Ceng 375 Numerical Computing
Midterm
Nov 9, 2004 13.40–15.30
Good Luck!

1 (15 Pts) In Newton's method the approximation x_{n+1} to a root of $f(x) = 0$ is computed from the approximation x_n using the equation

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

i Derive the above formula, using a Taylor series of $f(x)$.

ii For $f(x) = x - 3^{-x}$, refine the approximation $x_0 = 0.54$ to the unique root of $f(x)$ by carrying out one iteration of Newton's method.

Hints: $3^{-0.54} = 0.55253, 3^{-1.54} = 0.18418$

2 (15 Pts) Consider the function:

$$f(x) = cos(x) - x$$

i Show that this function has a simple root in the interval $0 < x < 1$

ii Estimate this root using two iterations of the Secant Method.

iii Estimate the error in your answer to part ii.

iv Would Newton's method have been preferable in this problem? (Briefly explain your answer!)

Hints: $cos(0) = 1, cos(1) = 0.5403$

3 (15 Pts) Illustrate graphically the cases of monotonic convergence, oscillatory convergence and divergence for the fixed-point ($x = g(x)$) iteration method.

4 (15 Pts) Let

$$A = \begin{bmatrix} 2 & 9 \\ 3 & -10 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 6 & 2 \\ 4 & -1 & 3 \\ 1 & -3 & -1 \end{bmatrix}$$

i Find the characteristic polynomials of both A and B.

ii Find the eigenvalues of both A and B.

iii Is $[0.2104, 0.8401]$ an eigenvector of A?

5 (20 Pts) Solve this system by Gaussian elimination with pivoting

$$\begin{bmatrix} 1 & -2 & 4 & 6 \\ 8 & -3 & 2 & 2 \\ -1 & 10 & 2 & 4 \end{bmatrix}$$

i How many row interchanges are needed?

ii Repeat without any row interchanges. Do you get the same results?

iii You could have saved the row multipliers and obtained a LU equivalent of the coefficient matrix. Use this LU to solve but with right-hand sides of $[1, -3, 5]^T$

6 (20 Pts) Consider the matrix

$$A = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 1 & 3 \\ -3 & 0 & 5 \end{bmatrix}$$

i Use the Gaussian elimination method to triangularize this matrix and from that gets its determinant.

ii Get the inverse of the matrix through Gaussian elimination.

iii Get the inverse of the matrix through Gauss-Jordan method.

7 (20 Pts) Consider the linear system

$$\begin{aligned} 7x_1 - 3x_2 + 4x_3 &= 6 \\ -3x_1 + 2x_2 + 6x_3 &= 2 \\ 2x_1 + 5x_2 + 3x_3 &= -5 \end{aligned}$$

i Solve this system with the Jacobi method. First rearrange to make it dioganally dominant if possible. Use $[0, 0, 0]$ as the starting vector. Proceed only 1 iteration.

ii Repeat with Gauss-Seidel method. Compare with Jacobi method.