# Ceng 375 Numerical Computing
# Final
# Jan 15, 2009 13.00–15.00
# Good Luck!

1. **(10pts)** Choose only **two** questions.

   i What are the advantages and disadvantages of numerical analysis?

   ii Describe truncation and round-off errors. Give example.

   iii Describe the concept of ill-conditionness. Give an example.

   iv What information can be obtained from the determinant of a matrix?

   v Why do we need pivoting while solving sets of equations by elimination methods? Can we skip pivoting and under which circumstances?

   vi What does singularity mean for a matrix? Make a comparison of singular and nonsingular matrices.

   vii What information can be obtained from the condition number of a matrix?

   viii What are the differences between the interpolation and curve fitting?

2. **(20pts)** Choose only **two** questions.

i For the given data points; we suggest the relation $y(x) = \alpha e^{\beta x}$.

(a) First, construct the normal equations.

(b) Then, describe the remaining steps.

ii Solve the following linear system by either by Jacobi or Gauss-Seidel iterations;

$$\begin{aligned} 4x - y + z &= 7 \\ -2x + y + 5z &= 15 \\ 4x - 8y + z &= -21 \end{aligned}$$

(a) Start by $P_0 = (1, 2, 2)$. Iterate only two steps.

(b) Compare Jacobi or Gauss-Seidel methods.

iii Consider the function:

$$f(x) = cos(x) - x$$

(a) Show that this function has a simple root in the interval $0 < x < 1$

(b) Estimate this root using two iterations of the Secant Method. The secant algorithm is

$$x_{n+1} = x_n - f(x_n)\frac{(x_{n-1} - x_n)}{f(x_{n-1}) - f(x_n)}$$

(c) Estimate the error in your answer to part ii.

3. **(20pts)** Consider the matrix

$$A = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 1 & 3 \\ -3 & 0 & 5 \end{bmatrix}$$

i Use the Gaussian elimination method to triangularize this matrix and from that gets its determinant.

ii Get the inverse of the matrix through Gauss-Jordan method.

4. **(20pts)**

i Find the Fourier coefficients for $f(x) = x^2 - 1$ if it is periodic and one period extends from $x = -1$ to $x = 2$. Do not evaluate the integrals.

ii Write the Fourier series expansion for this function until $3^{rd}$ term.

5. **(20pts)** Write the expression to economize the the Maclaurin series for $e^{2x}$ with the precision 0.008 by using Chebyshev polynomials.
   **Hint:** The two-term recursion formula

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$$
$$T_0(x) = 1$$
$$T_1(x) = x$$

6. **(20pts)** Consider the function $f(x) = x^3$. Following table within the five digit accuracy is given.

| $x_i$ | $f_i$ |
|---|---|
| 0.00000 | 0.00000 |
| 0.20000 | 0.00800 |
| 0.40000 | 0.06400 |
| 0.60000 | 0.21600 |
| 0.80000 | 0.51200 |
| 1.00000 | 1.00000 |
| 1.20000 | 1.72800 |

i Approximate $\int_0^{1.2} f(x)\text{dx}$ using the *Trapezoidal Rule* and a step size of $h = 0.2$.

ii Approximate $\int_0^{1.2} f(x)\text{dx}$ using the *Trapezoidal Rule* and a step size of $h = 0.4$.

iii Estimate the *error* in your answers;

(a) Find the exact value of the integral simply by integrating the given function. Then, find the errors for parts i and ii.

(b) Also use the following global error formula to find the errors for parts i and ii.

$$Global\ error = (-1/12)h^3 n f''(\xi)$$

(c) Analyze and compare these error values.