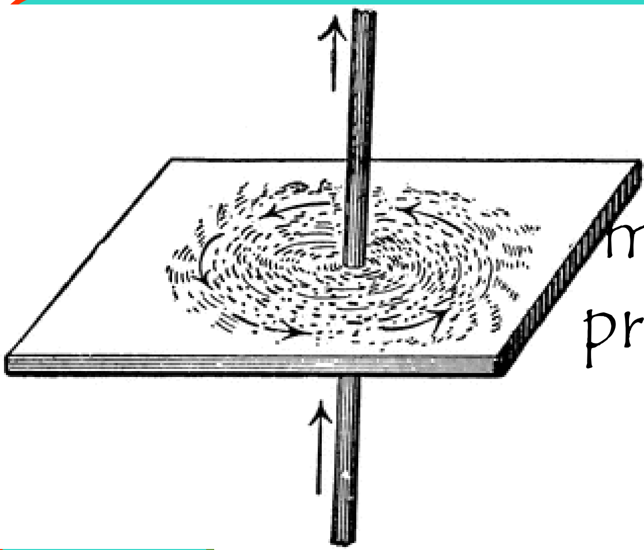
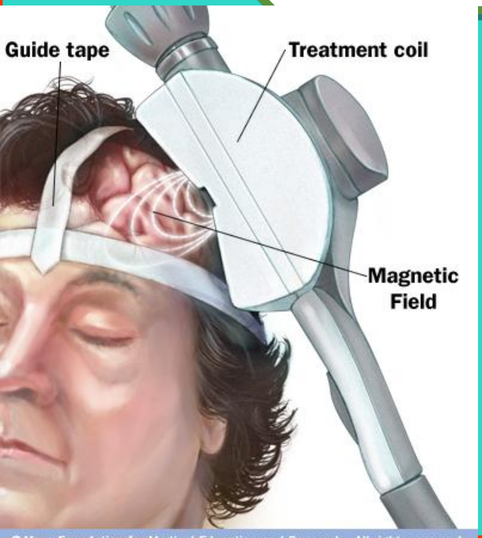


<u>Observation:</u>

a current of moving charged particles produces a magnetic field around the current.

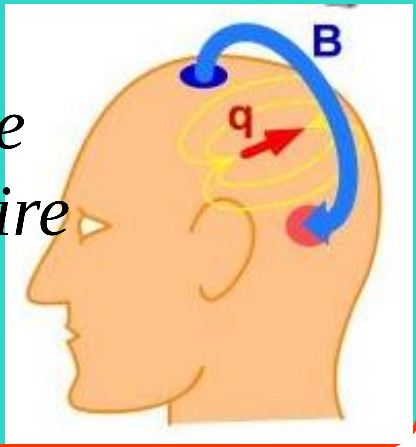
Chapter 29

Magnetic Fields due to Currents



Magnetic field due to

- *a current in a long straight wire*
- *a current in a circular arc of wire*
- *Brain activity...*



29 MAGNETIC FIELDS DUE TO CURRENTS 764

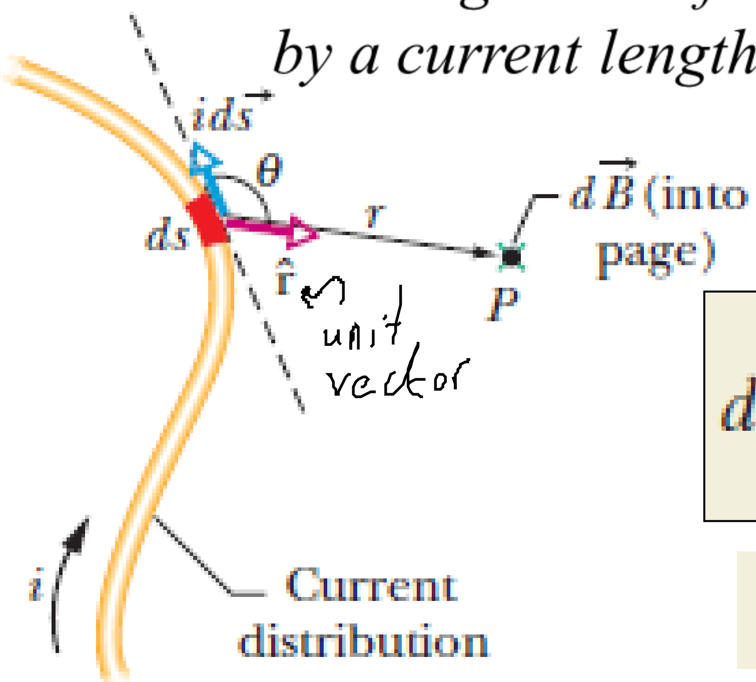
How to find the magnetic field at a nearby point *P*

1. first mentally divide the wire into differential elements *ds*
2. define for each element a length vector $d\vec{s}$ that has length *ds* and whose direction is the direction of the current in *ds*.
3. define a differential *current-length element* to be $i\, d\vec{s}$
4. calculate the net field $\vec{B}$ at *P* by summing, via integration

$i\,d\vec{s}$: current-length element → $d\vec{B}$

The magnitude of the field $d\vec{B}$ produced at P at distance r, by a current length element i:

$\vec{B}_P = \int d\vec{B}$



$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2}$$

Θ: angle btw $d\vec{s}$ and r

μo: permeability const.

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\hat{r}}{r^2} \quad \text{(Biot–Savart law).}$$

$\varepsilon_0 \sim \vec{E}$

$$\mu_0 = 4\pi\times10^{-7}\ \mathrm{T\cdot m/A} \approx 1.26\times10^{-6}\ \mathrm{T\cdot m/A}$$

↳ $\vec{B}$

Biot-Savart Law for B-Fields

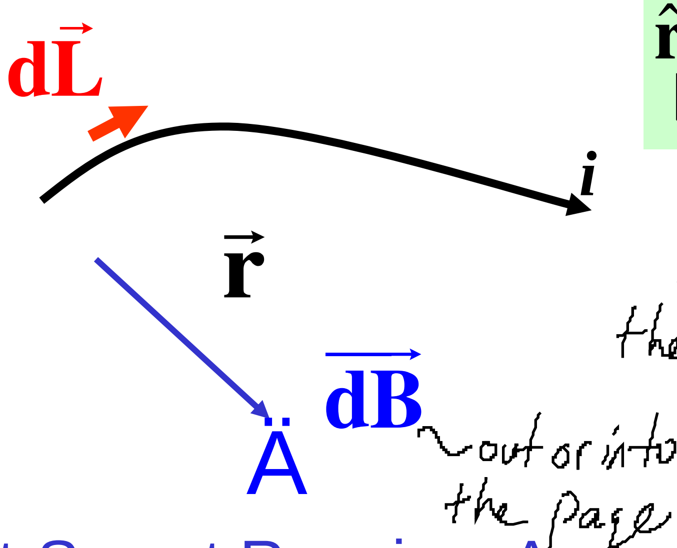
$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{\mathbf{L}} \times \hat{\mathbf{r}}}{r^2}$$

Coulomb Law for E-Fields

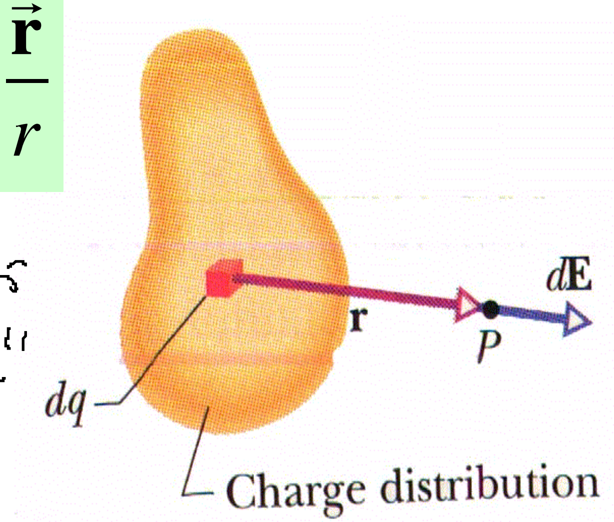
$$d\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{\mathbf{r}}$$

$$\hat{\mathbf{r}} = \frac{\vec{\mathbf{r}}}{|\vec{r}|} = \frac{\vec{\mathbf{r}}}{r}$$

Just shows the "direction"



Biot-Savart Requires A Right-Hand Rule

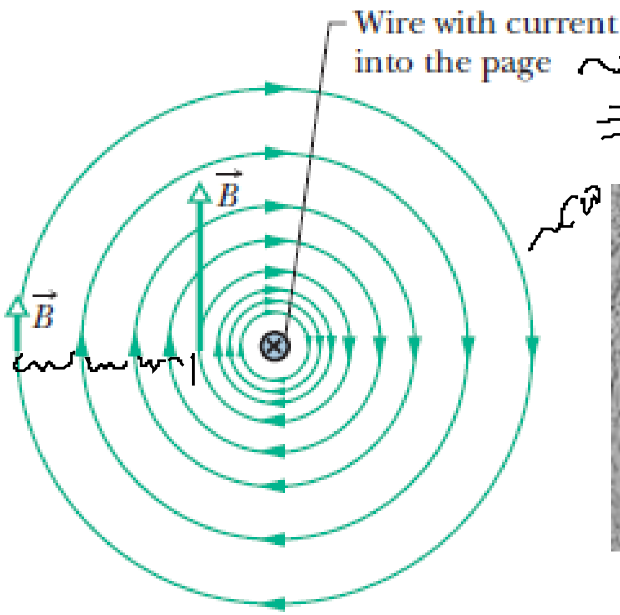


Both Are $1/r^2$ Laws!
The $\hat{\mathbf{r}}$ has no units.

http://www.phys.lsu.edu/~jdowling/PHYS21132-SP15/lectures/index.html

Magnetic field due to a current in a long straight wire

1. The field magnitude *B*:



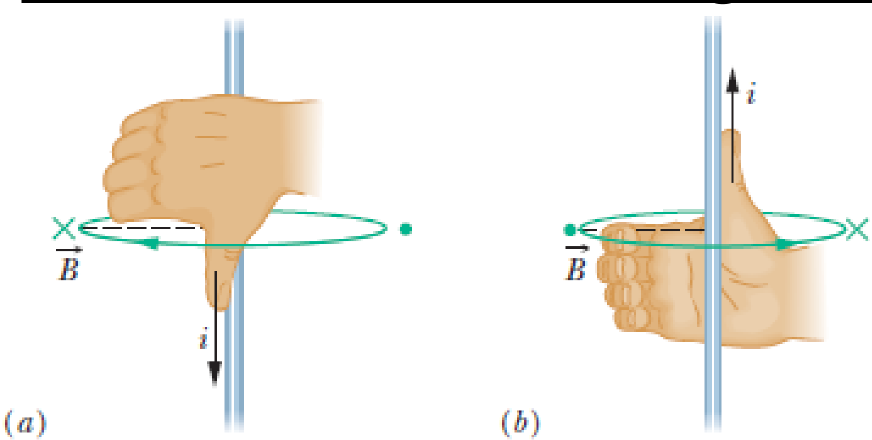
$$B = \frac{\mu_0 i}{2\pi R} \qquad \text{(long straight wire)}$$

R: perpendicular distance of the point from the wire

μ_o: permeability constant

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{i\, d\vec{s} \times \hat{r}}{r^2}$$

2.The direction of the magnetic field – Right Hand Rule



Grasp the element in your right hand with your extended thumb pointing in the direction of the current. Your fingers will then naturally curl around in the direction of the magnetic field lines due to that element.

