

Ceng272 -Exercise Set 3

3.5 Determine the value c so that each of the following functions can serve as a probability distribution of the discrete random variable X :

(a) $f(x) = c(x^2 + 4)$, for $x = 0, 1, 2, 3$;

(b) $f(x) = c \binom{2}{x} \binom{3}{3-x}$, for $x = 0, 1, 2$.

3.6 The shelf life, in days, for bottles of a certain prescribed medicine is a random variable having the density function

$$f(x) = \begin{cases} \frac{20,000}{(x+100)^3}, & x > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the probability that a bottle of this medicine will have a shelf life of

(a) at least 200 days;

(b) anywhere from 80 to 120 days.

3.12 An **investment** firm offers its customers municipal bonds that mature after varying numbers of years. Given that the cumulative distribution function of T , the number of years to maturity for a randomly selected bond, is,

$$F(t) = \begin{cases} 0, & t < 1, \\ \frac{1}{4}, & 1 \leq t < 3, \\ \frac{3}{4}, & 3 \leq t < 5, \\ \frac{5}{8}, & 5 \leq t < 7, \\ 1, & t \geq 7, \end{cases}$$

find

- (a) $P(T = 5)$;
- (b) $P(T > 3)$;
- (c) $P(1.4 < T < 6)$.

3.21 Consider the density function

$$f(x) = \begin{cases} k\sqrt{x}, & 0 < X < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Evaluate k .
- (b) Find $F(x)$ and use it to evaluate

$$P(0.3 < X < 0.6).$$

3.38 If the joint probability distribution of X and Y is given by

$$f(x, y) = \frac{x + y}{30}, \quad \text{for } x = 0, 1, 2, 3; y = (1, 1, 2,$$

find

- (a) $P(X \leq 2, Y = 1)$;
- (b) $P(X > 2, Y \leq 1)$;
- (c) $P(X > Y)$;
- (d) $P(X + Y = 4)$.