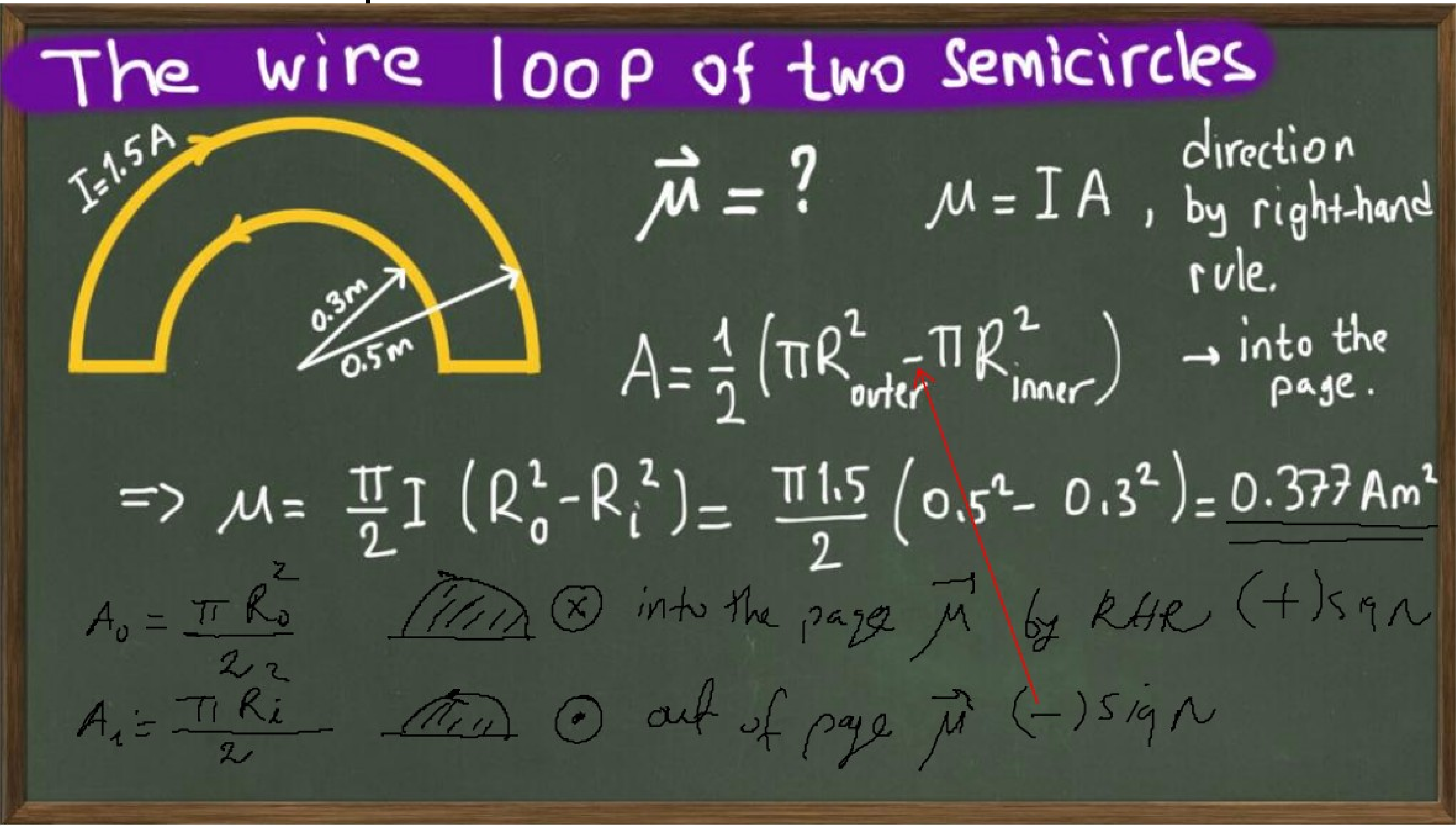


10 (60)



$$\vec{M} = \vec{\mu}_1 + \vec{\mu}_2 = iA_1(-\hat{k}) + iA_2(\hat{j})$$
$$= (5A)(0.2m)(0.3m)(-\hat{k}) + (5A)(0.3m)(0.1m)(\hat{j})$$
$$= 0.3(-\hat{k}) + 0.15(\hat{j})$$

10. The wire loop of two semicircles



The Magnetic Field *B*

Defined in terms of the force F_B acting on a test particle with charge *q* moving through the field with velocity ***v***

$$\vec{F}_B = q\vec{v} \times \vec{B}. \qquad \textbf{Eq. 28-2}$$

A Charge Particle Circulating in a Magnetic Field

Applying Newton's second law to the circular motion yields

$$|q|vB = \frac{mv^2}{r} \qquad \textbf{Eq. 28-15}$$

from which we find the radius r of the orbit circle to be

$$r = \frac{mv}{|q|B}. \qquad \textbf{Eq. 28-16}$$

Magnetic Force on a Current Carrying wire

A straight wire carrying a current *i* in a uniform magnetic field experiences a sideways force

$$\vec{F}_B = i\vec{L} \times \vec{B}. \qquad \textbf{Eq. 28-26}$$

The force acting on a current element i dL in a magnetic field is

$$d\vec{F}_B = i\,d\vec{L} \times \vec{B}. \qquad \textbf{Eq. 28-28}$$

Torque on a Current Carrying Coil

A coil (of area *A* and *N* turns, carrying current *i*) in a uniform magnetic field ***B*** will experience a torque ***τ*** given by

$$\vec{\tau} = \vec{\mu} \times \vec{B}. \qquad \textbf{Eq. 28-37}$$

The Hall Effect

When a conducting strip carrying a current *i* is placed in a uniform magnetic field ***B***, some charge carriers (with charge *e*) build up on one side of the conductor, creating a potential difference *V* across the strip. The polarities of the sides indicate the sign of the charge carriers.

Orientation Energy of a Magnetic Dipole

The orientation energy of a magnetic dipole in a magnetic field is

$$U(\theta) = -\vec{\mu} \cdot \vec{B}. \qquad \textbf{Eq. 28-38}$$

If an external agent rotates a magnetic dipole from an initial orientation θ_i to some other orientation θ_f and the dipole is stationary both initially and finally, the work W_a done on the dipole by the agent is

$$W_a = \Delta U = U_f - U_i. \qquad \textbf{Eq. 28-39}$$

Additional Materials