Is the B-Field From a Power Line Dangerous?

A power line carries a current of 500 A.

What is the magnetic field in a house located 100 m away from the power line?

$$B = \frac{\mu_0 i}{2\pi R}$$

$$B = \frac{\mu_0 i}{2\pi R}$$

$$= \frac{(4\pi \times 10^{-7}\, T \cdot m / A)(500\, A)}{2\pi (100\, m)}$$

$= 1\ \mu T$

Recall that the earth's magnetic field is $\sim 10^{-4} T = 100\ \mu T$

Probably not dangerous!

Magnetic field due to a current in a long straight wire

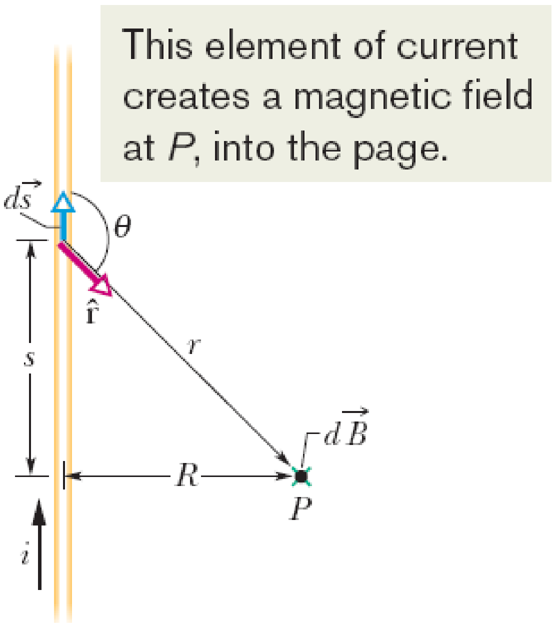


Fig. 29-5 Calculating the magnetic field produced by a current i in a long straight wire. The field $d\vec{B}$ at P associated with the current-length element $i\,d\vec{s}$ is directed into the page, as shown.

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2}.$$

$$B = 2\int_0^\infty dB = \frac{\mu_0 i}{2\pi}\int_0^\infty \frac{\sin\theta\,ds}{r^2}.$$

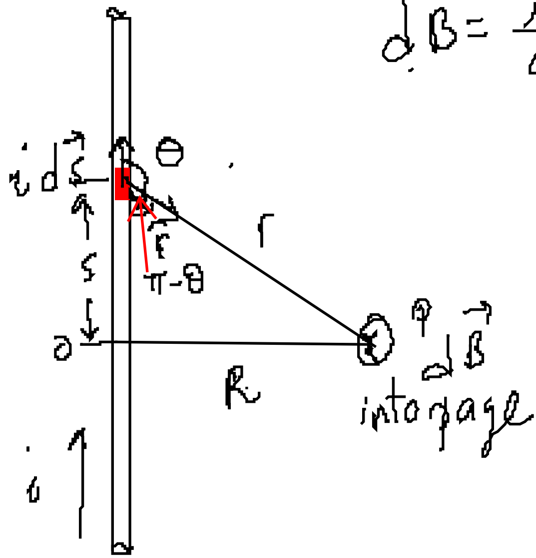
$$r = \sqrt{s^2 + R^2}$$

$$\sin\theta = \sin(\pi - \theta) = \frac{R}{\sqrt{s^2 + R^2}}.$$

$$B = \frac{\mu_0 i}{2\pi}\int_0^\infty \frac{R\,ds}{(s^2 + R^2)^{3/2}}$$

$$= \frac{\mu_0 i}{2\pi R}\left[\frac{s}{(s^2 + R^2)^{1/2}}\right]_0^\infty = \frac{\mu_0 i}{2\pi R}$$

$$B = \frac{\mu_0 i}{4\pi R} \qquad \text{(semi-infinite straight wire).}$$



$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\,d\vec{s} \times \hat{r}}{r^2}$$

$$\int_{-\infty}^{+\infty} = 2\int_0^{\infty}$$

$$\vec{B}_p = 2\int_0^{\infty} d\vec{B} = 2\int_0^{\infty} \frac{\mu_0}{4\pi} \frac{i\,ds\,\sin\theta}{r^2}$$

{ eliminate unknowns

$$r^2 = s^2 + R^2$$

$$\sin\theta = \sin(\pi-\theta) \checkmark$$

$$= \frac{R}{\sqrt{s^2+R^2}}$$

from Integral Table

$$B = \frac{2\mu_0 i}{4\pi} \int_0^{\infty} \frac{ds}{s^2+R^2} \frac{R}{\sqrt{s^2+R^2}} = \frac{\mu_0 i}{2\pi R} \left[\frac{s}{(s^2+R^2)^{1/2}} \right]_0^{\infty}$$

$$\boxed{B = \frac{\mu_0 i}{2\pi R}}$$

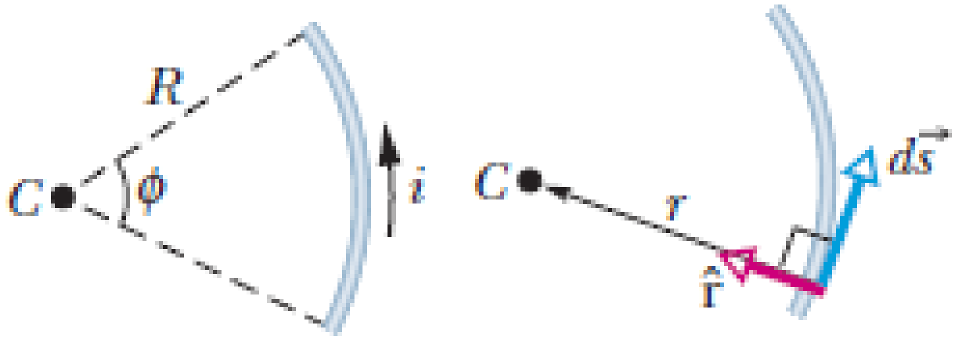
: infinitely long wire

Biot–Savart Law

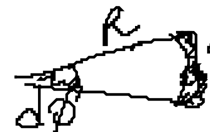
later Ampere's law

İZMİR KATİP ÇELEBİ ÜNİVERSİTESİ

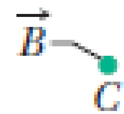
Magnetic field due to a current in circular arc of wire

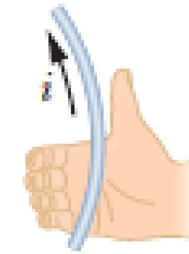


$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin 90^\circ}{R^2} = \frac{\mu_0}{4\pi}\frac{i\,ds}{R^2}$$



$ds \approx R\,d\phi$





- Apply right-hand rule anywhere along the wire → all the differential fields have the same direction at *C* (out of the page).
- Total field at *C* is simply the sum (via integration) of all the differential fields:

$$B = \int dB = \int_0^\phi \frac{\mu_0}{4\pi}\frac{iR\,d\phi}{R^2} = \frac{\mu_0 i}{4\pi R}\int_0^\phi d\phi$$

$$B = \frac{\mu_0 i\phi}{4\pi R} \qquad \text{(at center of circular arc).}$$

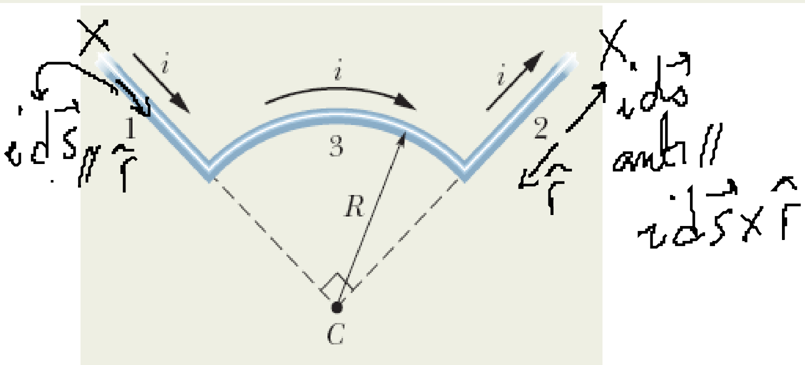
- When you insert data into the equation, be careful to express ϕ in radians!

Ex: at the center of a full circle of current ~ $\phi = 2\pi$

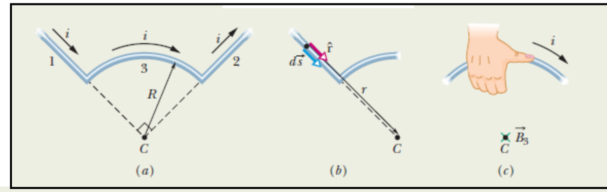
$$B = \frac{\mu_0 i(2\pi)}{4\pi R} = \frac{\mu_0 i}{2R}$$

Example, Magnetic field at the center of a circular arc of a circle

The wire in Fig. 29-7*a* carries a current i and consists of a circular arc of radius R and central angle $\pi/2$ rad, and two straight sections whose extensions intersect the center C of the arc. What magnetic field $\vec{B}$ (magnitude and direction) does the current produce at C?



(a)



Straight sections: For any current-length element in section 1, the angle θ between $d\vec{s}$ and $\hat{r}$ is zero (Fig. 29-7*b*); so Eq. 29-1 gives us

$$dB_1 = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{i\,ds\sin 0}{r^2} = 0.$$

Thus, the current along the entire length of straight section 1 contributes no magnetic field at C:

$$B_1 = 0.$$

The same situation prevails in straight section 2, where the angle θ between $d\vec{s}$ and $\hat{r}$ for any current-length element is 180°. Thus,

$$B_2 = 0.$$

Circular arc: Application of the Biot–Savart law to evaluate the magnetic field at the center of a circular arc leads to Eq. 29-9 ($B = \mu_0 i\phi/4\pi R$). Here the central angle ϕ of the arc is $\pi/2$ rad. Thus from Eq. 29-9, the magnitude of the magnetic field $\vec{B}_3$ at the arc's center C is

$$B_3 = \frac{\mu_0 i(\pi/2)}{4\pi R} = \frac{\mu_0 i}{8R}.$$

ϕ: in radian (not degree)

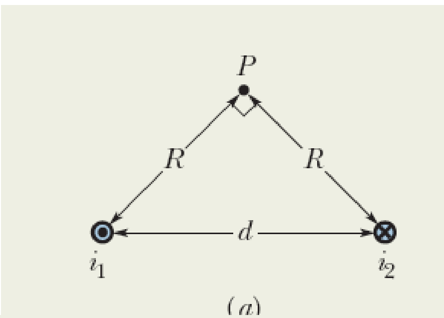
To find the direction of $\vec{B}_3$, we apply the right-hand rule displayed in Fig. 29-4. Mentally grasp the circular arc with your right hand as in Fig. 29-7*c*, with your thumb in the direction of the current. The direction in which your fingers curl around the wire indicates the direction of the magnetic field lines around the wire. They form circles around the wire, coming out of the page above the arc and going into the page inside the arc. In the region of point C (inside the arc), your fingertips point *into the plane* of the page. Thus, $\vec{B}_3$ is directed into that plane.

