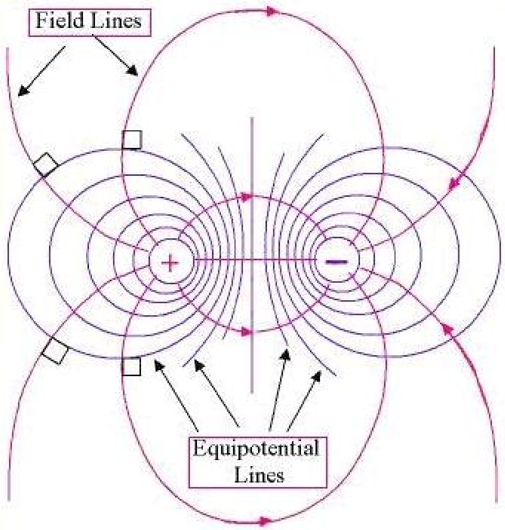
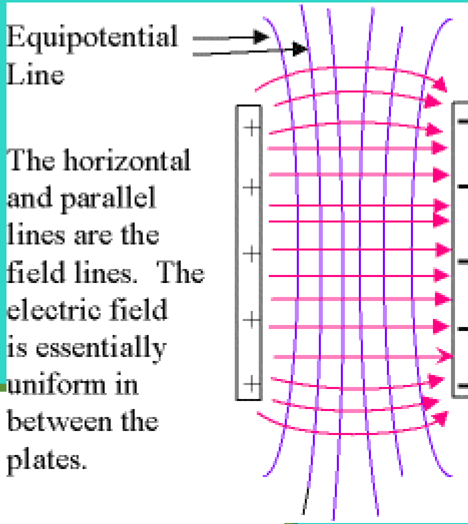
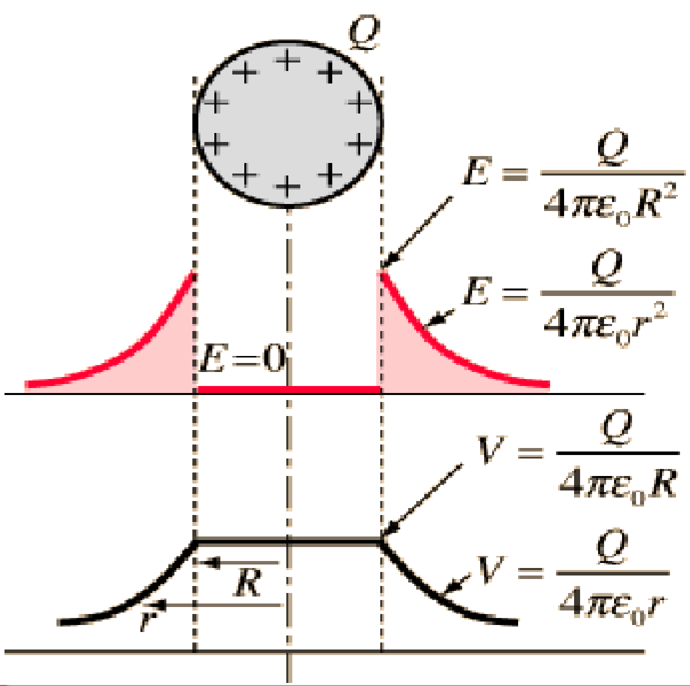




Chapter 24

Electric Potential



24 ELECTRIC POTENTIAL 628

- Experimentally, physicists and engineers discovered that the **electric force is conservative** and thus has an associated electric potential energy.

gravitational field → force ↔ path independency

- *The motivation for associating a potential energy with a force is that we can then apply the principle of the conservation of mechanical energy to closed systems involving the force.*

$W = -\Delta U$

$W = +\Delta K$

$\Delta K + \Delta U = 0$

$K_f - K_i + U_f - U_i$

$K_f + U_f = K_i + U_i$

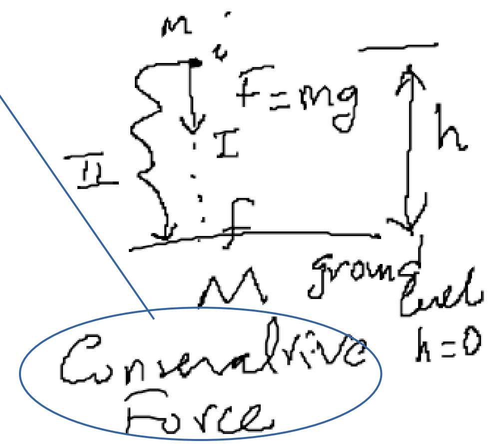
Closed system

$$dW = \vec{F} \bullet d\vec{s}$$

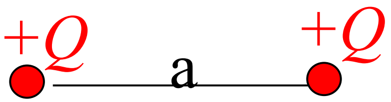
$$dW = q_0 \vec{E} \bullet d\vec{s}$$

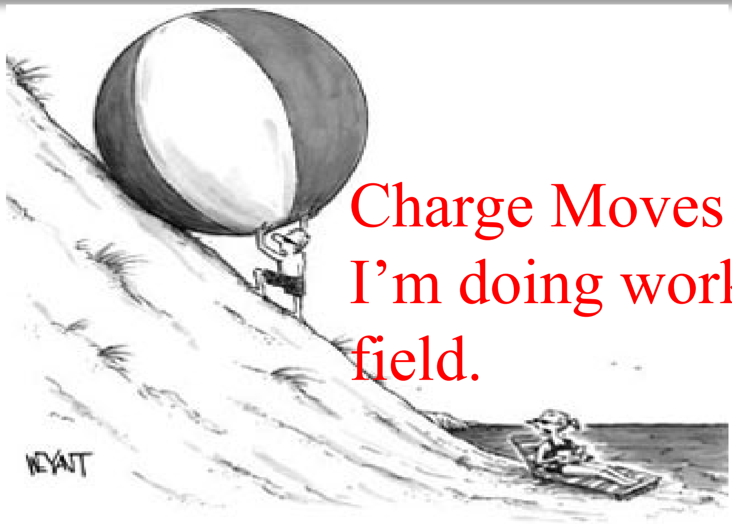
$$W = \int_i^f dW = \int_i^f q_0 \vec{E} \bullet d\vec{s}$$

$$\Delta V = V_f - V_i = -\frac{W}{q_0} = -\int_i^f \vec{E} \bullet d\vec{s}$$



Applied Positive Work: Potential Energy Increases

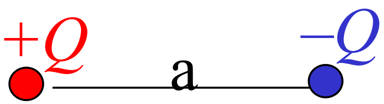




"Can't you ever relax?"

Charge Moves Uphill:
I'm doing work Against field.

Applied Negative Work: Potential Energy Decreases



Work done by field is negative of applied work done by me.

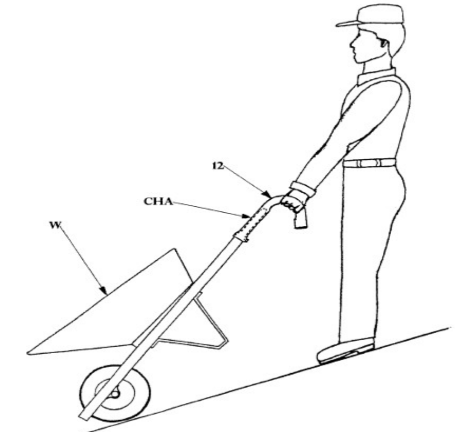


FIGURE 8

Charge Moves Downhill:
I'm doing work With field

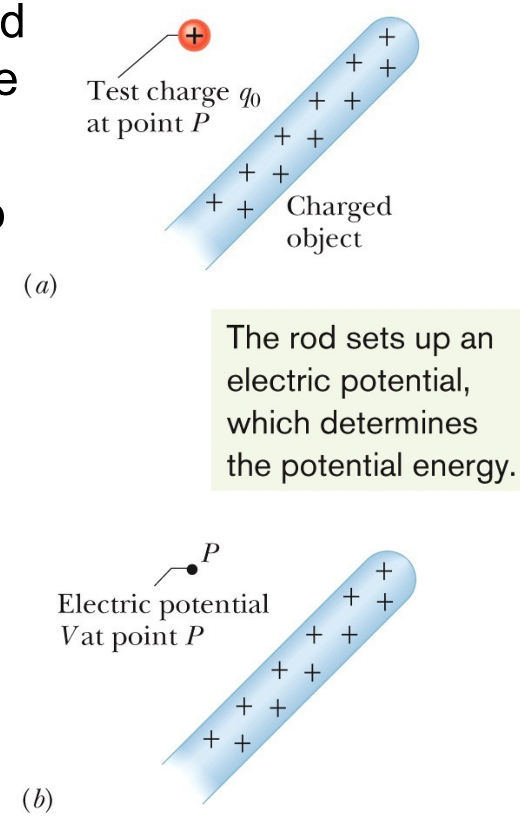
http://www.phys.lsu.edu/~jdowling/PHYS21132-SP15/lectures/index.html

- When an electrostatic force acts between two or more charged particles, we can assign an **electric potential energy *U*** to the system.
- If the system changes its configuration from an initial state *i* to a different final state *f*, the electrostatic force does work *W* on the particles. If the resulting change is ΔU, then

$$\Delta U = U_f - U_i = -W.$$

- As with other **conservative forces**, the work done by the electrostatic force is *path independent*.
- Usually the reference configuration of a system of charged particles is taken to be that in which the particles are all infinitely separated from one another. +q far away -q
- The corresponding reference potential energy is usually set be zero. Therefore, $U = -W_\infty.$

$$U_\infty = 0 = U_i \qquad U_f - 0 = -W_\infty$$



a) A test charge has been brought in from infinity to point P in the electric field of the rod.
b) We define an electric potential V at P based on the potential energy of the configuration in (a).

Example, Work and potential energy in an electric field:

Electrons are continually being knocked out of air molecules in the atmosphere by cosmic-ray particles coming in from space. Once released, each electron experiences an electrostatic force $\vec{F}$ due to the electric field $\vec{E}$ that is produced in the atmosphere by charged particles already on Earth. Near Earth's surface the electric field has the magnitude $E = 150$ N/C and is directed downward. What is the change ΔU in the electric potential energy of a released electron when the electrostatic force causes it to move vertically upward through a distance $d = 520$ m (Fig. 24-1)?

KEY IDEAS

(1) The change ΔU in the electric potential energy of the electron is related to the work W done on the electron by the electric field. Equation 24-1 ($\Delta U = -W$) gives the relation.

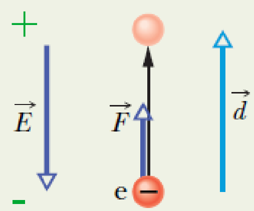


Fig. 24-1 An electron in the atmosphere is moved upward through displacement $\vec{d}$ by an electrostatic force $\vec{F}$ due to an electric field $\vec{E}$.

