

i. Generate the Chebyshev Polynomials up to order 10

Given $T_0(x) = 1$
 $T_1(x) = x$
 $T_2(x) = 2x^2 - 1$ } $T_{n+1} = 2xT_n - T_{n-1}$ General Term

$$T_3 = 2x(2x^2 - 1) - x$$

$$T_4 = 2x(2x(2x^2 - 1) - x) - (2x^2 - 1)$$

$$T_5 = 2x(2x(2x(2x^2 - 1) - x) - (2x^2 - 1)) - (2x(2x^2 - 1) - x)$$

$$T_6 = 2x[2x(2x(2x(2x^2 - 1) - x) - (2x^2 - 1)) - (2x(2x^2 - 1) - x)] - (2x(2x(2x^2 - 1) - x) - (2x^2 - 1))$$

$$T_7 = 2xT_6 - T_5$$

$$T_8 = 2xT_7 - T_6$$

$$T_9 = 2xT_8 - T_7$$

$$T_{10} = 2xT_9 - T_8$$

ii. Sixth degree Maclaurin Series & Chebyshev Polynomials

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} \quad \& \quad T_6(x) = 32x^6 - 48x^4 + 18x^2 - 1$$

Economize Maclaurin

$$\rightarrow 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} - \frac{1}{32} + \frac{1}{6!} (32x^6 - 48x^4 + 18x^2 - 1)$$