Net field: Generally, when we must combine two or more magnetic fields to find the net magnetic field, we must combine the fields as vectors and not simply add their magnitudes. Here, however, only the circular arc produces a magnetic field at point C. Thus, we can write the magnitude of the net field $\vec{B}$ as

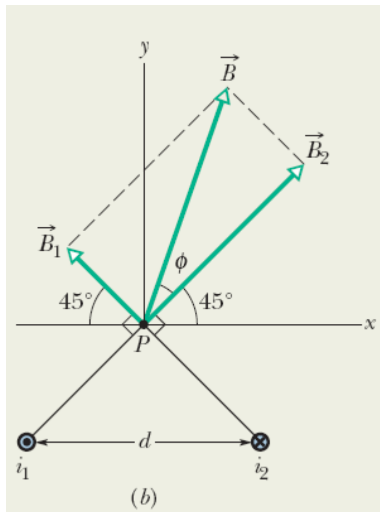
$$B = B_1 + B_2 + B_3 = 0 + 0 + \frac{\mu_0 i}{8R} = \frac{\mu_0 i}{8R}. \qquad \text{(Answer)}$$

Example, Magnetic field off to the side of two long straight currents

Figure 29-8*a* shows two long parallel wires carrying currents i_1 and i_2 in opposite directions. What are the magnitude and direction of the net magnetic field at point P? Assume the following values: $i_1 = 15$ A, $i_2 = 32$ A, and $d = 5.3$ cm.



The two currents create magnetic fields that must be added as vectors to get the net field.



Finding the vectors: In Fig. 29-8*a*, point P is distance R from both currents i_1 and i_2. Thus, Eq. 29-4 tells us that at point P those currents produce magnetic fields $\vec{B}_1$ and $\vec{B}_2$ with magnitudes

$$B_1 = \frac{\mu_0 i_1}{2\pi R} \quad \text{and} \quad B_2 = \frac{\mu_0 i_2}{2\pi R}.$$

In the right triangle of Fig. 29-8*a*, note that the base angles (between sides R and d) are both 45°. This allows us to write $\cos 45° = R/d$ and replace R with $d \cos 45°$. Then the field magnitudes B_1 and B_2 become

$$B_1 = \frac{\mu_0 i_1}{2\pi d \cos 45°} \quad \text{and} \quad B_2 = \frac{\mu_0 i_2}{2\pi d \cos 45°}.$$

Adding the vectors: We can now vectorially add $\vec{B}_1$ and $\vec{B}_2$ to find the net magnetic field $\vec{B}$ at point P, either by using a vector-capable calculator or by resolving the vectors into components and then combining the components of $\vec{B}$. However, in Fig. 29-8*b*, there is a third method: Because $\vec{B}_1$ and $\vec{B}_2$ are perpendicular to each other, they form the legs of a right triangle, with $\vec{B}$ as the hypotenuse. The Pythagorean theorem then gives us

$$B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2\pi d(\cos 45°)} \sqrt{i_1^2 + i_2^2}$$

$$= \frac{(4\pi \times 10^{-7}\ \mathrm{T\cdot m/A})\sqrt{(15\ \mathrm{A})^2 + (32\ \mathrm{A})^2}}{(2\pi)(5.3 \times 10^{-2}\ \mathrm{m})(\cos 45°)}$$

$$= 1.89 \times 10^{-4}\ \mathrm{T} \approx 190\ \mu\mathrm{T}. \qquad \text{(Answer)}$$

The angle ϕ between the directions of $\vec{B}$ and $\vec{B}_2$ in Fig. 29-8*b* follows from

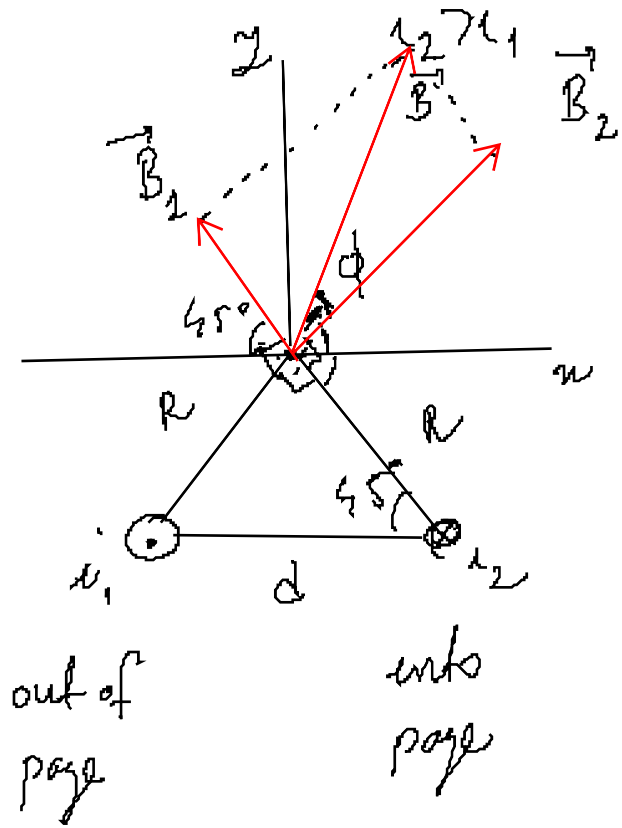
$$\phi = \tan^{-1}\frac{B_1}{B_2},$$

which, with B_1 and B_2 as given above, yields

$$\phi = \tan^{-1}\frac{i_1}{i_2} = \tan^{-1}\frac{15\ \mathrm{A}}{32\ \mathrm{A}} = 25°.$$

The angle between the direction of $\vec{B}$ and the x axis shown in Fig. 29-8*b* is then

$$\phi + 45° = 25° + 45° = 70°. \qquad \text{(Answer)}$$



B_p ✓

$\vec{B}_p = \vec{B}_1 + \vec{B}_2$

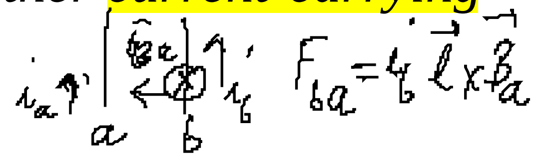
$|\vec{B}_p| = \sqrt{B_1^2 + B_2^2}$

$\phi = \tan \frac{B_1}{B_2}$

$B_1 = \frac{\mu_0 i_1}{2\pi R}$ & $B_2 = \frac{\mu_0 i_2}{2\pi R}$

what about R? $R = d \cos 45$

To find the **force on a current-carrying wire (b)** *due to another current-carrying wire (a):*



1. find the field due to «wire a» at the site of «wire b».
2. find the force on «wire b» due to magnetic field produced by «wire a»

1 ~ B_a
2 ~ F_{ba}

$$\vec{F}_{ba} = i_b \vec{L} \times \vec{B}_a$$

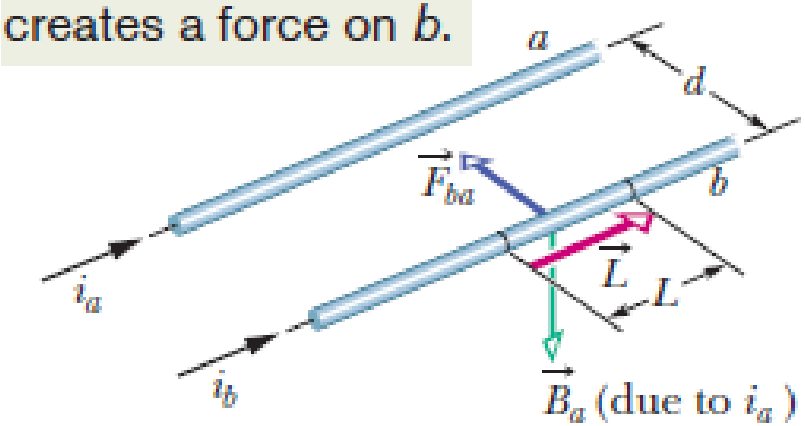
→ **Magnitude:** $B_a = \dfrac{\mu_0 i_a}{2\pi d}$.

length vector of the wire b

Direction: from curled–straight right-hand rule it is down

$|\vec{F}_{ba}| = |\vec{F}_{ab}|$

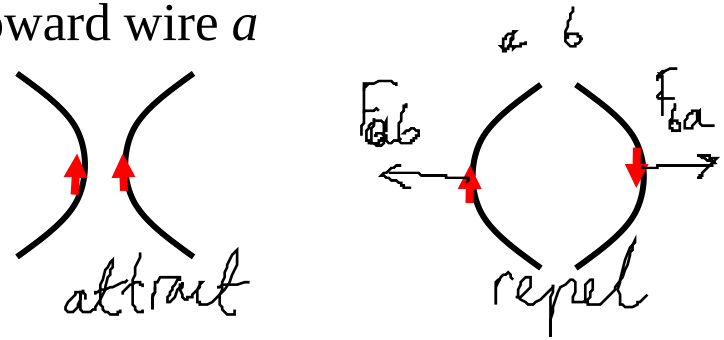
The field due to a at the position of b creates a force on b.



Mag.: $F_{ba} = i_b L B_a \sin 90° = \dfrac{\mu_0 L i_a i_b}{2\pi d}$

Direction: Apply right-hand rule for cross products → toward wire *a*

Parallel currents attract each other, and antiparallel currents repel each other



29-4 Ampere's Law

- We can find the net magnetic field due to *any* distribution of currents by first write the differential magnetic field due to a current-length:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s} \times \hat{r}}{r^2} \qquad \text{(Biot–Savart law)}$$

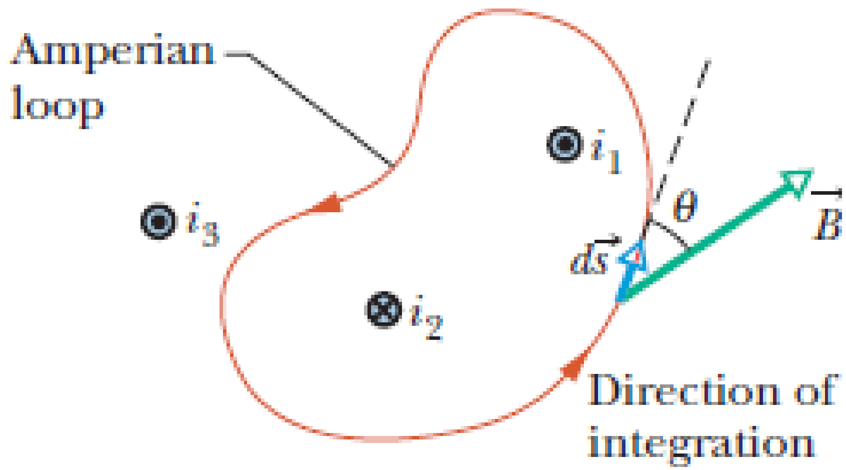
then sum the contributions from all the elements. Another approach;

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc} \qquad \text{(Ampere's law)}$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \quad \text{Gauss's Law}$$

- The line integral in this equation is evaluated around a closed loop called an *Amperian* loop.
- The current *i* on the right side is the net current encircled by the loop.