(2) The work done by a constant force $\vec{F}$ on a particle undergoing a displacement $\vec{d}$ is

$$W = \vec{F} \cdot \vec{d}. \qquad (24\text{-}3)$$

(3) The electrostatic force and the electric field are related by the force equation $\vec{F} = q\vec{E}$, where here q is the charge of an electron ($= -1.6 \times 10^{-19}$ C).

Calculations: Substituting for $\vec{F}$ in Eq. 24-3 and taking the dot product yield

$$W = q\vec{E} \cdot \vec{d} = qEd \cos\theta, \qquad (24\text{-}4)$$

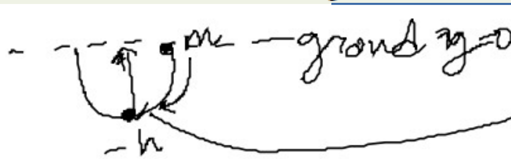
where θ is the angle between the directions of $\vec{E}$ and $\vec{d}$. The field $\vec{E}$ is directed downward and the displacement $\vec{d}$ is directed upward; so $\theta = 180°$. Substituting this and other data into Eq. 24-4, we find

$$W = (-1.6 \times 10^{-19}\ \text{C})(150\ \text{N/C})(520\ \text{m}) \cos 180°$$
$$= 1.2 \times 10^{-14}\ \text{J}.$$

Equation 24-1 then yields

$$\Delta U = -W = -1.2 \times 10^{-14}\ \text{J}. \qquad \text{(Answer)}$$

This result tells us that during the 520 m ascent, the electric potential energy of the electron *decreases* by 1.2×10^{-14} J.



to a better configuration (Potential Energy)

24-3 Electric Potential

- The potential energy per unit charge at a point in an electric field is called the **electric potential *V (or simply the potential)*** at that point. This is a scalar quantity. Thus,

$$V = \frac{U}{q}. \qquad \frac{U}{q} \equiv \frac{Fd}{|q|} \equiv \frac{\frac{k|q||Q|}{d^2}d}{|q|} \equiv Ed$$

- The *electric potential difference V* between any two points *i* and *f* in an electric field is equal to the difference in potential energy per unit charge between the two points. Thus,

$$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q}. \quad = -\frac{W}{q} \qquad \text{(potential difference defined).}$$

- The potential difference between two points is thus the *negative of the work done* (ΔU=-W) by the electrostatic force to move a unit charge from one point to the other.
- If we set *U_i =0* at infinity as our reference potential energy, then the electric potential V must also be zero there. Therefore, the electric potential at any point in an electric field can be defined to be

$$V = -\frac{W_\infty}{q} \qquad \text{(potential defined)}$$

- Here *W_∞* is the work done by the electric field on a charged particle as that particle moves in *from infinity to point f (V_i=0 → V_f=V).*

24-3 Electric Potential: Units

- The SI unit for potential is the joule per coulomb. This combination is called the *volt (abbreviated V).*

$$1 \text{ volt} = 1 \text{ joule per coulomb.}$$

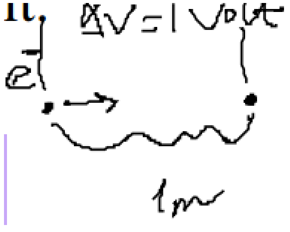
- This unit of volt allows us to adopt a more conventional unit for the electric field, E, which is expressed in newtons per coulomb.

$$1\ \text{N/C} = \left(1\frac{\text{N}}{\text{C}}\right)\left(\frac{1\ \text{V}\cdot\text{C}}{1\ \text{J}}\right)\left(\frac{1\ \text{J}}{1\ \text{N}\cdot\text{m}}\right) = 1\ \text{V/m}.$$

$$E \to \frac{V}{d} \Rightarrow V = -\int \vec{E}\cdot d\vec{s}$$

- We can now define an energy unit that is a convenient one for energy measurements in the atomic/subatomic domain:
  - *One electron-volt (eV)* is the energy equal to the work required to move a single elementary charge *e*, such as that of the electron or the proton, through a potential difference of exactly one volt.
  - The magnitude of this work is *qΔV*, and

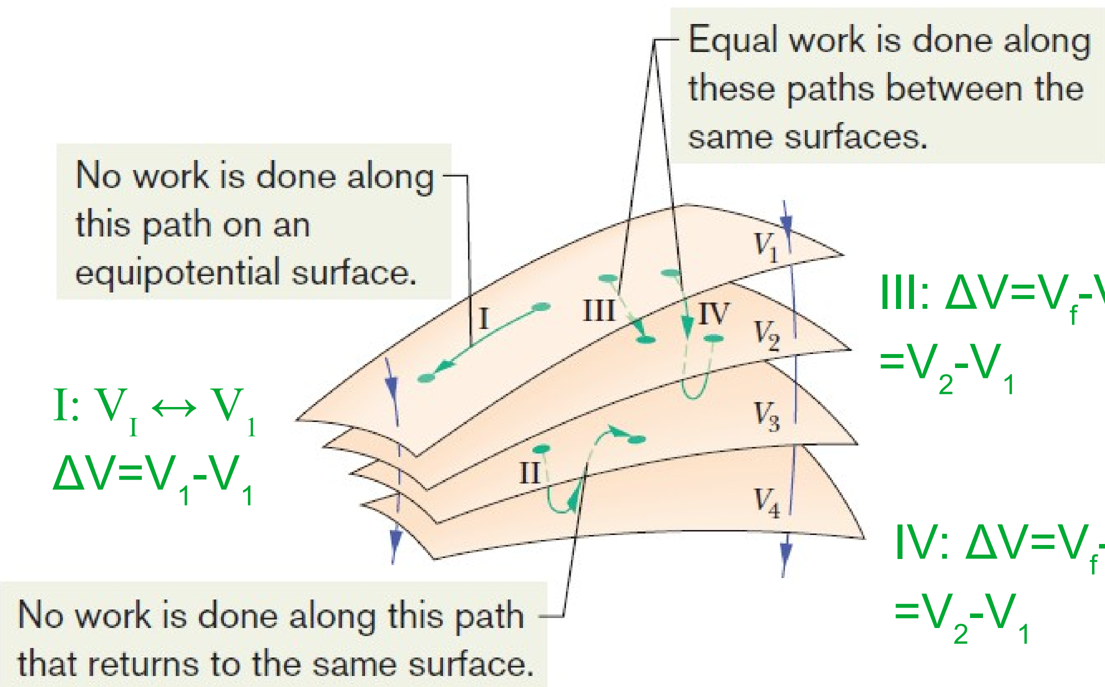
$$1\ \text{eV} = e(1\ \text{V}) = (1.60 \times 10^{-19}\ \text{C})(1\ \text{J/C}) = 1.60 \times 10^{-19}\ \text{J}.$$



24-3 Electric Potential: Work done by an Applied Force

- If a particle of charge *q* is moved from point *i* to point *f* **in an electric field by applying a force** to it, the applied force does work W_{app} on the charge while the electric field does work *W* on it. The change *K* in the kinetic energy of the particle is $$\Delta K = K_f - K_i = W_{\text{app}} + W.$$
- If the particle is stationary before and after the move, Then K_f and K_i are both zero. $v_i = v_f = 0$ $$W_{\text{app}} = -W.$$
- Relating the work done by our applied force to the change in the potential energy of the particle during the move, one has $$\Delta U = U_f - U_i = W_{\text{app}}.$$
- We can also relate W_{app} to the electric potential difference ΔV between the initial and final locations of the particle: $$W_{\text{app}} = q\,\Delta V.$$

- Adjacent points that have the same **electric potential form** an **equipotential surface**, which can be either an imaginary surface or a real, physical surface.
- No net work *W* is done on a charged particle by an electric field when the particle moves between two points *i* and *f* on the same equipotential surface.



I: $V_I \leftrightarrow V_1$

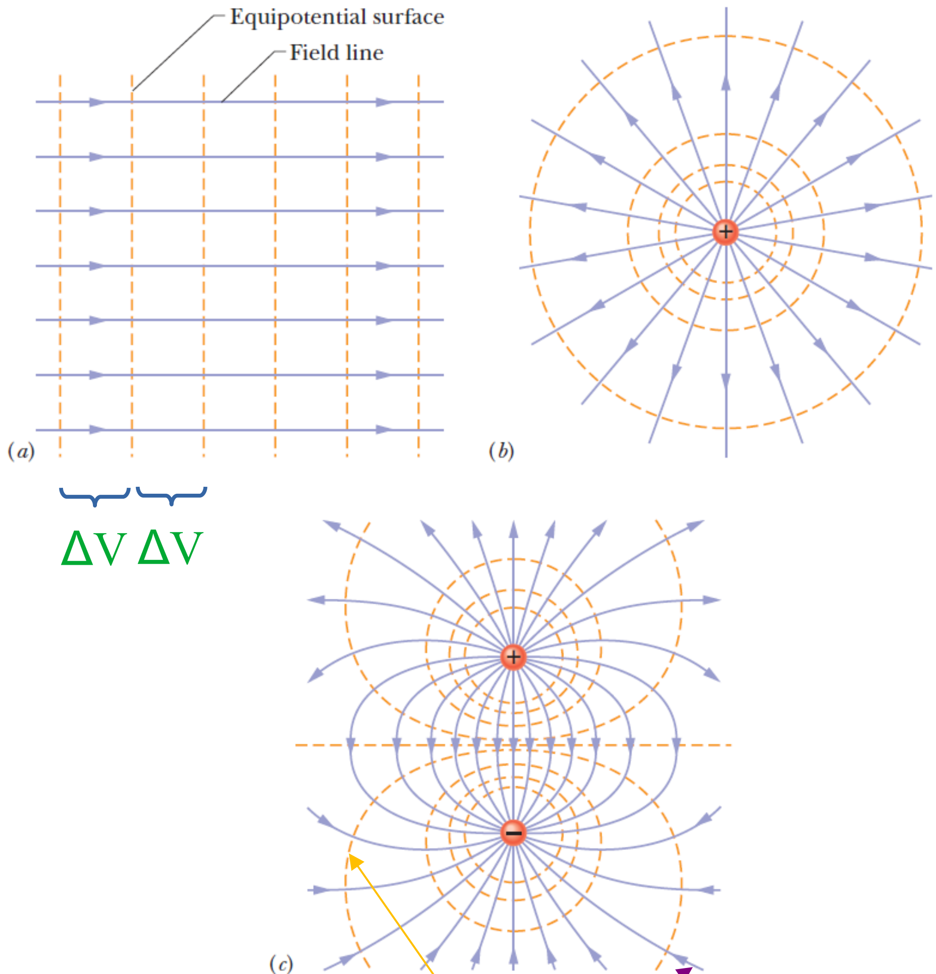
$\Delta V = V_1 - V_1$

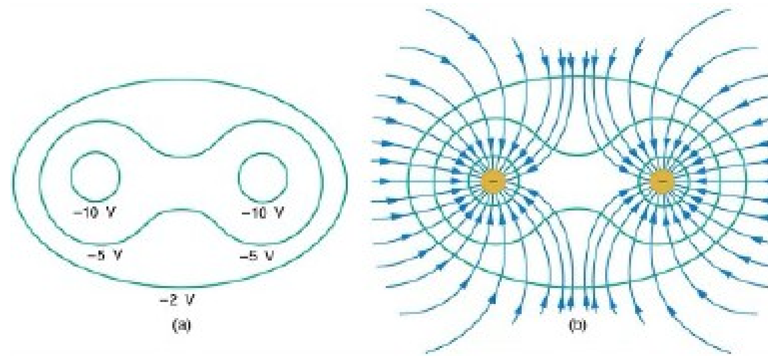
III: $\Delta V = V_f - V_i = V_2 - V_1$

IV: $\Delta V = V_f - V_i = V_2 - V_1$

II: $\Delta V = V_f - V_i = V_3 - V_3 = 0$

Fig. 24-2 Portions of four equipotential surfaces at electric potentials $V_1 = 100\ V,\ V_2 = 80\ V, V_3 = 60\ V$, and $V_4 = 40\ V$. Four paths along which a test charge may move are shown. Two electric field lines are also indicated.





