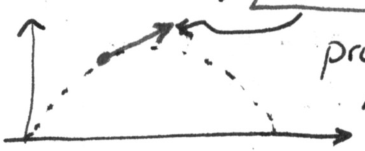
Chapter 3 – Vectors

* Language of vectors to describe physical quantities
* Vectors follow certain rules of combination
* Motion along a straight line: ∓ sign is enough to indicate the direction
* Motion in three dimensions: ∓ sign is not enough to indicate the direction of motion $\Rightarrow$ **use vectors**



position, displacement, velocity, acceleration } All defined by means of vectors

vector
· magnitude
· direction

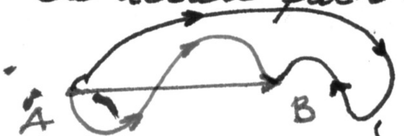
scalar
· magnitude

time, speed, temperature } Quantities which only indicate magnitude

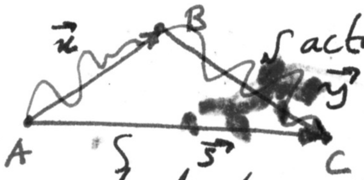
Vectors: shown by arrows

The head of arrow signifies direction.
The length of arrow signifies magnitude, $|\vec{a}|$ or a

Displacement vector: Change of position. Does not tell us the actual path that particle takes.

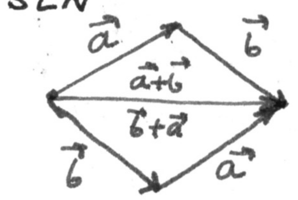


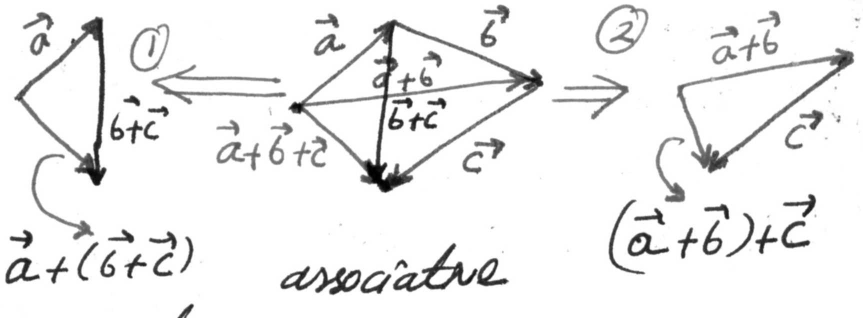
Adding Vectors Geometrically

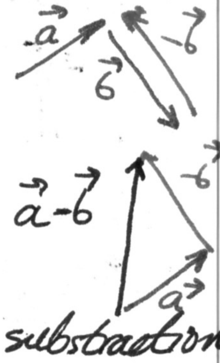


$$\vec{s} = \vec{x} + \vec{y}$$

net displacement is the $\underbrace{\text{vector sum}}_{\text{resultant}}$. not the usual algebraic sum.



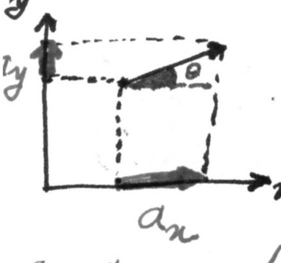




* multiplication by a scalar, s

$s(\vec{a}+\vec{b}) = s\vec{a} + s\vec{b}$ Distrubutive low

Component of vectors



a_x, a_y : scalar quantity

The component of a vector along an axis is the projection of the vector onto that axis.

$a_x = |\vec{a}| \cos\theta$
$a_y = |\vec{a}| \sin\theta$

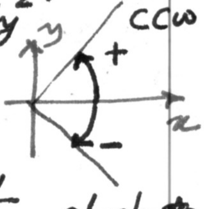
Construction of a vector from its components

Suppose that components are a_x & a_y

Magnitude $|\vec{a}| = \sqrt{a_x^2 + a_y^2}$

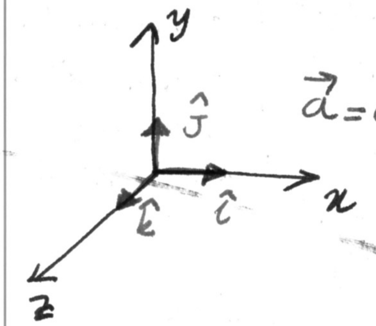
Direction (angle) $\tan\theta = \dfrac{a_y}{a_x}$

$\theta = \tan^{-1} \dfrac{a_y}{a_x}$



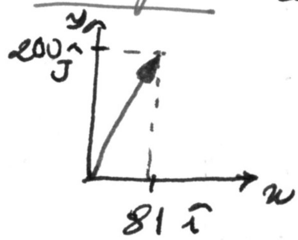
Unit Vectors

A unit vector is a vector that has a magnitude of exactly 1 and points in a particular direction.