Ex: An arbitrary Amperian loop lying in the plane of the page encircles two of the currents but not the third.



29-4 Ampere's Law

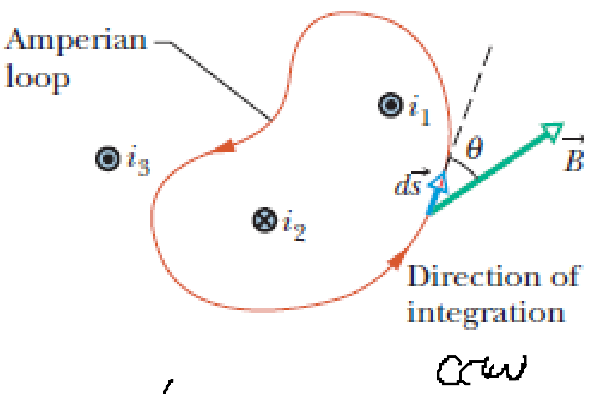
1. Arbitrarily choose the direction of integration: CCW

2. Divide the loop into differential vector elements (**ds**)

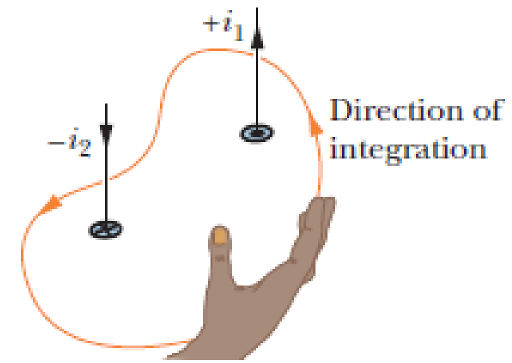
3. Assume that at the location of the element, the net magnetic field due to the three currents is $\vec{B}$. (Because the wires are perpendicular to the page, we know that the magnetic field at due to each current is in the plane, However, we do not know the orientation of $\vec{B}$ within the plane.)

4. Write the Ampere's Law: $$\oint \vec{B} \cdot d\vec{s} = \oint B \cos\theta \, ds = \mu_0 i_{enc}$$

5. At first, no need to know the direction of before integrating; arbitrarily assume $\vec{B}$ to be in the direction of integration. Then use the **curled–straight right-hand rule** to assign a + sign or a (-) sign to each of the currents that make up the net encircled current i_{enc}.



- Curl your right hand around the Amperian loop, with the fingers pointing in the direction of integration.
- A current through the loop in the general direction of your outstretched thumb is assigned a plus sign, and a current generally in the opposite direction is assigned a minus sign.



6. Finally, solve the eqn. for the magnitude.

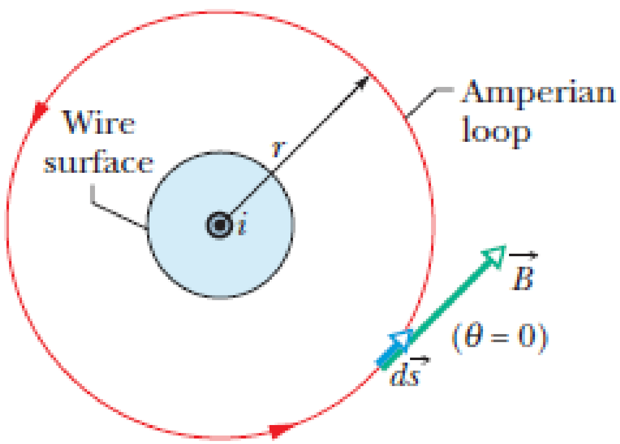
- If B turns out positive, then the direction we assumed for is correct.
- If it turns out negative, we neglect the minus sign and redraw in the opposite direction.

$$\oint \vec{B}\cdot d\vec{s} = \oint B\cos\theta\, ds = \mu_0 i_{enc} \quad i_{enc} = i_1 - i_2 \quad \oint B\cos\theta\, ds = \mu_0(i_1 - i_2)$$

- The contributions of current i_3 to the magnetic field cancel out because the integration in eqn is made around the full loop.
- In contrast, the contributions of an encircled current to the magnetic field do not cancel out.

Magnetic Field Outside a Long Straight Wire with Current

All of the current is encircled and thus all is used in Ampere's law.



1. Look for the symmetry to simplify the integral:

B has cylindrical symmetry about the wire.

2. Encircle the wire with a concentric circular Amperian loop of radius r → magnetic field has the same magnitude *B* at every point on the loop.

3. Integrate CCW

4. *Note that mag. Field is tangent to the loop at every point along the loop →* $\vec{B}$ *and* $d\vec{s}$ *are either parallel or antiparallel at each point of the loop.*

arbitrarily assume that $\vec{B}$ and $d\vec{s}$ are parallel → Θ=0 → cos0=1

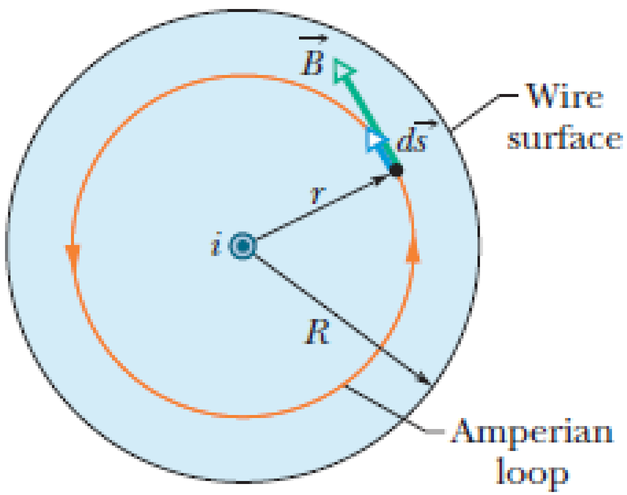
$$\oint \vec{B}\cdot d\vec{s} = \oint B\cos\theta\, ds = B\oint ds = B(2\pi r)$$

r: radius of Amperian loop

right-hand rule gives us a plus sign for the current → $B(2\pi r) = \mu_0 i$ → $B = \dfrac{\mu_0 i}{2\pi r}$

Magnetic Field Inside a Long Straight Wire with Current

Only the current encircled by the loop is used in Ampere's law.



1. current is uniformly distributed over a cross section of the wire → the magnetic field produced by the current must be cylindrically symmetrical.

2. use an Amperian loop of radius $r < R$

3. Symmetry again suggests that B vector is tangent to the loop

$$\oint \vec{B} \cdot d\vec{s} = B \oint ds = B(2\pi r)$$

radius of Amperian loop

i πR^2
i_{enc} πr^2

4. current is uniformly distributed, the current i_{enc} encircled by the loop is proportional to the area encircled by the loop

$$i_{enc} = i \frac{\pi r^2}{\pi R^2}$$

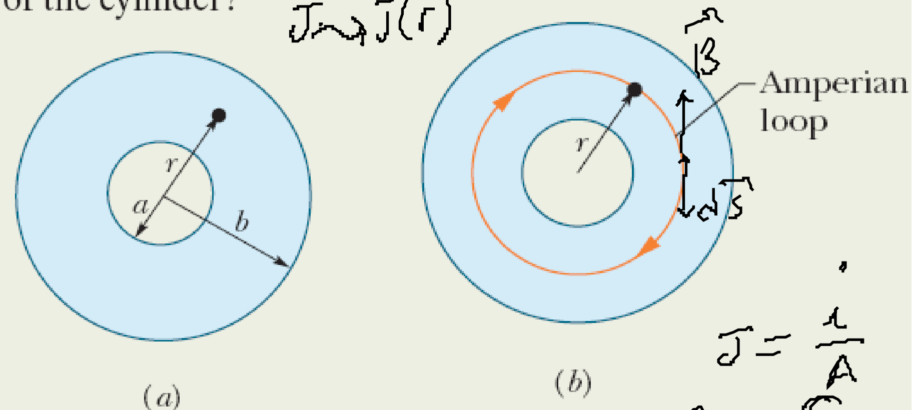
5. right-hand rule tells i_{enc} gets a plus sign. Then Ampere's law gives:

$$B(2\pi r) = \mu_0 i \frac{\pi r^2}{\pi R^2}$$

$$B = \left(\frac{\mu_0 i}{2\pi R^2} \right) r \quad \text{(inside straight wire)}$$

Example, Ampere's Law to find the magnetic field inside a long cylinder of current

Figure 29-15a shows the cross section of a long conducting cylinder with inner radius $a = 2.0$ cm and outer radius $b = 4.0$ cm. The cylinder carries a current out of the page, and the magnitude of the current density in the cross section is given by $J = cr^2$, with $c = 3.0 \times 10^6$ A/m^4 and r in meters. What is the magnetic field $\vec{B}$ at the dot in Fig. 29-15a, which is at radius $r = 3.0$ cm from the central axis of the cylinder?