AK Lecture Notes

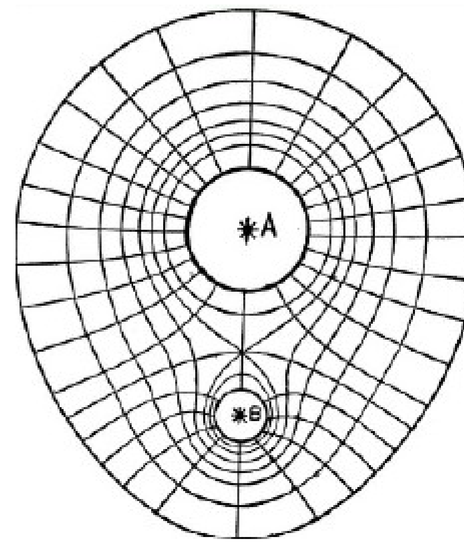
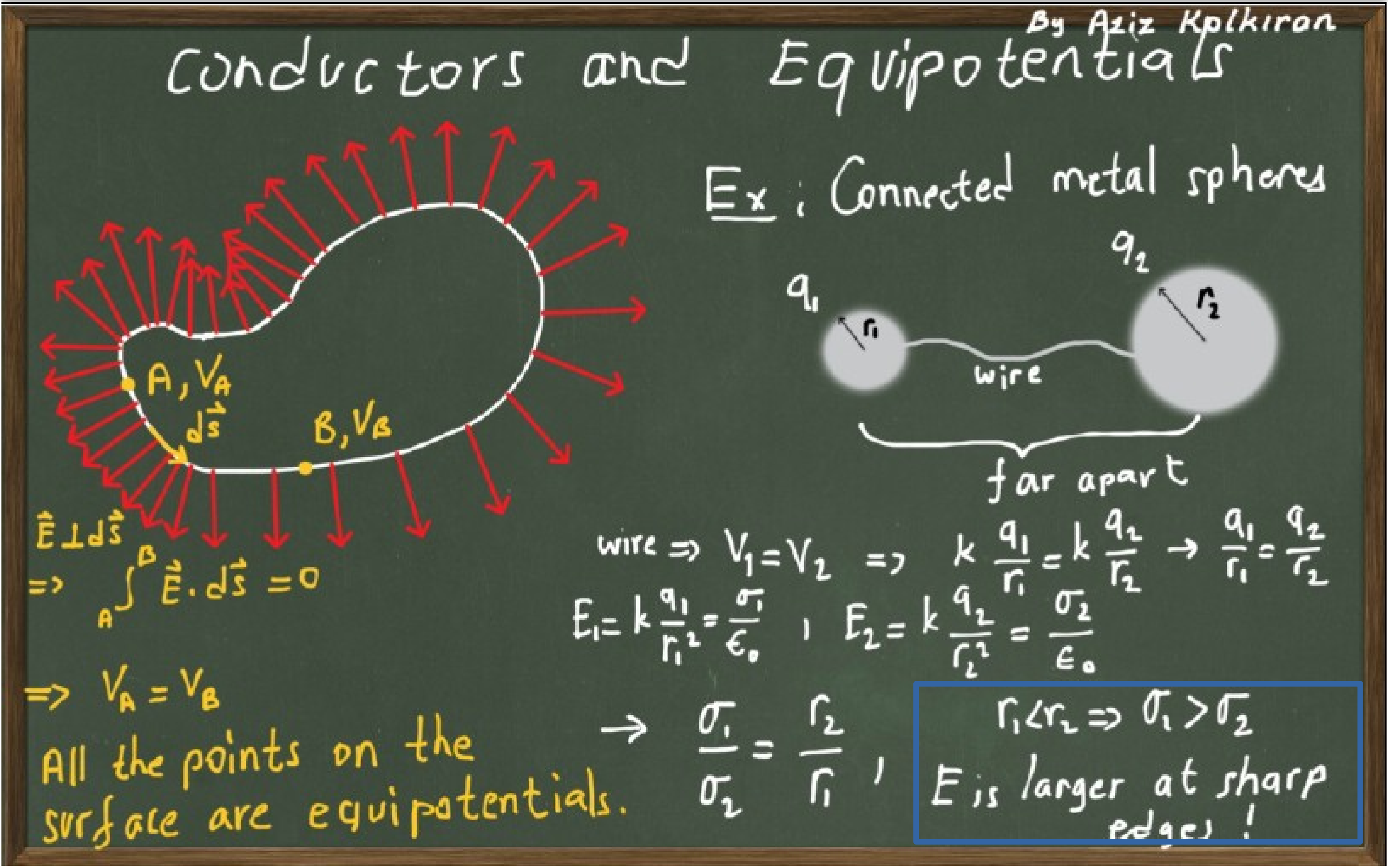


Fig. 24-3 Electric field lines (purple) and cross sections of equipotential surfaces (gold) for *(a)* a uniform electric field, *(b)* the field due to a point charge, and *(c)* the field due to an electric dipole.

İZMİR KATİP ÇELEBİ ÜNİVERSİTESİ



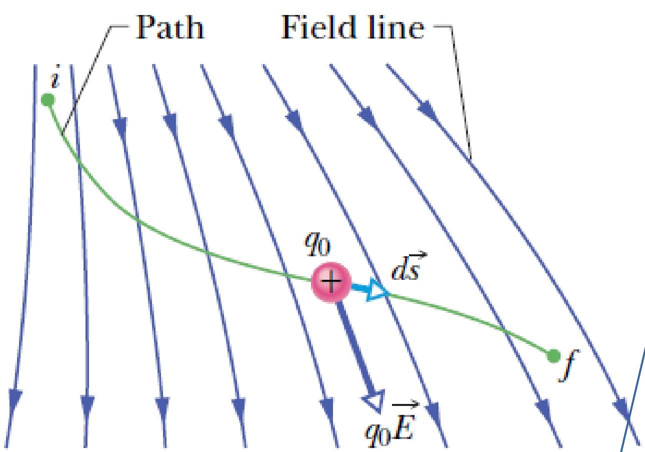


Fig. 24-4 A test charge q_0 moves from point i to point f along the path shown in a nonuniform electric field. During a displacement $d\vec{s}$, an electrostatic force $q_0\vec{E}$ acts on the test charge. This force points in the direction of the field line at the location of the test charge.

$$dW = \vec{F}\cdot d\vec{s}.$$

$$dW = q_0\vec{E}\cdot d\vec{s}.$$

Total work:

$$W = q_0\int_i^f \vec{E}\cdot d\vec{s}.$$

$\Delta V = -W/q = \Delta U$

$$V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s}.$$

- Thus, the potential difference $V_f - V_i$ between any two points *i* and *f* in an electric field is equal to the negative of the line integral from *i* to *f.*
- *Since the electrostatic force is conservative, all paths yield the same result.*
- If we set potential $V_i = 0$, then

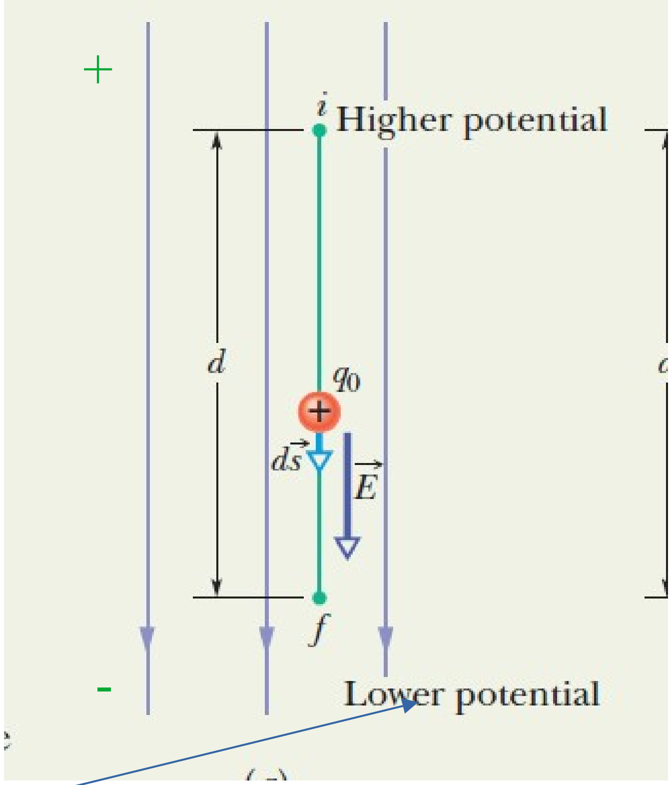
$$V = -\int_i^f \vec{E}\cdot d\vec{s},$$

bring the charge from ∞

This is the potential *V* at any point *f* in the electric field relative to the zero potential at point *i.*

Example, Finding the Potential change from the Electric Field:

(a) Figure 24-5*a* shows two points *i* and *f* in a uniform electric field $\vec{E}$. The points lie on the same electric field line (not shown) and are separated by a distance *d*. Find the potential difference $V_f - V_i$ by moving a positive test charge q_0 from *i* to *f* along the path shown, which is parallel to the field direction.



Calculations: We begin by mentally moving a test charge q_0 along that path, from initial point *i* to final point *f*. As we move such a test charge along the path in Fig. 24-5*a*, its differential displacement $d\vec{s}$ always has the same direction as $\vec{E}$. Thus, the angle θ between $\vec{E}$ and $d\vec{s}$ is zero and the dot product in Eq. 24-18 is

$$\vec{E}\cdot d\vec{s} = E\,ds\cos\theta = E\,ds. \qquad (24\text{-}20)$$

Equations 24-18 and 24-20 then give us

$$V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s} = -\int_i^f E\,ds. \qquad (24\text{-}21)$$

Since the field is uniform, *E* is constant over the path and can be moved outside the integral, giving us

$$V_f - V_i = -E\int_i^f ds = -Ed, \qquad \text{(Answer)}$$

(b) Now find the potential difference $V_f - V_i$ by moving the positive test charge q_0 from i to f along the path icf shown in Fig. 24-5b.

