Ceng 375 Numerical Computing
Midterm
Nov 16, 2005 10.40–12.30
Good Luck!

1 (20 Pts)

i Under what conditions can parallel processing not be used to speed up a computation?

ii How many iterations of bisection will be required to attain an accuracy of 10^{-4} if the starting interval is $[a, b]$?

2 (20 Pts) The function $f(x) = 4x^3 - 1 - e^{x^2/2}$ has values of zero near $x = 1.0$ and $x = 3.0$.

i What is the derivative of $f(x)$?

ii If you begin Newton's method at $x = 2$, which root is reached? How many iterations to achieve an error less than 10^{-5}?

3 (20 Pts) Solve this system by Gaussian elimination with pivoting

$$\begin{bmatrix} 1 & -2 & 4 & 6 \\ 8 & -3 & 2 & 2 \\ -1 & 10 & 2 & 4 \end{bmatrix}$$

i How many row interchanges are needed?

ii Repeat without any row interchanges. Do you get the same results?

iii You could have saved the row multipliers and obtained a LU equivalent of the coefficient matrix. Use this LU to solve but with right-hand sides of $[-3, 7, -2]^T$

iv Solve the second item again but use only three significant digits of precision.

4 (20 Pts) Consider the linear system

$$\begin{aligned} 7x_1 - 3x_2 + 4x_3 &= 6 \\ -3x_1 + 2x_2 + 6x_3 &= 2 \\ 2x_1 + 5x_2 + 3x_3 &= -5 \end{aligned}$$

i Solve this system with the Jacobi method. First rearrange to make it dioganally dominant if possible. Use $[0, 0, 0]$ as the starting vector.

ii Repeat with Gauss-Seidel method. Compare with Jacobi method.

5 (20 Pts) For the given data points;

x	y
2.1	-12.4
4.1	7.3
7.1	10.1

1. Write out the Lagrangian polynomial from this table

   i confirm that it reproduces the y's for each x-value.

   ii interpolate with it to estimate y at $x = 3$.

   iii extrapolate with it to estimate y at $x = 8$.

2. Suppose in previous item that the y-value for $x = 4.1$ is mistakenly entered as 7.2 rather than 7.3. Repeat the previous item with this incorrect value. How much difference does this make?

3. Expand the Lagrangian polynomials in the previous items to get the quadratics in the form $ax^2 + bx + c$. How different are the values for a,b, and c?

6 (20 Pts) For the given data points;

x	y
1	1.06
2	1.12
3	1.34
5	1.78

i construct the divided-difference table.

ii interpolate for $x = 4$.

iii extrapolate for $x = 5.5$.