(a) (b)

Calculations: We write the integral as

$$i_{enc} = \int J\,dA = \int_a^r cr^2(2\pi r\,dr)$$

$$= 2\pi c \int_a^r r^3\,dr = 2\pi c\left[\frac{r^4}{4}\right]_a^r$$

$$= \frac{\pi c(r^4 - a^4)}{2}.$$

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc},$$

gives us

$$B(2\pi r) = -\frac{\mu_0 \pi c}{2}(r^4 - a^4).$$

Solving for B and substituting known data yield

$$B = -\frac{\mu_0 c}{4r}(r^4 - a^4)$$

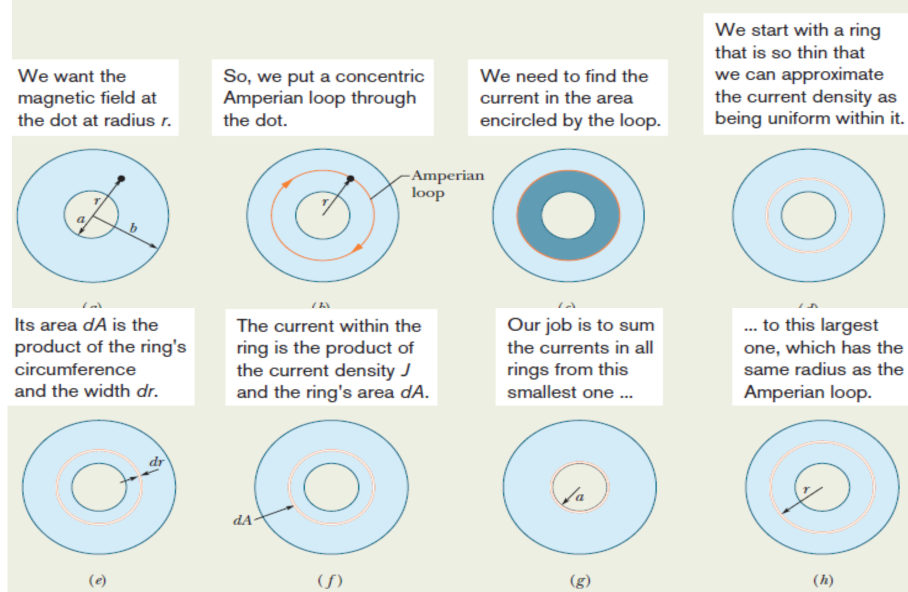
$$= -\frac{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(3.0 \times 10^6\,\text{A/m}^4)}{4(0.030\,\text{m})}$$

$$\times [(0.030\,\text{m})^4 - (0.020\,\text{m})^4]$$

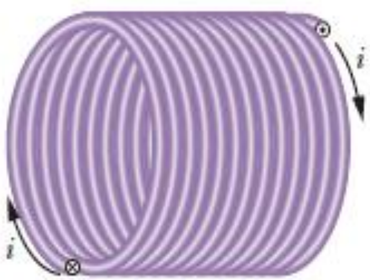
$$= -2.0 \times 10^{-5}\,\text{T}.$$

Thus, the magnetic field $\vec{B}$ at a point 3.0 cm from the central axis has magnitude

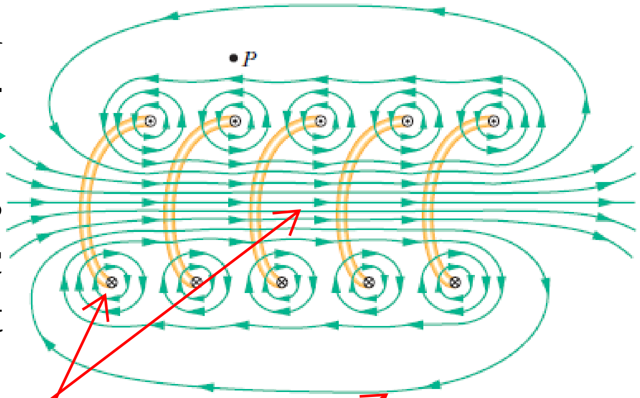
$$B = 2.0 \times 10^{-5}\,\text{T} \qquad \text{(Answer)}$$



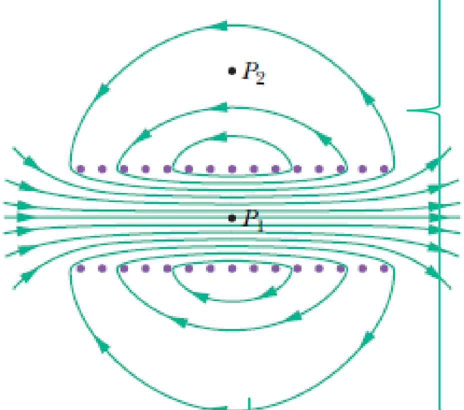
29-5 Solenoids and Toroids



- A vertical cross section through the central axis of a "stretched-out" solenoid.
- The back portions of five turns are shown, as are the magnetic field lines due to a current through the solenoid.



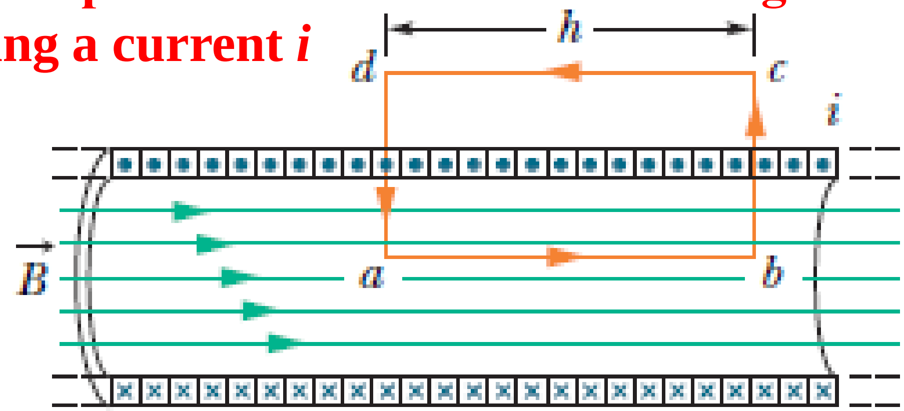
Solenoid: tightly wound helical coil of wire



- The solenoid's magnetic field is the **vector sum of** the fields produced by the **individual turns** (windings) that make up the solenoid.
- Each turn produces circular magnetic field lines near itself and the lines of ***B*** there are almost **concentric circles**.
- *Near the solenoid's axis (reasonably far from the wire), the field lines combine into a net magnetic field that is directed along the axis.* ***B*** *is approximately parallel to the (central) solenoid axis.*
- The closely spaced field lines there indicate a strong magnetic field.
- **Outside** the solenoid the field lines are widely spaced; the field there is **very weak**.

- **Magnetic field lines for a real solenoid of finite length: The field is strong and uniform at interior points such as P_1 but relatively weak at external points such as P_2.**

Application of Ampere's law to a section of a long ideal solenoid carrying a current *i*



$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$$

$$i_{enc} = i(nh)$$

The **Amperian loop** is the rectangle *abcda*

Here *n* be the number of turns per unit length of the solenoid

- We write $\oint \vec{B}\cdot d\vec{s}$ as the **sum of four integrals, one for each loop**

$$\oint \vec{B}\cdot d\vec{s} = \int_a^b \vec{B}\cdot d\vec{s} + \int_b^c \vec{B}\cdot d\vec{s} + \int_c^d \vec{B}\cdot d\vec{s} + \int_d^a \vec{B}\cdot d\vec{s}.$$

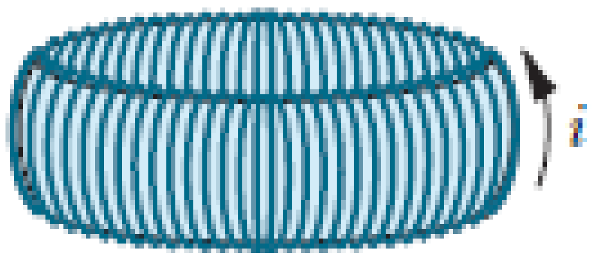
1. The **first** integral on the right of equation is *Bh*, where *B* is the magnitude of the uniform field ***B*** inside the solenoid and *h* is the (arbitrary) length of the segment from a to b.
2. The **second** and **fourth** integrals are **zero** because for every element *ds* of these segments, ***B*** either is perpendicular to *ds* or is zero, and thus the product ***B*•*ds*** is zero.
3. The **third** integral, which is taken along a segment that lies outside the solenoid, is **zero** because *B=0* at all external points.

Thus, $\oint \vec{B}\cdot d\vec{s}$ for the entire rectangular loop has the value *Bh*.

$$Bh = \mu_0 inh$$

- **Inside a long solenoid carrying current i, at points not near its ends, the magnitude B of the magnetic field is**

$$B = \mu_0 in \qquad \text{(ideal solenoid)}$$



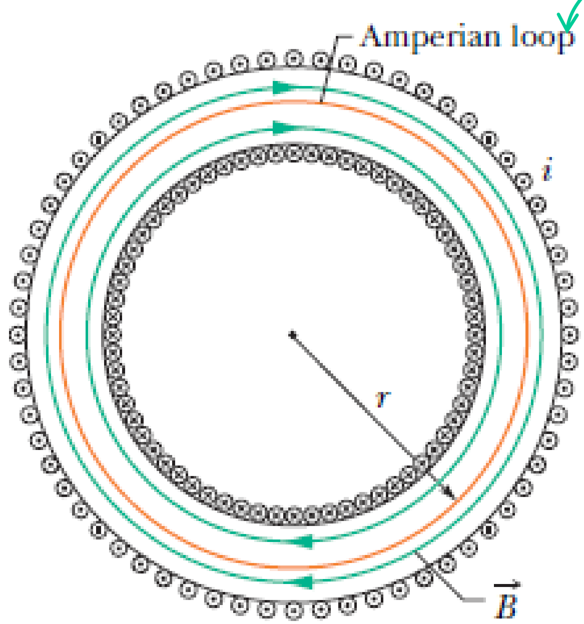
Toroid: hollow solenoid curved until its two ends meet, forming a sort of hollow bracelet.

- What magnetic field is set up inside the toroid (inside the hollow of the bracelet)?
- We can find out from Ampere's law and the symmetry of the bracelet.
- Let us choose a concentric circle of radius *r* as an Amperian loop and traverse it in the clockwise direction. Ampere's law yields

$$(B)(2\pi r) = \mu_0 iN, \quad B = \frac{\mu_0 iN}{2\pi}\frac{1}{r}$$

i: current in the toroid windings
N: total number of turns

- In contrast to the situation for a solenoid, *B* is **not constant** over the cross section of a toroid.



Horizontal cross section of the toroid.

Sample Problem

The field inside a solenoid (a long coil of current)

A solenoid has length $L = 1.23$ m and inner diameter $d = 3.55$ cm, and it carries a current $i = 5.57$ A. It consists of five close-packed layers, each with 850 turns along length L. What is B at its center?

KEY IDEA

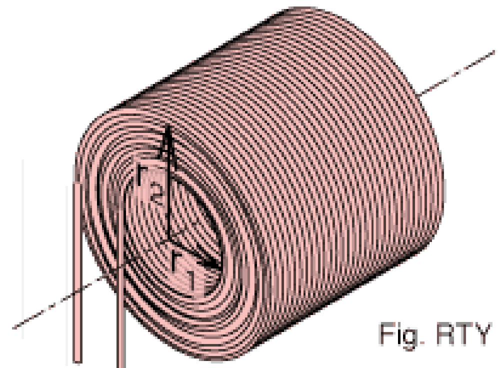
The magnitude B of the magnetic field along the solenoid's central axis is related to the solenoid's current i and number of turns per unit length n by Eq. 29-23 ($B = \mu_0 in$).