Calculations: The Key Idea of (a) applies here too, except now we move the test charge along a path that consists of two lines: ic and cf. At all points along line ic, the displacement $d\vec{s}$ of the test charge is perpendicular to $\vec{E}$. Thus, the angle θ between $\vec{E}$ and $d\vec{s}$ is 90°, and the dot product $\vec{E}\cdot d\vec{s}$ is 0. Equation 24-18 then tells us that points i and c are at the same potential: $V_c - V_i = 0$.

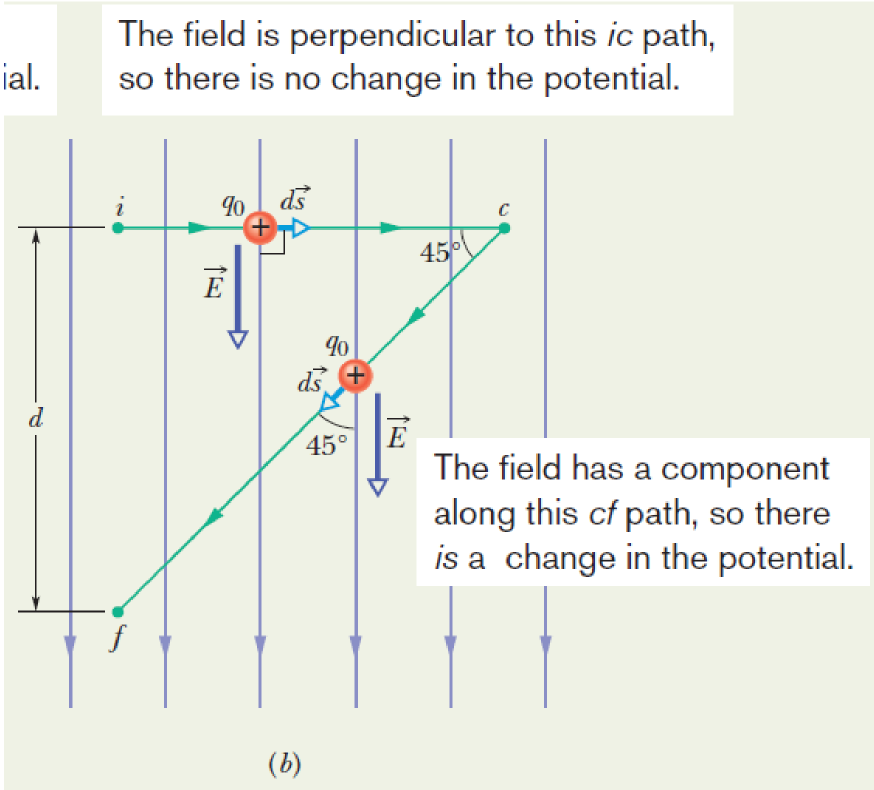
For line cf we have $\theta = 45°$ and, from Eq. 24-18,

$$V_f - V_i = -\int_c^f \vec{E}\cdot d\vec{s} = -\int_c^f E(\cos 45°)\, ds$$

$$= -E(\cos 45°)\int_c^f ds.$$

The integral in this equation is just the length of line cf; from Fig. 24-5b, that length is $d/\cos 45°$. Thus,

$$V_f - V_i = -E(\cos 45°)\frac{d}{\cos 45°} = -Ed. \quad \text{(Answer)}$$



(b)

Same result
→ path independence

We know that the electric potential difference between two points *i* and *f* is

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s},$$

For radial path

$$V_f - V_i = -\int_R^\infty E\, dr.$$

The magnitude of the electric field at the site of the test charge

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}.$$

We set $V_f = 0$ (at ∞) and $V_i = V$ (at R

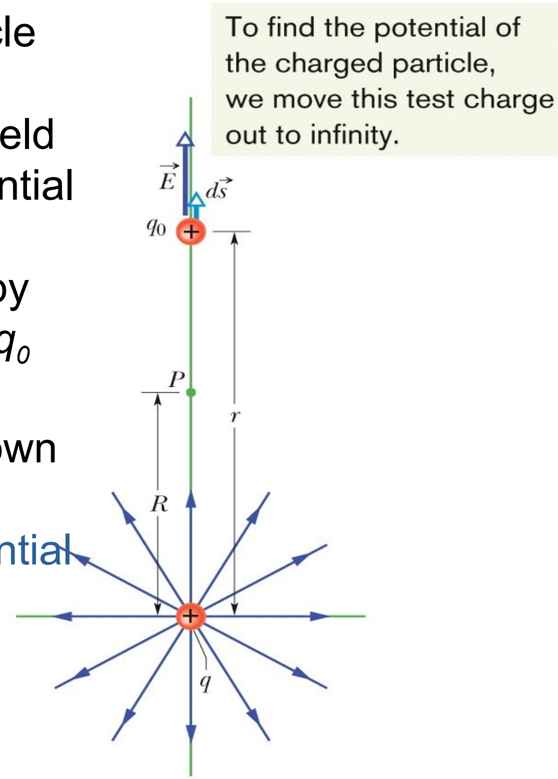
$$0 - V = -\frac{q}{4\pi\varepsilon_0}\int_R^\infty \frac{1}{r^2}\, dr = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_R^\infty$$

Solving for V and switching R to r, we get

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

as the electric potential V due to a particle of charge q at any radial distance r from the particle.

- In this figure the particle with positive charge *q* produces an electric field ***E*** and an electric potential *V* at point *P*.
- We find the potential by moving a test charge q_0 from *P* to infinity.
- The test charge is shown at distance *r* from the particle, during differential displacement *d**s***.



$$F = k\frac{q^2}{r^2} \rightarrow E = k\frac{q}{r^2} \rightarrow V = k\frac{q}{r}$$

vector — vector — scalar

24-7 Potential due to a Group of Point Charges

- The net potential at a point due to a group of point charges can be found with the help of the superposition principle.
  - First the individual potential resulting from each charge is considered at the given point.
  - Then we sum the potentials.
  - For n charges, the net potential is

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i} \qquad (n \text{ point charges}).$$

Example: Net Potential of Several Charged Particles

What is the electric potential at point P, located at the center of the square of point charges shown in Fig. 24-8a? The distance d is 1.3 m, and the charges are

$$q_1 = +12 \text{ nC}, \qquad q_3 = +31 \text{ nC},$$
$$q_2 = -24 \text{ nC}, \qquad q_4 = +17 \text{ nC}.$$

KEY IDEA

The electric potential V at point P is the algebraic sum of the electric potentials contributed by the four point charges.

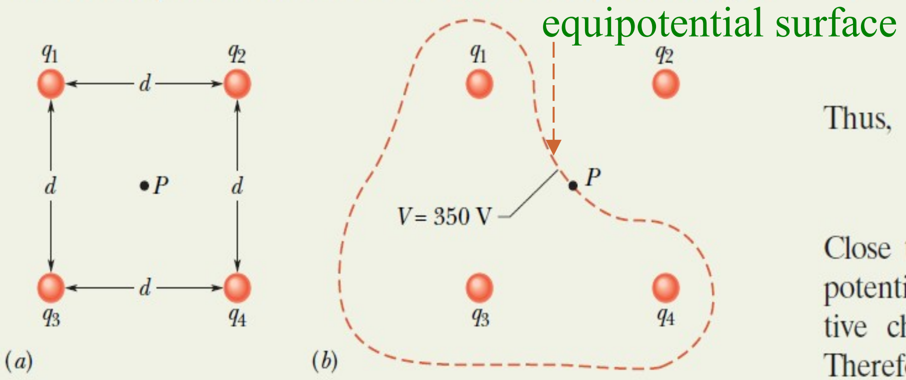


Fig. 24-8 (a) Four point charges are held fixed at the corners of a square. (b) The closed curve is a cross section, in the plane of the figure, of the equipotential surface that contains point P. (The curve is drawn only roughly.)

(Because electric potential is a scalar, the orientations of the point charges do not matter.)

Calculations: From Eq. 24-27, we have

$$V = \sum_{i=1}^{4} V_i = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r}\right).$$

The distance r is $d/\sqrt{2}$, which is 0.919 m, and the sum of the charges is

$$q_1 + q_2 + q_3 + q_4 = (12 - 24 + 31 + 17) \times 10^{-9} \text{ C}$$
$$= 36 \times 10^{-9} \text{ C}.$$

Thus,

$$V = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(36 \times 10^{-9} \text{ C})}{0.919 \text{ m}}$$

$$\approx 350 \text{ V}. \qquad \text{(Answer)}$$

Close to any of the three positive charges in Fig. 24-8a, the potential has very large positive values. Close to the single negative charge, the potential has very large negative values. Therefore, there must be points within the square that have the same intermediate potential as that at point P. The curve in Fig. 24-8b shows the intersection of the plane of the figure with the equipotential surface that contains point P. Any point along that curve has the same potential as point P.

Potential is not a Vector

Potential is a scalar and orientation is irrelevant.

(a) In Fig. 24-9*a*, 12 electrons (of charge $-e$) are equally spaced and fixed around a circle of radius R. Relative to $V = 0$ at infinity, what are the electric potential and electric field at the center C of the circle due to these electrons?

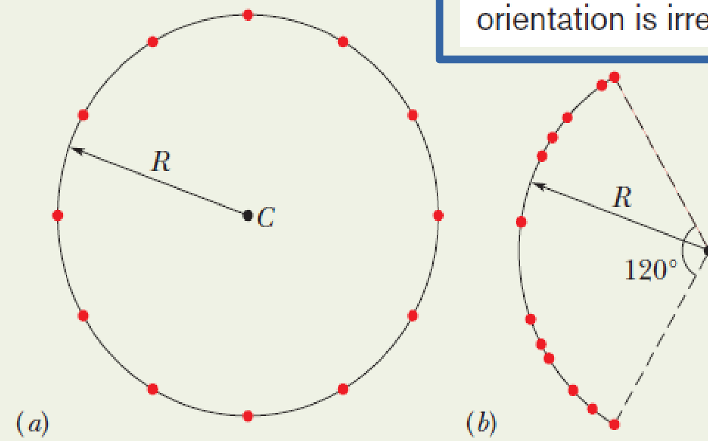


Fig. 24-9 (*a*) Twelve electrons uniformly spaced around a circle. (*b*) The electrons nonuniformly spaced along an arc of the original circle.

KEY IDEAS

(1) The electric potential V at C is the algebraic sum of the electric potentials contributed by all the electrons. (Because electric potential is a scalar, the orientations of the electrons do not matter.) (2) The electric field at C is a vector quantity and thus the orientation of the electrons *is* important.

