1 In Newton's method the approximation x_{n+1} to a root of $f(x) = 0$ is computed from the approximation x_n using the equation

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

i Derive the above formula, using a Taylor series of $f(x)$.

ii For $f(x) = x - 3^{-x}$, refine the approximation $x_0 = 0.54$ to the unique root of $f(x)$ by carrying out one iteration of Newton's method.

2 Consider the difference approximation

$$f'_n = \frac{-f_{n+2} + 4f_{n+1} - 3f_n}{2h}$$

where f_n means $f(x)$ and f_{n+1} means $f(x+h)$

i Use this formula to approximate the derivative of $f(x) = cos(x)$ at $x = 0$ using step sizes of $h = 0.10$ and 0.20.

ii Make an error analysis. Estimate the order of error $(O(h^?))$.
Hints: The ratio of errors and the difference with the exact value.

3

$$f(x) = 3 * x + sin(x) - e^x$$

This nonlinear equation is solved by using three methods, namely *Bisection*, *Newton's*, *Muller's* methods. Then, the following tables are obtained.

iteration	$(x)_1$	$(x)_2$	$(x)_3$
1	0.50000000000000	0.33333333333333	0.50000000000000
2	0.25000000000000	0.36017071357763	0.35491389049015
3	0.37500000000000	0.36042168047602	0.36046467792776
4	0.31250000000000	0.36042170296032	0.36042169766326
5	0.34375000000000	0.36042170296032	0.36042170296032

iteration	$(f(x))_1$	$(f(x))_2$	$(f(x))_3$
1	3.3070e-01	-1.0000e+00	3.3070e-01
2	-2.8662e-01	-6.8418e-02	-1.3807e-02
3	3.6281e-02	-6.2799e-04	1.0751e-04
4	-1.2190e-01	-5.6252e-08	-1.3252e-08
5	-4.1956e-02	-6.6613e-16	2.2204e-16

i If the exact value is given as 0.36042170296032, fill the following tables (use scientific notation as %12.4e, see the table above);

iteration	$Error_1$	$Error_2$	$Error_3$	$ErrorRatio_1$	$ErrorRatio_2$	$ErrorRatio_3$
1						
2						
3						
4						
5						

ii Analyze the obtained tables. Is the convergence sustained for the each methods? For the sustained ones; at which iteration and why?

iii What can you say about the speed of convergences for each method?

iv By using your answers for the previous items, fill the following table. You should explain your decision.

	$Method_1$	$Method_2$	$Method_3$
Name			

v Which method is the best one? Why?

4 Solve this system by Gaussian elimination with pivoting

$$\begin{bmatrix} 1 & -2 & 4 & 6 \\ 8 & -3 & 2 & 2 \\ -1 & 10 & 2 & 4 \end{bmatrix}$$

i How many row interchanges are needed?

ii Repeat without any row interchanges. Do you get the same results?

iii You could have saved the row multipliers and obtained a LU equivalent of the coefficient matrix. Use this LU to solve but with right-hand sides of $[-3, 7, -2]^T$

iv Solve the second item again but use three significant digits of precision.

5 Consider the matrix

$$A = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 1 & 3 \\ -3 & 0 & 5 \end{bmatrix}$$

i Use the Gaussian elimination method to triangularize this matrix and from that gets its determinant.

ii Get the inverse of the matrix through Gaussian elimination.

iii Get the inverse of the matrix through Gauss-Jordan method.

6 Find the power fit $y = Ax^2$ for the following data,

x_k	y_k
2.0	5.1
2.3	7.5
2.6	10.6
2.9	14.4
3.2	10.0

7 For the given set of data, find the least-squares curve, $f(x) = Ce^{Ax}$, by using the change of variables $X = x, Y = ln(y)$, and $C = e^B$, to linearize the data points.

x_k	y_k
1	0.6
2	1.9
3	4.3
4	7.6
5	12.6

8 Write the expression to economize the the Maclaurin series for e^{2x} with the precision 0.008 by using Chebyshev polynomials. Do not perform the calculation.

9 Find the Fourier series representation of the given function.

$$f(x) = \begin{cases} -1 & for \ -\pi < x < 0 \\ 1 & for \ 0 < x < \pi \end{cases}$$

10 Find the Fourier series representation of the given function.

$$f(x) = \begin{cases} -1 & for \ \pi/2 < x < \pi \\ 1 & for \ -\pi/2 < x < \pi/2 \\ -1 & for \ -\pi < x < -\pi/2 \end{cases}$$

11 Consider the following table of data

x_i	f_i
0.0000	0.0000
0.2000	0.5879
0.4000	1.0637
0.6000	1.3927
0.8000	1.5573
1.0000	1.5575
1.2000	1.4091

i Approximate $\int_0^{1.2} f(x)\mathrm{dx}$ using the *Trapezoidal Rule* and a step size of $h = 0.6$.

ii Approximate $\int_0^{1.2} f(x)\mathrm{dx}$ using the *Trapezoidal Rule* and a step size of $h = 0.2$.

iii Estimate the *error* in your answer to previous item.
Hint: Use the procedure to estimate the proportionality factor, C.

12 Consider the function $f(x) = x^2$;

i Fill the following table within the five digit accuracy

x_i	f_i
0.00000	0.00000
1.20000	

ii Approximate $\int_0^{1.2} f(x)\mathrm{dx}$ using the *Trapezoidal Rule* and a step size of $h = 0.2$.

iii Approximate $\int_0^{1.2} f(x)\mathrm{dx}$ using the *Trapezoidal Rule* and a step size of $h = 0.4$.

iv Analyze and compare your results. Estimate the *error* in your answers.