Calculation: Because B does not depend on the diameter of the windings, the value of n for five identical layers is simply five times the value for each layer. Equation 29-23 then tells us

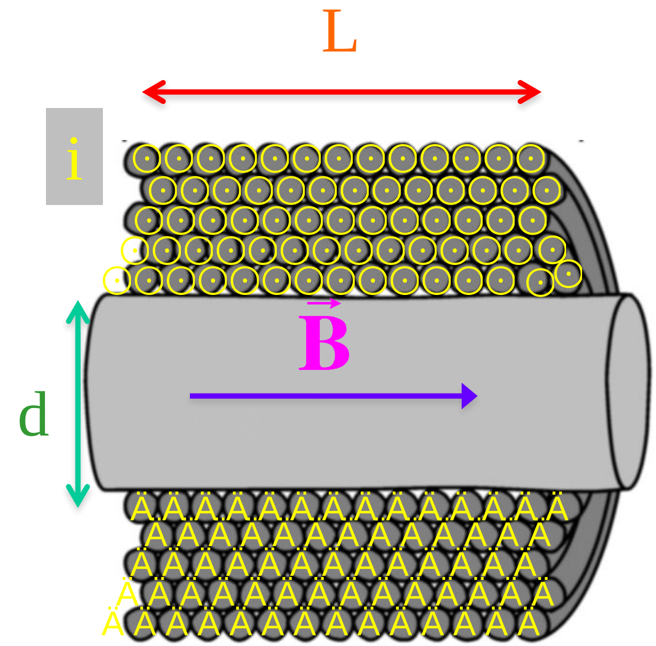
$$B = \mu_0 in = (4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(5.57\ \text{A})\frac{5 \times 850\ \text{turns}}{1.23\ \text{m}}$$

$$= 2.42 \times 10^{-2}\ \text{T} = 24.2\ \text{mT}. \qquad \text{(Answer)}$$

To a good approximation, this is the field magnitude throughout most of the solenoid.



A radially thick, multilayer solenoid.



The Biot-Savart Law

The magnetic field set up by a current-carrying conductor can be found from the Biot–Savart law.

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s} \times \hat{r}}{r^2} \qquad \textbf{Eq. 29-3}$$

The quantity μ_0, called the permeability constant, has the value

$$4\pi \times 10^{-7}\ \mathrm{T \cdot m/A} \approx 1.26 \times 10^{-6}\ \mathrm{T \cdot m/A}.$$

Magnetic Field of a Long Straight Wire

For a long straight wire carrying a current *i*, the Biot–Savart law gives,

$$B = \frac{\mu_0 i}{2\pi R} \qquad \textbf{Eq. 29-4}$$

Magnetic Field of a Circular Arc

The magnitude of the magnetic field at the center of a circular arc,

$$B = \frac{\mu_0 i \phi}{4\pi R} \qquad \textbf{Eq. 29-9}$$

Force Between Parallel Currents

The magnitude of the force on a length L of either wire is

$$F_{ba} = i_b L B_a \sin 90° = \frac{\mu_0 L i_a i_b}{2\pi d} \qquad \textbf{Eq. 29-13}$$

Ampere's Law

Ampere's law states that,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\mathrm{enc}} \qquad \textbf{Eq. 29-14}$$

Fields of a Solenoid and a Toroid

Inside a long solenoid carrying current *i*, at points not near its ends, the magnitude *B* of the magnetic field is

$$B = \mu_0 i n \qquad \textbf{Eq. 29-23}$$

At a point inside a toroid, the magnitude *B* of the magnetic field is

$$B = \frac{\mu_0 i N}{2\pi}\frac{1}{r} \qquad \textbf{Eq. 29-24}$$

Field of a Magnetic Dipole

The magnetic field produced by a current-carrying coil, which is a magnetic dipole, at a point P located a distance *z* along the coil's perpendicular central axis is parallel to the axis and is given by

$$\vec{B}(z) = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3} \qquad \textbf{Eq. 29-9}$$

1. AK

28 • A small current element $I\,d\vec{\ell}$ with $d\vec{\ell} = 2\text{ mm}\hat{k}$ and $I = 2$ A, is centered at the origin. Find the magnetic field $d\vec{B}$ at the following points: (*a*) on the x axis at $x = 3$ m, (*b*) on the x axis at $x = -6$ m, (*c*) on the z axis at $z = 3$ m, and (*d*) on the y axis at $y = 3$ m.

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{\ell} \times \hat{r}}{r^2}$$

a)

$$= \left(10^{-7}\text{ N/A}^2\right)\frac{(2\text{ A})(2\text{ mm})\hat{k}\times\hat{r}}{r^2}$$

$$= \left(0.400\text{ nT}\cdot\text{m}^2\right)\frac{\hat{k}\times\hat{r}}{r^2}$$

$\vec{r} = (3\text{m})\hat{i}$, $r = 3\text{m}$, and $\hat{r} = \hat{i}$

$$d\vec{B}(3\text{m},0,0) = \left(0.400\text{ nT}\cdot\text{m}^2\right)\frac{\hat{k}\times\hat{i}}{(3\text{m})^2}$$

$$= \boxed{(44.4\text{pT})\hat{j}}$$

b) $\vec{r} = -(6\text{m})\hat{i}$, $r = 6\text{m}$, and $\hat{r} = -\hat{i}$

$$d\vec{B}(-6\text{m},0,0) = \left(0.400\text{ nT}\cdot\text{m}^2\right)\frac{\hat{k}\times(-\hat{i})}{(6\text{m})^2}$$

$$= \boxed{-(11.1\text{pT})\hat{j}}$$

c)

$\vec{r} = (3\text{m})\hat{k}$, $r = 3\text{m}$, and $\hat{r} = \hat{k}$

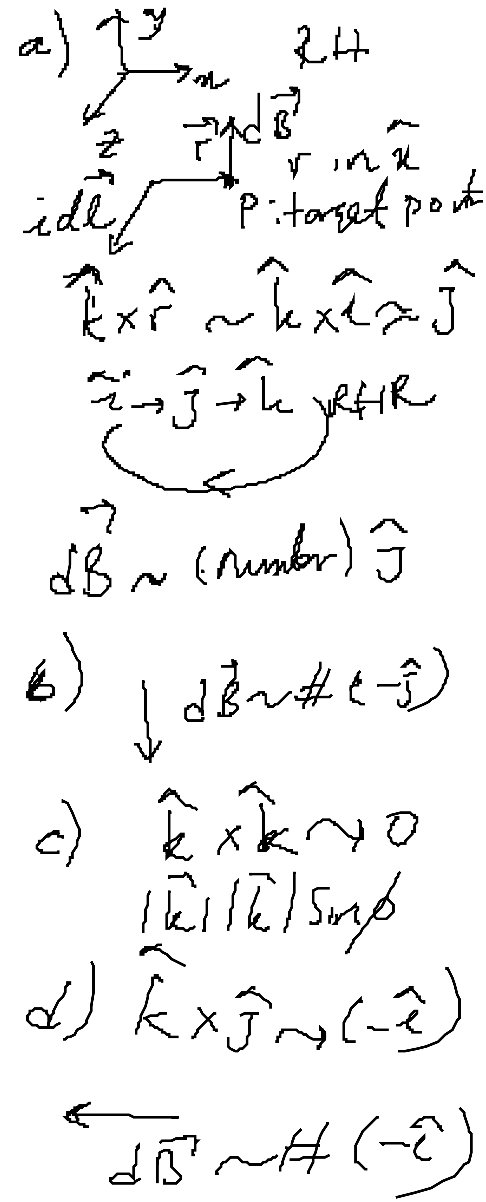
$$d\vec{B}(0,0,3\text{m}) = \left(0.400\text{ nT}\cdot\text{m}^2\right)\frac{\hat{k}\times\hat{k}}{(3\text{m})^2}$$

$$= \boxed{0}$$

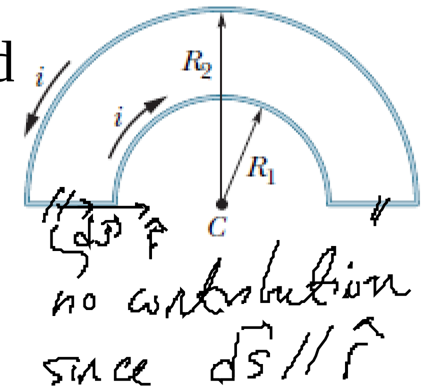
d) $\vec{r} = (3\text{m})\hat{j}$, $r = 3\text{m}$, and $\hat{r} = \hat{j}$

$$d\vec{B}(0,3\text{m},0) = \left(0.400\text{ nT}\cdot\text{m}^2\right)\frac{\hat{k}\times\hat{j}}{(3\text{m})^2}$$

$$= \boxed{-(44.4\text{pT})\hat{i}}$$

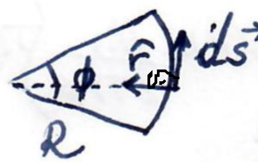


2. In Figure, two semicircular arcs have radii R_2=7.80 cm and R_1=3.15 cm, carry current i=0.281 A, and share the same center of curvature C. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at C?



2(8)

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\hat{r}}{r^2}$$
(Biot-Savart law) →



$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin 90^\circ}{R^2} = \frac{\mu_0}{4\pi}\frac{i\,ds}{R^2} \quad \{ ds = R\,d\phi$$

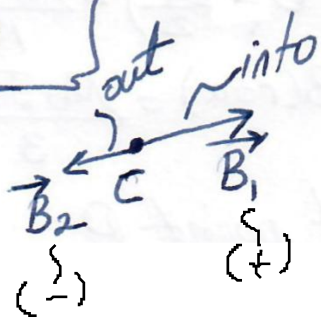
$$B = \int dB = \frac{\mu_0}{4\pi} i \int_0^{\phi} \frac{R\,d\phi}{R^2} = \boxed{\frac{\mu_0 i}{4\pi R}\phi \sim \text{arc angle}}$$

⇒ Solution

$\phi = \pi$, $R_2 = 7.8\times10^{-2}\,m$
$R_1 = 3.15\times10^{-2}\,m$
$i = 0.281\,A$

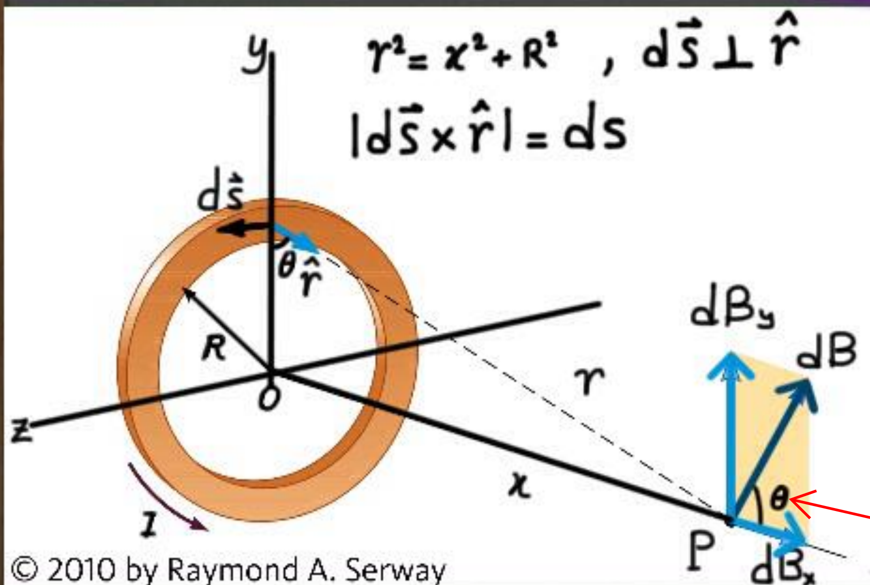
i) $$B = B_1 + B_2 = \frac{\mu_0 i}{4\pi R_1}\pi - \frac{\mu_0 i}{4\pi R_2}\pi$$
$$= \frac{(4\pi\times10^{-7}\,Tm/A)(0.281\,A)}{4}\left(\frac{1}{3.15\times10^{-2}\,m} - \frac{1}{7.8\times10^{-2}\,m}\right)$$
$$= 1.67\times10^{-6}\,T$$

ii) into the page



3. AK

B on the axis of a circular loop



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$$|d\vec{B}| = \frac{\mu_0 I}{4\pi}\frac{|d\vec{s}\times\hat{r}|}{r^2} = \frac{\mu_0 I}{4\pi}\frac{ds}{x^2+R^2}$$

by symmetry $B_y = \oint dB_y = 0$

$\rightarrow \vec{B} = B_x \hat{i}$, $B_x = \int dB_x$, $dB_x = dB\cos\theta$

$$B_x = \oint dB\cos\theta = \frac{\mu_0 I}{4\pi}\oint\frac{ds\cos\theta}{x^2+R^2}$$