Calculations: Because the electrons all have the same negative charge $-e$ and are all the same distance R from C, Eq. 24-27 gives us

$$V = -12 \frac{1}{4\pi\varepsilon_0} \frac{e}{R}. \qquad \text{(Answer)} \quad (24\text{-}28)$$

Because of the symmetry of the arrangement in Fig. 24-9*a*, the electric field vector at C due to any given electron is canceled by the field vector due to the electron that is diametrically opposite it. Thus, at C,

$$\vec{E} = 0. \qquad \text{(Answer)}$$

(b) If the electrons are moved along the circle until they are nonuniformly spaced over a 120° arc (Fig. 24-9*b*), what then is the potential at C? How does the electric field at C change (if at all)?

Reasoning: The potential is still given by Eq. 24-28, because the distance between C and each electron is unchanged and orientation is irrelevant. The electric field is no longer zero, however, because the arrangement is no longer symmetric. A net field is now directed toward the charge distribution.

24-8 Potential due to an Electric Dipole

- At *P*, the positive point charge (at distance $r_{(+)}$) sets up potential $V_{(+)}$ and the negative point charge (at distance $r_{(-)}$) sets up potential $V_{(-)}$. Then the net potential at P is:

$$V = \sum_{i=1}^{2} V_i = V_{(+)} + V_{(-)} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_{(+)}} + \frac{-q}{r_{(-)}}\right)$$

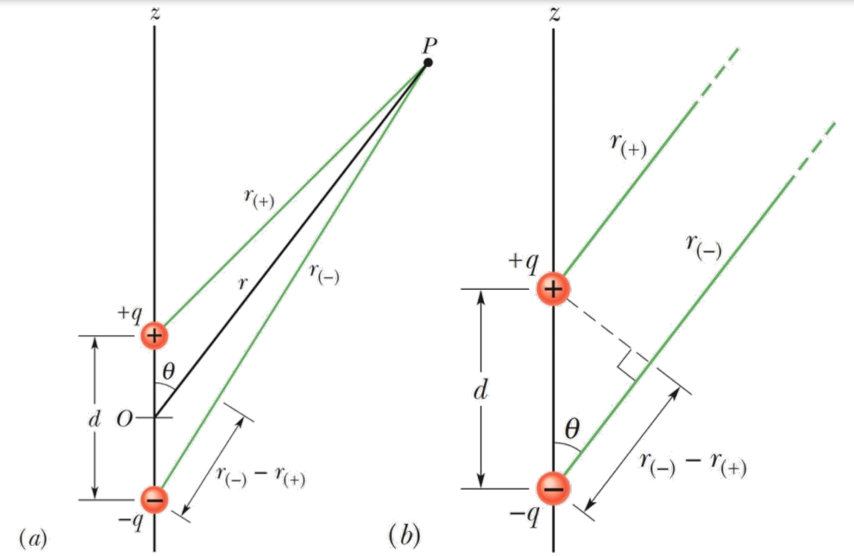
$$= \frac{q}{4\pi\varepsilon_0}\frac{r_{(-)} - r_{(+)}}{r_{(-)}r_{(+)}}.$$

- Naturally occurring dipoles are quite small; so we are usually interested only in points that are relatively far from the dipole, such that d<<r, where *d* is the distance between the charges. If *p* = *qd*,

$$r_{(-)} - r_{(+)} \approx d\cos\theta \quad \text{and} \quad r_{(-)}r_{(+)} \approx r^2.$$

$$V = \frac{q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2} \qquad V \propto \frac{1}{r^2} \quad E \propto \frac{1}{r^3}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} \qquad \text{(electric dipole)},$$

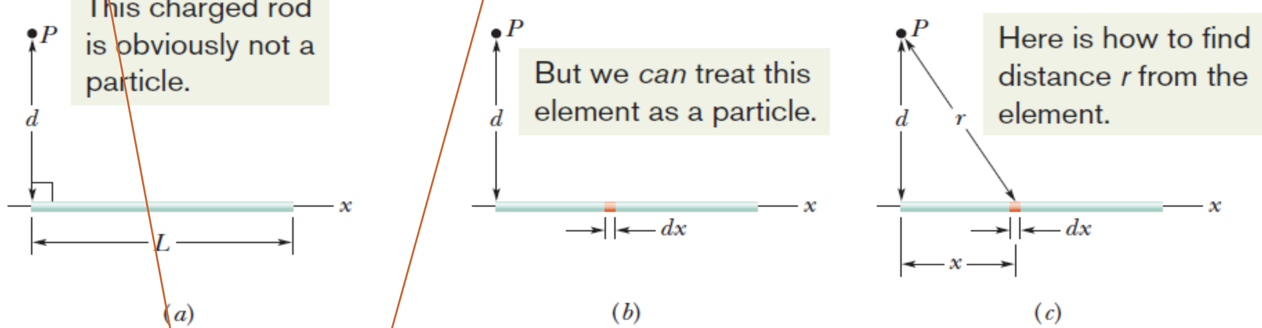


a) Point P is a distance *r* from the midpoint O of a dipole. The line OP makes an angle *θ* with the dipole axis.

b) If P is far from the dipole, the lines of lengths $r_{(+)}$ and $r_{(-)}$ are approximately parallel to the line of length *r*, and the dashed black line is approximately perpendicular to the line of length $r_{(-)}$.

V Due to a Line of Charge:

In Figure *(a)* A thin, uniformly charged rod produces an electric potential V at point *P.* *(b) An* element can be treated as a particle. (*c)* The potential at *P* due to the element depends on the distance *r. We* need to sum the potentials due to all the elements, from the left side (*d)* to the right side (*e).*

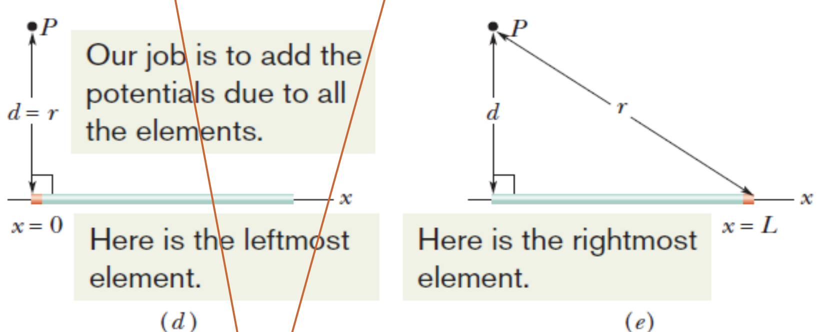


$$V = k\frac{q}{r}$$

(b) If λ is the charge per unit length, then the charge on length *dx* is: $dq = \lambda\, dx.$

(c) $r = (x^2 + d^2)^{1/2}$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{(x^2 + d^2)^{1/2}}.$$



$$V = \int dV = \int_0^L \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{(x^2 + d^2)^{1/2}}\, dx = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{(x^2 + d^2)^{1/2}} = \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(x + (x^2 + d^2)^{1/2}\right)\right]_0^L$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(L + (L^2 + d^2)^{1/2}\right) - \ln d\right]. = \frac{\lambda}{4\pi\varepsilon_0}\ln\left[\frac{L + (L^2 + d^2)^{1/2}}{d}\right].$$

V Due to a Line of Charge:

- Uniformly charged rod
- Total charge +Q
- Length L
- What is V at position P?

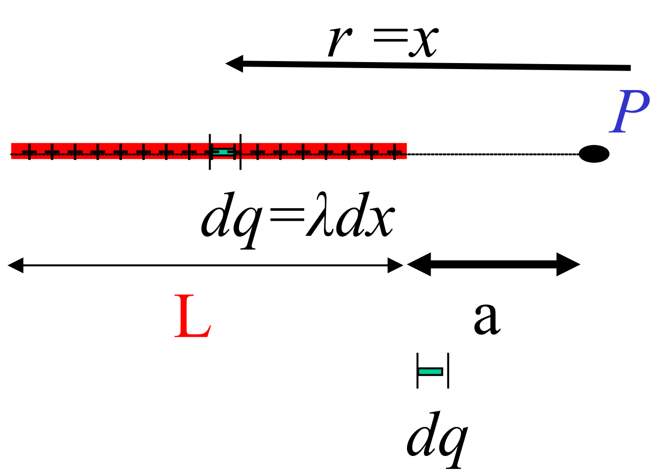
$$\lambda = Q/L \qquad dq = \lambda\, dx$$

$$V_P = \int_{\text{rod!}} \frac{k\,dq}{r} = \int_a^{L+a} \frac{k\lambda\, dx}{x}$$

$$= k\lambda \Big[\ln(x)\Big]_a^{L+a}$$

$$V_P = k\lambda\left[\ln(L+a) - \ln(a)\right]$$

$$= k\lambda \ln\left(\frac{L+a}{a}\right) = k\lambda \ln(1 + L/a)$$



$$\underset{a>>L}{\cong} k\lambda \frac{L}{a} = k\frac{\lambda L}{a} = \frac{kQ}{a}$$

by a>> L becomes point charge

Units:
[Nm²/C²][C/m]=[Nm/C]=[J/C]=[V]

First term of Taylor series expansion of ln(1+L/a)

http://www.phys.lsu.edu/~jdowling/PHYS21132-SP15/lectures/index.html

Charged Disk:

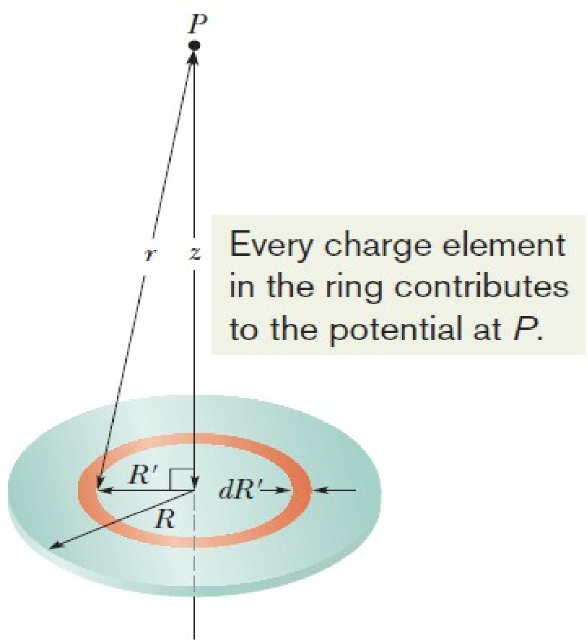


Fig. 24-13 A plastic disk of radius R, charged on its top surface to a uniform surface charge density σ. We wish to find the potential V at point P on the central axis of the disk.