$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

Example: $\vec{a} = 81\,\hat{i} + 200\,\hat{j}$



$|\vec{a}| = \sqrt{81^2 + 200^2} \cong 215$

$\tan\theta = \frac{a_y}{a_x} = \frac{200}{81}$

$\Rightarrow \theta \sim 68^\circ$

Adding vectors by components

$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$
$\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$

$\vec{r} = \vec{a} + \vec{b}$

$\vec{r} = \underbrace{(a_x + b_x)}_{r_x}\hat{i} + \underbrace{(a_y + b_y)}_{r_y}\hat{j} + \underbrace{(a_z + b_z)}_{r_z}\hat{k}$

Example:

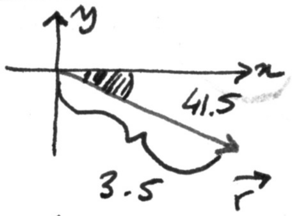
$\vec{a} = 4.2\,\hat{i} - 1.5\,\hat{j}$
$\vec{b} = -1.6\,\hat{i} + 2.9\,\hat{j}$
$c = -3.7\,\hat{j}$

$\vec{r} = \vec{a} + \vec{b} + \vec{c}$
magnitude & angle

$\vec{r} = (4.2 - 1.6)\,\hat{i} + (-1.5 + 2.9 - 3.7)\,\hat{j} + (0+0+0)\,\hat{k}$

$\boxed{\vec{r} = 2.6\,\hat{i} + (-2.3)\,\hat{j}}$

$|\vec{r}| = \sqrt{(2.6)^2 + (-2.3)^2} \cong 3.5$



$\theta = \tan^{-1}\frac{-2.3}{2.6} = -41.5^\circ$

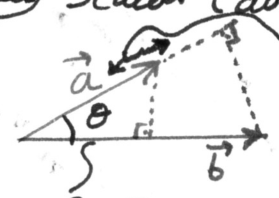
SLN

Multiplying Vectors

① Multiplying or dividing vector by a scalar. $\vec{a}(s) = s\vec{a}$ $\vec{a} = 3\hat{i} + 5\hat{j}$, $2\vec{a} = 6\,\hat{i} + 10\,\hat{j}$

② " " " " vector

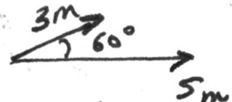
(2a) Scalar (dot) product



$b\cos\theta$

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$(a\cos\theta)b = a(b\cos\theta)$



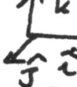
$\vec{a}\cdot\vec{b} = (3(\cos 60))5 = 3(5\cos 60^\circ)$

properties

$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$
$c\vec{a}\cdot\vec{b} = c(\vec{a}\cdot\vec{b})$
$(\vec{a}+\vec{b})\cdot\vec{c} = \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{c}$

in cartesian coor.

$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$
$\hat{i}\cdot\hat{j} = \hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{k} = 0$



$\hat{i}\cdot\hat{i} = 1\,1\cos 0$
$\hat{i}\cdot\hat{j} = 1\,1\cos 90^\circ$

Example. $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$, $\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$

$\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z$

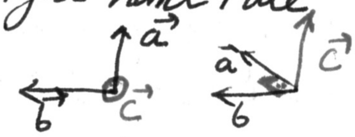
produces a scalar

(2b) Vector (cross) product

Produces a new vector

$|\vec{a}\times\vec{b}| = \underbrace{|\vec{a}||\vec{b}|\sin\theta}_{\text{magnitude of vector}}$, $\vec{a}\times\vec{b} = \vec{c}$

* The direction of third vector is detemined by right hand rule



properties

$\vec{a}\times\vec{b} = -(\vec{b}\times\vec{a})$
$c(\vec{a}\times\vec{b}) = c\vec{a}\times\vec{b} = \vec{a}\times c\vec{b}$
$(\vec{a}+\vec{b})\times\vec{c} = (\vec{a}\times\vec{c}) + (\vec{b}\times\vec{c})$

$\hat{i}\times\hat{j} = \hat{k}$ | $\hat{i}\times\hat{i} = 0$
$\hat{j}\times\hat{k} = \hat{i}$ | $\hat{j}\times\hat{j} = 0$
$\hat{k}\times\hat{i} = \hat{j}$ | $\hat{k}\times\hat{k} = 0$

$i \rightarrow j \rightarrow k$

otherwise minus sign

2b Components of cross product

$$\left.\begin{aligned}\vec{a} &= a_x\hat{i} + a_y\hat{j} + a_z\hat{k}\\ \vec{b} &= b_x\hat{i} + b_y\hat{j} + b_z\hat{k}\end{aligned}\right\} \quad \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} \sim \text{determinant}$$

$$\begin{aligned}\vec{a}\times\vec{b} = &\left(\cancelto{0}{a_x b_x\, \hat{i}\times\hat{i}} + a_x b_y\, \hat{i}\times\hat{j} + a_x b_z\, \hat{i}\times\hat{k}\right) + \\ &\left(a_y b_x\, \hat{j}\times\hat{i} + \cancelto{0}{a_y b_y\, \hat{j}\times\hat{j}} + a_y b_z\, \hat{j}\times\hat{k}\right) + \\ &\left(a_z b_x\, \hat{k}\times\hat{i} + a_z b_y\, \hat{k}\times\hat{j} + \cancelto{0}{a_z b_z\, \hat{k}\times\hat{k}}\right)\end{aligned}$$

$i \to j \to k \to i$

otherwise minus sign

$\hat{i}\times\hat{j} = \hat{k}$

$\hat{j}\times\hat{i} = -\hat{k}$

$$= a_x b_y\,\hat{k} + a_x b_z(-\hat{j}) + a_y b_x(-\hat{k}) + a_y b_z\,\hat{i} + a_z b_x\,\hat{j} + a_z b_y(-\hat{i})$$

$$\boxed{\vec{a}\times\vec{b} = (a_y b_z - a_z b_y)\,\hat{i} + (a_z b_x - a_x b_z)\,\hat{j} + (a_x b_y - a_y b_x)\,\hat{k}}$$

Components