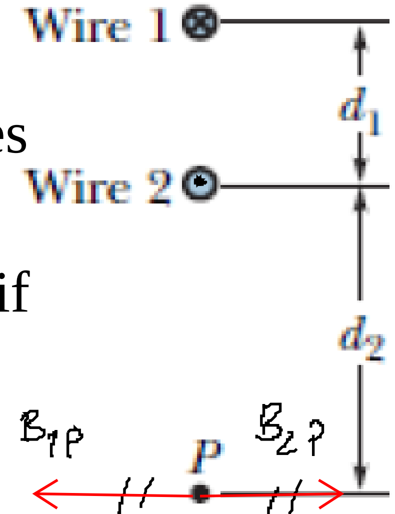
θ, x and R : const. of integ.

$$B_x = \frac{\mu_0 I R}{4\pi(x^2+R^2)^{3/2}}\oint ds \quad (\oint ds = 2\pi R)$$

where $\cos\theta = \frac{R}{\sqrt{x^2+R^2}}$

$$B_x = \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}$$

4. In Figure, two long straight wires are perpendicular to the page and separated by distance $d_1 = 0.75$ cm. Wire 1 carries 6.5 A into the page. What are the (a) magnitude and (b) direction (into or out of the page) of the current in wire 2 if the net magnetic field due to the two currents is zero at point P located at distance $d_2 = 1.50$ cm from wire 2?



3(11) $B = \frac{\mu_0 i}{2\pi R}$: magnetic field due to a current in a long straight wire

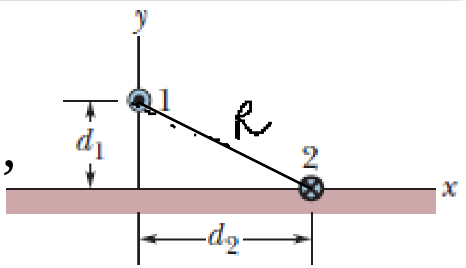
$d_1 = 0.75\times10^{-2}$ m
$d_2 = 1.50\times10^{-2}$ m
$i_1 = 6.5$ A

i) $|\vec{B}_{1P}| = |\vec{B}_{2P}|$, $B_{1P} = \frac{\mu_0 i_1}{2\pi(d_1+d_2)}$, $B_{2P} = \frac{\mu_0 i_2}{2\pi d_2}$

$$\frac{\mu_0 i_1}{2\pi(d_1+d_2)} = \frac{\mu_0 i_2}{2\pi d_2} \rightarrow i_2 = \frac{d_2}{(d_1+d_2)} i_1 = \frac{1.5}{2.25}(6.5A) = 4.3A$$

ii) wire 1 → B_1 is to left ⇒ B_2 should be in opposite direction → B_2 is to right ⇐ i_2 : out of page

5. In Figure shows wire 1 in cross section; the wire is long and straight, carries a current of 4.00 mA out of the page, and is at distance d_1 = 2.40 cm from a surface. Wire 2, which is parallel to wire 1 and also long, is at horizontal distance d_2 =5.00 cm from wire 1 and carries a current of 6.80 mA into the page. What is the x component of the magnetic force per unit length on wire 2 due to wire 1?



$\vec{F}_{21} = i_2 \vec{L} \times \vec{B}_1$

$\frac{F_{21}}{\text{length}}$ — find B_1 first

6(35) $\vec{F}_{ba} = i_b \vec{L} \times \vec{B}_a$: force on wire b due to magnetic field produced by a

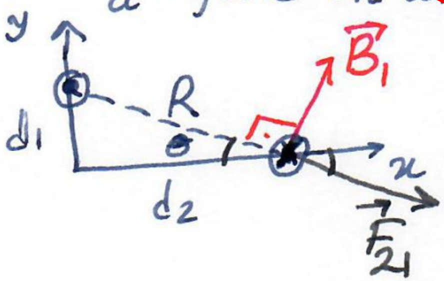
$i_1 = 4 \times 10^{-3}$ A

$d_1 = 2.4 \times 10^{-2}$ m

$d_2 = 5 \times 10^{-2}$ m

$i_2 = 6.8 \times 10^{-3}$ A

$\frac{F_{21,x}}{L} = ?$



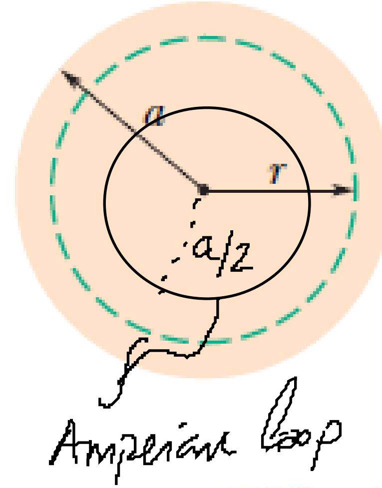
$$B_1 = \frac{\mu_0 i_1}{2\pi R} = \frac{\mu_0 i_1}{2\pi\sqrt{d_1^2 + d_2^2}}$$

$$F_{21} = i_2 L B_1 \sim \frac{F_{21}}{L} = \frac{i_2 \mu_0 i_1}{2\pi\sqrt{d_1^2+d_2^2}}$$

$$\frac{F_{21,x}}{L} = \frac{\mu_0 i_1 i_2}{2\pi\sqrt{d_1^2+d_2^2}} \cos\theta = \frac{\mu_0 i_1 i_2 d_2}{2\pi(d_1^2+d_2^2)}$$

$$= \frac{(4\pi \times 10^{-7}\,\text{Tm/A})(4\times 10^{-3}\,\text{A})(6.8\times 10^{-3}\,\text{A})(5\times 10^{-2}\,\text{m})}{2\pi((2.4\times 10^{-2}\,\text{m})^2 + (5\times 10^{-2}\,\text{m})^2)} = 8.84\times 10^{-11}\,\frac{\text{N}}{\text{m}}$$

6. Figure shows a cross section across a diameter of a long cylindrical conductor of radius a=2.00 cm carrying uniform current 170 A. What is the magnitude of the current's magnetic field at radial distance (a) 0, (b) 1.00 cm, (c) 2.00 cm (wire's surface), and (d) 4.00 cm?



7 (43)

long cylindrical conductor. $\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$

radius, $a = 2\times10^{-2}$ m

$i = 170$ A

$\pi a^2 \;\; i$, $\pi r^2 \;\; i_{enc}$ $\rightarrow i_{enc} = \frac{\pi r^2}{\pi a^2} i$

$B\, 2\pi r = \mu_0 \frac{\pi r^2}{\pi a^2} i \rightarrow B = \frac{\mu_0 i r}{2\pi a^2}$

i) $r = 0 \rightarrow B = 0$

ii) $r = 1\times10^{-2}$ m

$B = \frac{(4\pi\times10^{-7}\,\text{Tm/A})(170\,\text{A})(1\times10^{-2}\,\text{m})}{2\pi(2\times10^{-2}\,\text{m})^2} = 8.50\times10^{-4}$ T

iii) $r = 2\times10^{-2}$ m $\quad B = \frac{\mu_0 i}{2\pi a} = 1.70\times10^{-3}$ T (2 cm)

iv) $r = 4\times10^{-2}$ m $\quad B = \frac{\mu_0 i}{2\pi(4\times10^{-2}\,\text{m})} = 8.50\times10^{-4}$ T (4 cm)

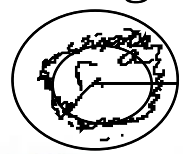
ii) $\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc} \sim B\oint ds = \mu_0 \frac{\pi (a/2)^2}{\pi a^2} \rightarrow B\, 2\pi \frac{a}{2} = \mu_0 \frac{a^2/4}{a^2} i$

$\oint ds = 2\pi a/2$

$B = \frac{\mu_0 i}{2\pi} \frac{a}{2}$

7. The current density J inside a long, solid, cylindrical wire of radius a=3.1 mm is in the direction of the central axis, and its magnitude varies linearly with radial distance r from the axis according to $J = J_0 r/a$, where $J_0 = 310$ A/m^2. Find the magnitude of the magnetic field at (a) r = 0, (b) r = a/2, and (c) r = a.

$J = i/A$



$J \sim J(r)$

$i_{enc} = ?$

$i_{enc} = \int J(r)\,dA = \int J(r)\,2\pi r\,dr$

8(47)

cylindrical wire
radius $a = 3.1\times10^{-3}$ m
$J \rightsquigarrow J(r) = \frac{J_0}{a} r$
$J_0 = 310$ A/m^2
$\rightarrow B \rightsquigarrow B(r)$

$B = \frac{\mu_0 i_{enc}}{2\pi R} \rightsquigarrow B(r) = \frac{\mu_0}{2\pi R}\int J(r)\,dA = \frac{\mu_0}{2\pi R}\int J(r)\,2\pi r\,dr$

$\rightsquigarrow B(r) = \frac{\mu_0}{2\pi R}\int_0^R \frac{J_0}{a} r \, 2\pi r\,dr = \frac{\mu_0 J_0}{a}\frac{R^2}{3}$

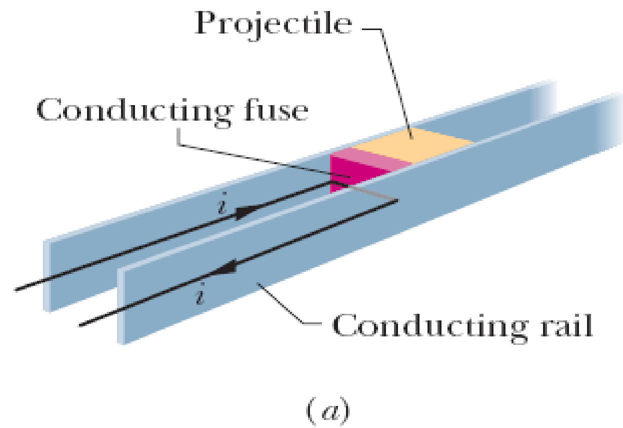
i) $r=0 \rightsquigarrow B=0$

ii) $r=a/2 \rightsquigarrow B(r=\frac{a}{2}) = \frac{\mu_0 J_0 a}{12} = 1\times10^{-7}$ T

iii) $r=a \rightsquigarrow B(r=a) = \frac{\mu_0 J_0 a}{3} = 4\times10^{-7}$ T

5(28)

Additional Materials



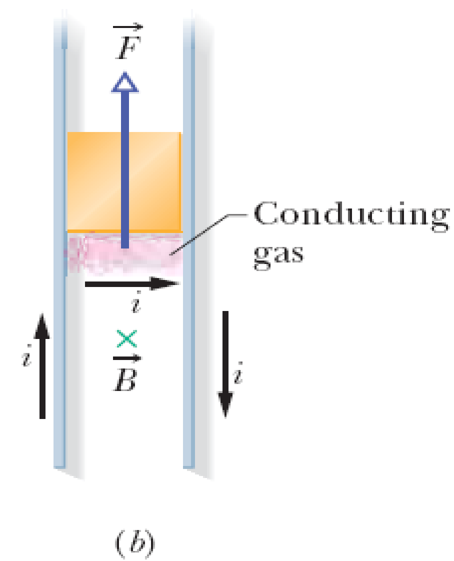


Fig. 29-10 (*a*) A rail gun, as a current i is set up in it. The current rapidly causes the conducting fuse to vaporize. (*b*) The current produces a magnetic field $\vec{B}$ between the rails, and the field causes a force $\vec{F}$ to act on the conducting gas, which is part of the current path. The gas propels the projectile along the rails, launching it.