- In Fig. 24-13, consider a differential element consisting of a flat ring of radius *R'* and radial width *dR'*. Its charge has magnitude

$$dq = \sigma(2\pi R')(dR')$$

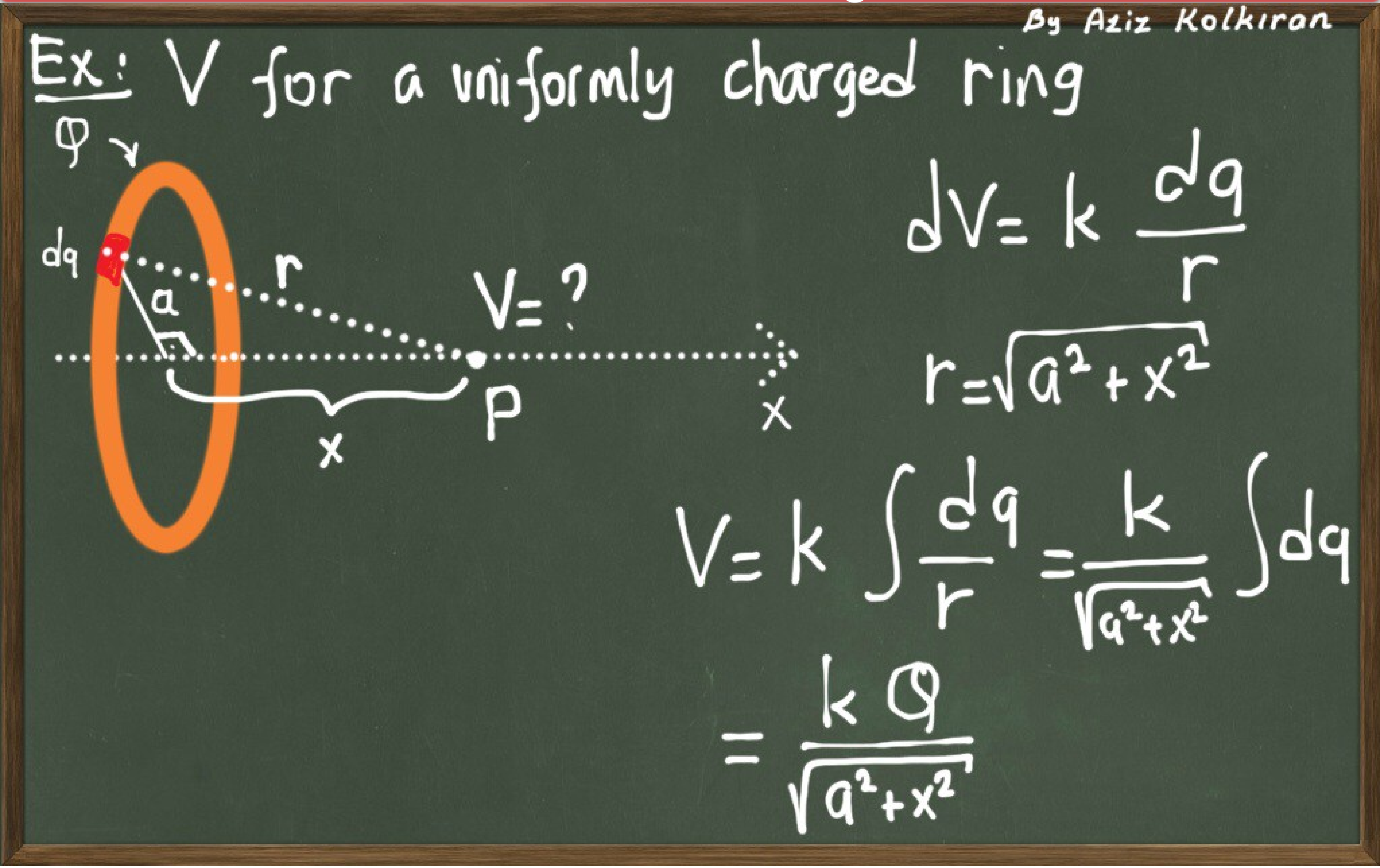
in which (*2πR'*)(*dR'*) is the upper surface area of the ring. $A = \pi r^2$; $dA = 2\pi r dr$

- The contribution of this ring to the electric potential at P is:

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma(2\pi R')(dR')}{\sqrt{z^2 + R'^2}}$$

- The net potential at *P* can be found by adding (via integration) the contributions of all the rings from *R'=0* to *R'= R:*

$$V = \int dV = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{R'\,dR'}{\sqrt{z^2 + R'^2}} = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + R^2} - z\right)$$


By Aziz Kolkıran
Ex: V for a uniformly charged ring
Q
dq
a
r
V= ?
x
P
x
dV= k dq/r
r=√(a²+x²)
V= k ∫ dq/r = k/√(a²+x²) ∫dq
= kQ/√(a²+x²)

- Suppose that a **positive test charge *q_0*** moves through a displacement from one *equipotential* surface to the adjacent surface.
- The work the electric field does on the test charge during the move is $-q_0\, dV$.
- The work done by the electric field may also be written as the scalar product of

$$(q_0\vec{E})\cdot d\vec{s} = q_0 E(\cos\theta)ds.$$

$\Delta U = q_0 \Delta V = -W$

$F\Delta s = q_0 E \Delta s = W$

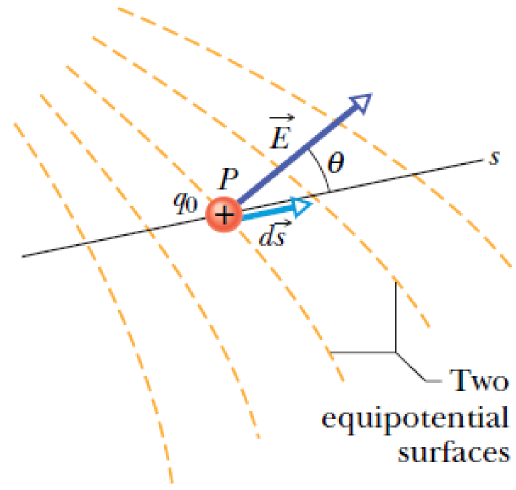


Fig. 24-14 A test charge q_0 moves a distance $d\vec{s}$ from one equipotential surface to another. (The separation between the surfaces has been exaggerated for clarity.) The displacement $d\vec{s}$ makes an angle θ with the direction of the electric field $\vec{E}$.

Therefore, $-q_0\, dV = q_0 E(\cos\theta)\, ds,$

That is, $E\cos\theta = -\dfrac{dV}{ds}$

$$E_s = -\frac{\partial V}{\partial s}$$

Since *E cos θ* is the component of **E** in the direction of **ds**,

If we take the *s* axis to be, in turn, the x, y, and z axes, the x, y, and z components of **E** at any point are

$$E_x = -\frac{\partial V}{\partial x}; \qquad E_y = -\frac{\partial V}{\partial y}; \qquad E_z = -\frac{\partial V}{\partial z}.$$

Therefore, the component of E in any direction is the negative of the rate at which the electric potential changes with distance in that direction.

Example:

The electric potential at any point on the central axis of a uniformly charged disk is given by Eq. 24-37,

$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + R^2} - z\right).$$

Starting with this expression, derive an expression for the electric field at any point on the axis of the disk.

KEY IDEAS

We want the electric field $\vec{E}$ as a function of distance z along the axis of the disk. For any value of z, the direction of $\vec{E}$ must be along that axis because the disk has circular symmetry about that axis. Thus, we want the component E_z of $\vec{E}$ in the direction of z. This component is the negative of the rate at which the electric potential changes with distance z.

Calculation: Thus, from the last of Eqs. 24-41, we can write

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\sigma}{2\varepsilon_0}\frac{d}{dz}\left(\sqrt{z^2 + R^2} - z\right)$$

$$= \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right). \quad \text{(Answer)}$$

Summary: we are given an expression for V(r) (or V(x,y,z)) and E=-∂V/∂s then find E.

Reminder: V is scalar E is vector

- The electric potential energy of a system of fixed point charges is equal to the work that must be done by an external agent to assemble the system, bringing each charge in from an infinite distance.

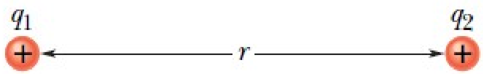
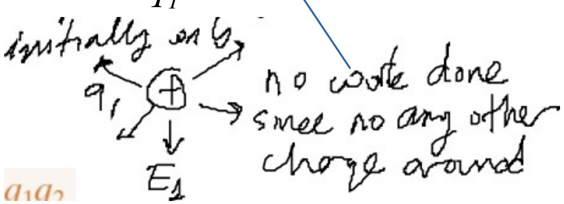


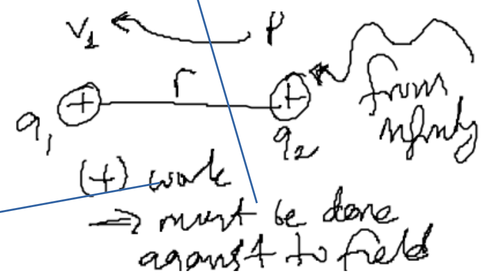
Fig. 24-15 Two charges held a fixed distance r apart.

Figure 24-15 shows two point charges q_1 *and* q_2, separated by a distance r.

When we bring q_1 in from infinity and put it in place, we do **no work** because no electrostatic force acts on q_1.



However, when we next bring q_2 in from infinity and put it in place, we must **do work** because q_1 exerts an electrostatic force on q_2 during the move.



- The work done is q_2V, where V is the potential that has been set up by q_1 at the point where we put q_2.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}$$

$$U = W = q_2V = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$$

Example, Potential Energy of a System of Three Charged Particles:

Figure 24-16 shows three point charges held in fixed positions by forces that are not shown. What is the electric potential energy U of this system of charges? Assume that $d = 12$ cm and that

$$q_1 = +q, \quad q_2 = -4q, \quad \text{and} \quad q_3 = +2q,$$

in which $q = 150$ nC.

