

Midterm
Nov 17, 2006 12.40–14.30
Good Luck!

1. **(30 Pts)** Consider the function:

$$f(x) = \cos(x) - 2x = 0$$

- i Use two iterations of Newton's method to estimate the root of this function between $x = 0.0$ and $x = 1.0$
- ii Estimate the error in your answer to part i.
- iii Approximately how many iterations of the bisection method would have been required to achieve the same error?

$$f(x) = \sqrt{x} - \cos(x)$$

- i Describe the general working of a bracketing method. What are the assumptions for this family of methods? Are these assumptions satisfied for $f(x)$?
- ii Describe how the secant method determine a smaller sub-interval containing a root.
- iii What is the advantages and disadvantages of the secant method?
- iv Apply the secant method to $f(x)$, to reduce the size of the original interval $[0, 1]$, twice.

$$A = \begin{bmatrix} 1 & 3 & 1 & 1 \\ 2 & 5 & 2 & 2 \\ -1 & -3 & -3 & 5 \\ 1 & 3 & 2 & 2 \end{bmatrix}$$

by Gaussian elimination (without pivoting). Clearly show how you get the entries of L and U .

$$\begin{aligned}4x_1 + x_2 &= 5 \\x_1 + 5x_2 &= 6\end{aligned}$$

- i Write down the Jacobi iteration formula for this problem given initial guess $x^{(0)} = 0.1$.
- ii Assume that the error (vector) at iteration k is denoted by $e^{(k)}$ and that $\|e^{(0)}\| = 0.9$. How many iterations do we need before $\|e^{(k)}\| \leq 10^{-4}$?