29-5 Solenoids and Toroids

The magnetic fields produced by current in a

- ✓long straight wire
- ✓solenoid
- ✓toroid

NEXT: the magnetic field produced by a coil carrying a current a coil behaves as a magnetic dipole in that, if we place it in an external magnetic field $\vec{B}$, a torque acts on it:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

magnetic dipole moment of the coil

N:number of turns,
i: current in each turn,
A:area enclosed by each turn.

Magnitude: NiA

Direction: by curled–straight right-hand rule:

- Grasp the coil so that the fingers of your right hand curl around it in the direction of the current;
- your extended thumb then points in the direction of the dipole moment

Magnetic Field of a Coil

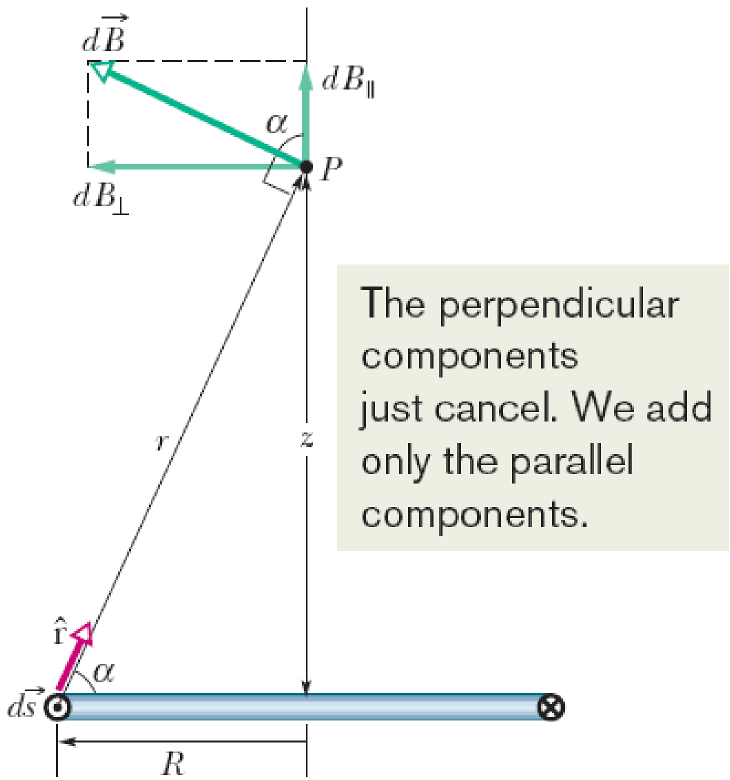


Fig. 29-22 Cross section through a current loop of radius R. The plane of the loop is perpendicular to the page, and only the back half of the loop is shown. We use the law of Biot and Savart to find the magnetic field at point P on the central perpendicular axis of the loop.

$$dB = \frac{\mu_0}{4\pi} \frac{i\,ds \sin 90°}{r^2}.$$

$$dB_{\parallel} = dB \cos \alpha. = \frac{\mu_0 i \cos \alpha\, ds}{4\pi r^2}.$$

$$r = \sqrt{R^2 + z^2}$$

$$\cos \alpha = \frac{R}{r} = \frac{R}{\sqrt{R^2 + z^2}}.$$

$$dB_{\parallel} = \frac{\mu_0 i R}{4\pi (R^2 + z^2)^{3/2}}\, ds.$$

$$B = \int dB_{\parallel}$$

$$= \frac{\mu_0 i R}{4\pi (R^2 + z^2)^{3/2}} \int ds$$

$$B(z) = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}}.$$

Magnetic Field of a Coil

- For simplicity, first consider only a coil with a single circular loop and only points on its perpendicular central axis, which is *z* axis.
- Magnitude of the magnetic field at such points is:

$$B(z) = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}}$$

R: radius of the circular loop

z: distance of the point in question from the center of the loop.

- The direction of the magnetic field is the same as the direction of the magnetic dipole moment of the loop.
- For axial points far from the loop, we have $z >> R$ → $B(z) \approx \frac{\mu_0 i R^2}{2z^3}$
- πR^2 is the area A of the loop and extending our result to include a coil of N turns,

$$B(z) = \frac{\mu_0}{2\pi} \frac{NiA}{z^3}$$

$\vec{B}$ and $\vec{\mu}$ have the same direction,

$\mu = NiA$

$$\vec{B}(z) = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{z^3}$$

$$\vec{B}(z) = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3} \qquad \text{(current-carrying coil)}$$

- We can regard a current-carrying coil as a magnetic dipole:

(1) it experiences a torque when we place it in an external magnetic field;
(2) it generates its own intrinsic magnetic field, given, for distant points along its axis

- *If we were to place a current- carrying coil in an external magnetic field, it would tend to rotate just like a bar magnet would.*

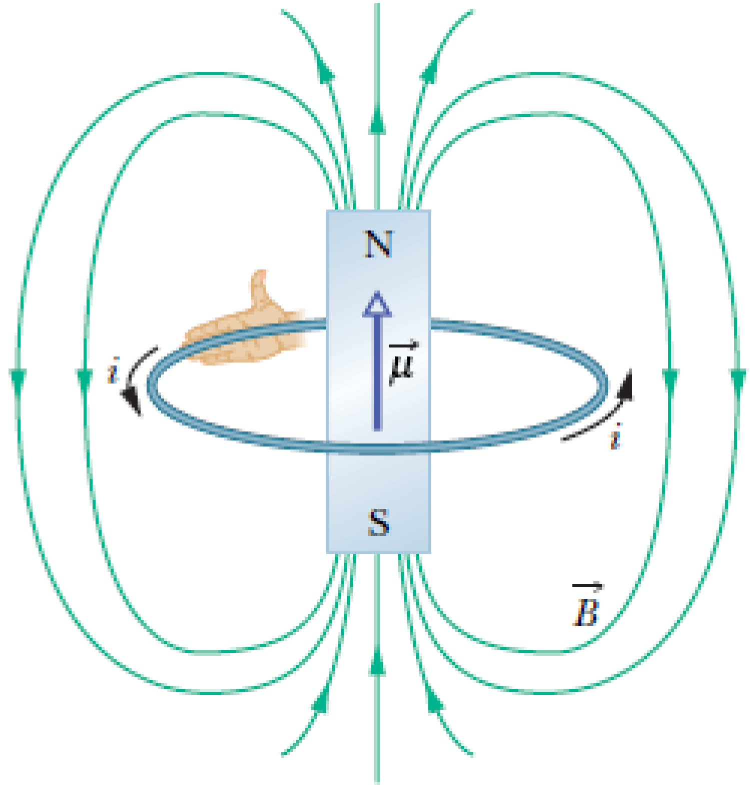


Fig. 29-21 A current loop produces a magnetic field like that of a bar magnet and thus has associated north and south poles. The magnetic dipole moment $\vec{\mu}$ of the loop, its direction given by a curled–straight right-hand rule, points from the south pole to the north pole, in the direction of the field $\vec{B}$ within the loop.

Learning Objectives

29.01 Sketch a current-length element in a wire and indicate the direction of the magnetic field that it sets up at a given point near the wire.

29.02 For a given point near a wire and a given current-element in the wire, determine the magnitude and direction of the magnetic field due to that element.

29.03 Identify the magnitude of the magnetic field set up by a current-length element at a point in line with the direction of that element.

29.04 For a point to one side of a long straight wire carrying current, apply the relationship between the magnetic field magnitude, the current, and the distance to the point.

29.05 For a point to one side of a long straight wire carrying current, use a right-hand rule to determine the direction of the magnetic field vector.

29.06 Identify that around a long straight wire carrying current, the magnetic field lines form circles.

29.07 For a point to one side of the end of a semi-infinite wire carrying current, apply the relationship between the magnetic field magnitude, the current, and the distance to the point.

29.08 For the center of curvature of a circular arc of wire carrying current, apply the relationship between the magnetic field magnitude, the current, the radius of curvature, and the angle subtended by the arc (in radians).

29.09 For a point to one side of a short straight wire carrying current, integrate the Biot–Savart law to find the magnetic field set up at the point by the current.

Learning Objectives

29.10 Given two parallel or anti-parallel currents, find the magnetic field of the first current at the location of the second current and then find the resulting force acting on that second current.

29.11 Identify that parallel currents attract each other, and anti-parallel currents repel each other.

29.12 Describe how a rail gun works.

Learning Objectives

29.13 Apply Ampere's law to a loop that encircles current.

29.14 With Ampere's law, use a right-hand rule for determining the algebraic sign of an encircled current.

29.15 For more than one current within an Amperian loop, determine the net current to be used in Ampere's law.

29.16 Apply Ampere's law to a long straight wire with current, to find the magnetic field magnitude inside and outside the wire, identifying that only the current encircled by the Amperian loop matters.

Learning Objectives

29.17 Describe a solenoid and a toroid and sketch their magnetic field lines.

29.18 Explain how Ampere's law is used to find the magnetic field inside a solenoid.

29.19 Apply the relationship between a solenoid's internal magnetic field B, the current i, and the number of turns per unit length n of the solenoid.

29.20 Explain how Ampere's law is used to find the magnetic field inside a toroid.

29.21 Apply the relationship between a toroid's internal magnetic field B, the current i, the radius r, and the total number of turns N.

Learning Objectives

29.22 Sketch the magnetic field lines of a flat coil that is carrying current.

29.23 For a current-carrying coil, apply the relationship between the dipole moment magnitude μ and the coil's current i, number of turns N, and area per turn A.

29.24 For a point along the central axis, apply the relationship between the magnetic field magnitude B, the magnetic moment μ, and the distance z from the center of the coil.