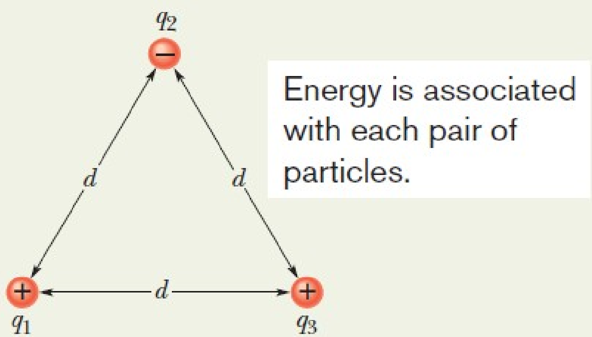


Fig. 24-16 Three charges are fixed at the vertices of an equilateral triangle. What is the electric potential energy of the system?

q_1 ⊕ $U_{01} = 0$

q_2 ⊖ U_{12} ✓ now we have 2 particle

q_3 ⊕ U_{13}, U_{23}

+ ______

$U_{12} + U_{13} + U_{23} = U$

Calculations: Let's mentally build the system of Fig. 24-16, starting with one of the point charges, say q_1, in place and the others at infinity. Then we bring another one, say q_2, in from infinity and put it in place. From Eq. 24-43 with d substituted for r, the potential energy U_{12} associated with the pair of point charges q_1 and q_2 is

$$U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d}.$$

We then bring the last point charge q_3 in from infinity and put it in place. The work that we must do in this last step is equal to the sum of the work we must do to bring q_3 near q_1 and the work we must do to bring it near q_2. From Eq. 24-43, with d substituted for r, that sum is

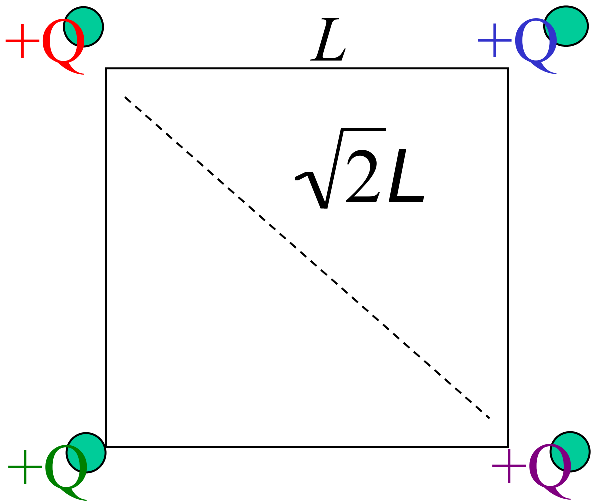
charges. This sum (which is actually independent of the order in which the charges are brought together) is

$$U = U_{12} + U_{13} + U_{23}$$
$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{(+q)(-4q)}{d} + \frac{(+q)(+2q)}{d} + \frac{(-4q)(+2q)}{d}\right)$$
$$= -\frac{10q^2}{4\pi\varepsilon_0 d}$$
$$= -\frac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10)(150 \times 10^{-9}\ \text{C})^2}{0.12\ \text{m}}$$
$$= -1.7 \times 10^{-2}\ \text{J} = -17\ \text{mJ}. \qquad \text{(Answer)}$$

lower potential → preferred configuration

- 4 point charges (each +Q and equal mass) are connected by strings, forming a square of side L.
- If all four strings suddenly snap, (i) what is the kinetic energy of each charge when they are very far apart? (ii) If each charge has mass m, find the velocity of each charge long after the string snaps.
- **Use conservation of energy ($K_f+U_f=K_i+U_i$):**

(Final K_f of all four charges) = (U_i stored) = (energy required to assemble the system of charges)



1. No energy needed to bring in first charge: $U_1=0$
2. Energy needed to bring in 2nd charge:

$$U_2 = QV_1 = \frac{kQ^2}{L}$$

3. Energy needed to bring in 3rd charge:

$$U_3 = QV = Q(V_1+V_2) = \frac{kQ^2}{L} + \frac{kQ^2}{\sqrt{2}L}$$

4. Energy needed to bring in 4th charge:

$$U_4 = QV = Q(V_1+V_2+V_3) = \frac{2kQ^2}{L} + \frac{kQ^2}{\sqrt{2}L}$$

- Total potential energy is sum of all the individual terms shown on right hand side:

$$\frac{kQ^2}{L}\left(4+\sqrt{2}\right)$$

- So, final kinetic energy of each charge =>K=mv²/2 =

$$\frac{kQ^2}{4L}\left(4+\sqrt{2}\right)$$

http://www.phys.lsu.edu/~jdowling/PHYS21132-SP15/lectures/index.html

Example:Conservation of Mechanical Energy with Electric Potential Energy

An alpha particle (two protons, two neutrons) moves into a stationary gold atom (79 protons, 118 neutrons), passing through the electron region that surrounds the gold nucleus like a shell and headed directly toward the nucleus (Fig. 24-17). The alpha particle slows until it momentarily stops when its center is at radial distance $r = 9.23$ fm from the nuclear center. Then it moves back along its incoming path. (Because the gold nucleus is much more massive than the alpha particle, we can assume the gold nucleus does not move.) What was the kinetic energy K_i of the alpha particle when it was initially far away (hence external to the gold atom)? Assume that the only force acting between the alpha particle and the gold nucleus is the (electrostatic) Coulomb force.

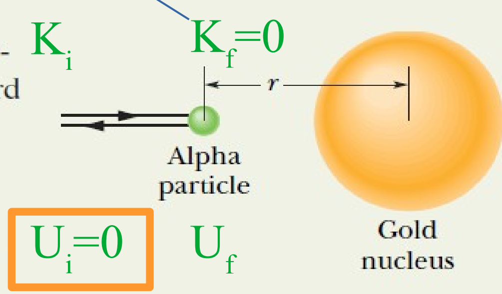


Fig. 24-17 An alpha particle, traveling head-on toward the center of a gold nucleus, comes to a momentary stop (at which time all its kinetic energy has been transferred to electric potential energy) and then reverses its path.

Reasoning: When the alpha particle is outside the atom, the system's initial electric potential energy U_i is zero because the atom has an equal number of electrons and protons, which produce a *net* electric field of zero. However, once the alpha particle passes through the electron region surrounding the nucleus on its way to the nucleus, the electric field due to the electrons goes to zero. The reason is that the electrons act like a closed spherical shell of uniform negative charge and, as discussed in Section 23-9, such a shell produces zero electric field in the space it encloses. The alpha particle still experiences the electric field of the protons in the nucleus, which produces a repulsive force on the protons within the alpha particle.

As the incoming alpha particle is slowed by this repulsive force, its kinetic energy is transferred to electric potential energy of the system. The transfer is complete when the alpha particle momentarily stops and the kinetic energy is $K_f = 0$.

Calculations: The principle of conservation of mechanical energy tells us that

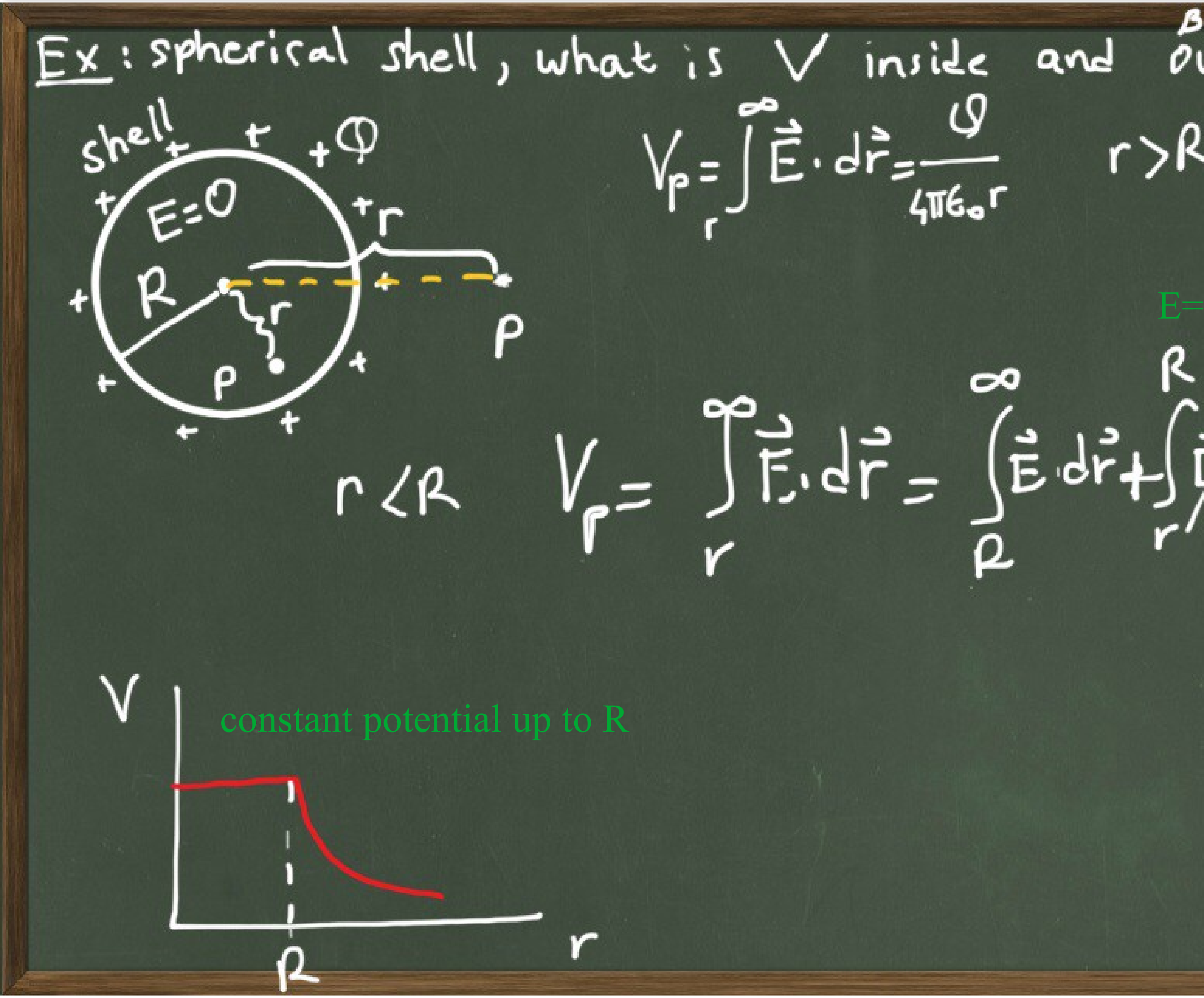
$$K_i + U_i = K_f + U_f. \qquad (24\text{-}44)$$

We know two values: $U_i = 0$ and $K_f = 0$. We also know that the potential energy U_f at the stopping point is given by the right side of Eq. 24-43, with $q_1 = 2e$, $q_2 = 79e$ (in which e is the elementary charge, 1.60×10^{-19} C), and $r = 9.23$ fm. Thus, we can rewrite Eq. 24-44 as

$$K_i = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(79e)}{9.23\text{ fm}}$$

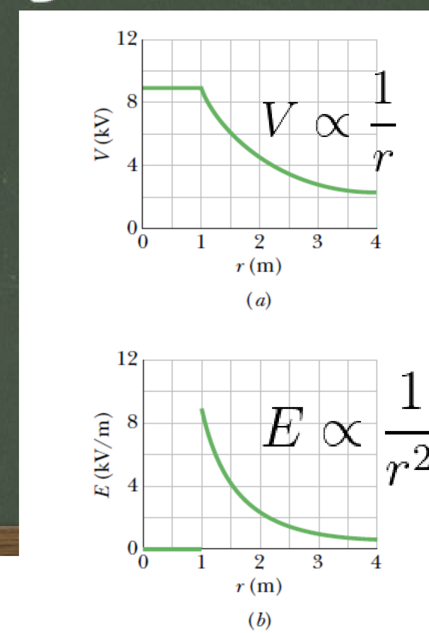
$$= \frac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(158)(1.60 \times 10^{-19}\ \text{C})^2}{9.23 \times 10^{-15}\ \text{m}}$$

$$= 3.94 \times 10^{-12}\ \text{J} = 24.6\ \text{MeV}. \qquad \text{(Answer)}$$



E=0 inside the conductor

constant potential up to R



1. An infinite <u>nonconducting sheet</u> has a surface charge density σ=0.10 μC/m² on one side. How far apart are equipotential surfaces whose potentials differ by 50 V?

1(s) nonconducting infinite sheet

$\sigma = 0.10\,\mu C/m^2$

$\Delta V = 50\ V$

$V_f - V_i = -\int E \cdot ds$ ($V_f - V_i$ = −50 V)

$50V = \int_0^y E\,dy \rightarrow 50V = \frac{\sigma}{2\varepsilon_0}\Delta y \rightarrow \Delta y = \frac{(50\,V)(8.85\times10^{-12}\,C^2/Nm^2)2}{0.10\times10^{-6}\,C/m^2}$

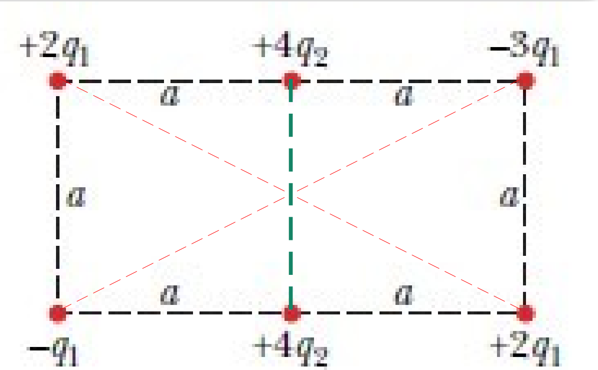
$\Delta y = 8.85\times10^{-3}\,m$

$E = \frac{\sigma}{2\varepsilon_0}$

$V_2 = 50\ V$, $V_1 = 100\ V$ } $\Delta V = V_f - V_i$

side view

2. Figure shows a rectangular array of charged particles fixed in place, with distance a = 39.0 cm and the charges shown as integer multiples of $q_1 = 3.40$ pC and $q_2 = 6.00$ pC. With $V = 0$ at infinity, what is the net electric potential at the rectangle's center? (*Hint:* Thoughtful examination can reduce the calculation.)



2 (16) ✓

$q_A = 3.40$ pC

$q_B = 6.00$ pC

$V_{at\ the\ center} = ?$

$a = 0.39$ m

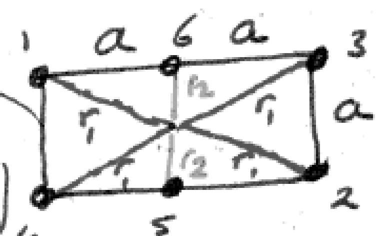
$V_{at\ the\ center} = V_1 + V_2 + V_3 + V_4 + V_5 + V_6$

$+2q_A + 2q_A - 3q_A - 1q_A$

$= \frac{1}{4\pi\varepsilon_0}\frac{1}{r_1}(q_1 + q_2 + q_3 + q_4) + \frac{1}{4\pi\varepsilon_0}\frac{1}{r_2}(q_5 + q_6)$

$= \frac{1}{4\pi\varepsilon_0}\frac{1}{r_2}(4q_B + 4q_B) = \frac{8 \times 6\times10^{-12}\,C}{4\pi\left(8.85\times10^{-12}\,C^2/N.m^2\right)\left(\frac{0.39\,m}{2}\right)}$

$= 2.21\ \frac{N\,m}{C} = 2.21\ V$



3. Figure shows a thin rod with a uniform charge density of 2.00 μC/m. Evaluate the electric potential at point P if d=D=L/4.00.

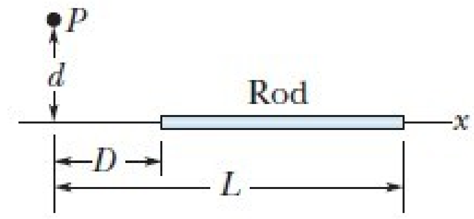


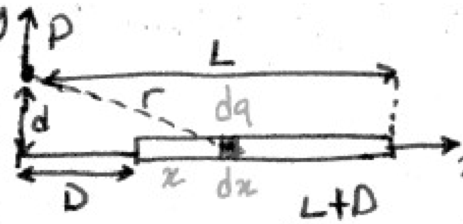
Fig. 24-40 Problem 26.

5(26)

a thin rod

$\lambda = 2\times10^{-6}$ C/m

$d = D = \frac{L}{4}$



$E_P = ?$ $V_P = ?$

$\left(dE = k\frac{dq}{r^2}\right)$ $\left(dV = k\frac{dq}{r}\right)$ $dV = k\frac{\lambda dx}{((D+x)^2+d^2)^{1/2}}$

$$V_P = \int dV = \int_0^{L+D} k\lambda \frac{dx}{((D+x)^2+d^2)^{1/2}} = k\lambda \ln\left(2x+2D+2\sqrt{(x^2+2Dx+D^2+d^2)}\right)\Big|_0^{L-D}$$

zero

$$= k\lambda \ln\left(2(L-D)+2D+2\sqrt{L^2-2LD+D^2+2DL-2D^2+D^2+d^2}\right)$$

$$-\ln\left(0+2D+2\sqrt{0+0+D^2+d^2}\right)$$

~~$\int\frac{dx}{\sqrt{x^2+a^2}} = \ln(x+\sqrt{x^2+a^2})$~~

$$\int\frac{dx}{x^2+2Dx+D^2+d^2} = \frac{1}{\sqrt{a}}\ln\left(2ax+b+2\sqrt{a(ax^2+bx+c)}\right)$$

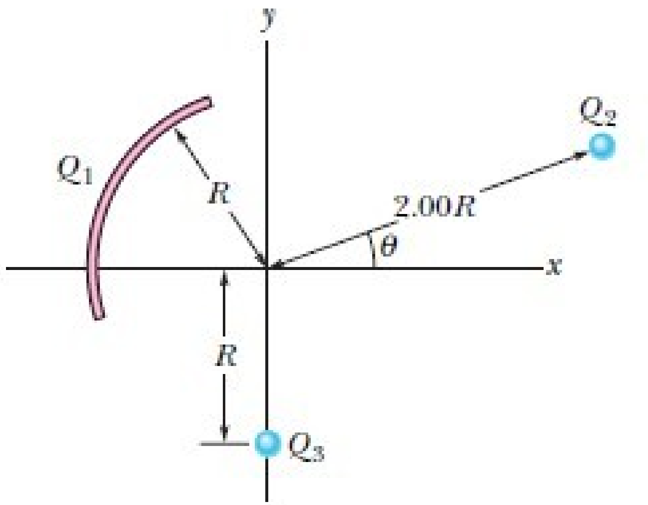
ax^2+bx+c (Integral Table)

$$= k\lambda\ln\left(\frac{L+\sqrt{L^2+d^2}}{D+\sqrt{D^2+d^2}}\right) = k\lambda\ln\left(\frac{L+\sqrt{17}\frac{L}{4}}{\frac{L}{4}+\sqrt{2}\frac{L}{4}}\right)$$

$$= k\lambda\ln\left(\frac{4+\sqrt{17}}{1+\sqrt{2}}\right) = 8.99\times10^9\frac{Nm^2}{C^2}\,2\times10^{-6}\frac{C}{m}\ln\left(\frac{4+\sqrt{17}}{1+\sqrt{2}}\right)$$

$$= 2.18\times10^4\ \text{V}$$

4. In Figure, what is the net electric potential at the origin due to the circular arc of charge Q_1=7.21 pC and the two particles of charges Q_2 = 4.00Q_1 and Q_3= -2.00Q_1? The arc's center of curvature is at the origin and its radius is R = 2.00 m; the angle indicated is θ=20.0°.



6 (29)

circular arc charge, particle charges; $V_{net,center}$ = ?

θ = 20°, R = 2 m

Q_1 = 7.21 pC (R); $Q_2 = 4Q_1$ (2R); $Q_3 = -2Q_1$ (R)

$V = k\frac{q}{r}$, charges on the arc are equidistant to center ⇒ behaves as a point charge

$V_{net} = \Sigma k \frac{q_i}{r_i}$ $\quad V_{net,center} = k\left(\frac{Q_1}{R} + \frac{4Q_1}{2R} + \frac{-2Q_1}{R}\right) = k\frac{Q_1}{R}$

$= 8.99\times10^{9}\frac{Nm^2}{C^2}\frac{7.21\times10^{-12}C}{2m} = 0.032\frac{Nm}{C}$

$\{F = qE,\ V = Ed,\ V = C\frac{Volt}{m} \rightarrow \frac{Nm}{C} \Rightarrow \frac{CVolt}{m}\frac{m}{C} \rightarrow Volt$ $\quad = 0.032\ Volt$

5. The electric potential at points in an xy plane is given by $V=(2.0\ V/m^2)x^2 - (3.0\ V/m^2)y^2$. In unit-vector notation, what is the electric field at the point (3.0 m, 2.0 m)?

V → V(x,y):scalar → E(x,y):vector

9(35) $V=\left(2\frac{V}{m^2}\right)x^2-\left(3\frac{V}{m^2}\right)y^2$; $E(3m,2m)=?$; $E_x=-\frac{\partial V}{\partial x}$, $E_y=-\frac{\partial V}{\partial y}$, $E_z=-\frac{\partial V}{\partial z}$

$\Rightarrow E_x=-4\frac{V}{m^2}x\Big|_{x=3m}$, $E_y=+6\frac{V}{m^2}y\Big|_{y=2m}$ $\Rightarrow \vec{E}=-12\frac{V}{m}\hat{i}+12\frac{V}{m}\hat{j}$

6. In Figure, how much work must we do to bring a particle, of charge Q=+16e and initially at rest, along the dashed line from infinity to the indicated point near two fixed particles of charges q_1=+4e and q_2=-q_1/2? Distance d=1.40 cm, θ_1=43°, and θ_2=60°.

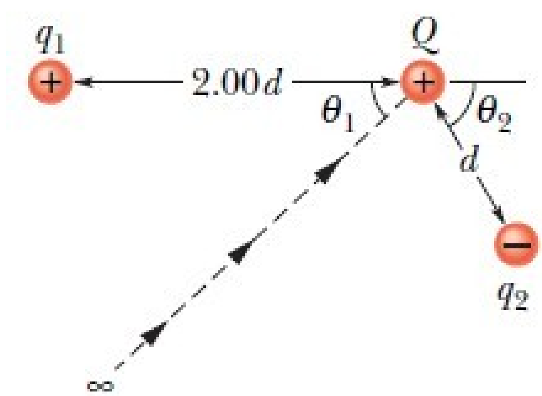


Fig. 24-49 Problem 50.

10 (50)
$Q = 16e$, $d = 1.4\times10^{-2}$ m
initally at rest $\theta_1 = 43°$
$\theta_2 = 60$
from infinity
to the target point
$q_1 = 4e$ $q_2 = \frac{-q_1}{2}$

Work required? $w = \Delta U = U_f - U_i$ { i → infinity, $U_i \to 0$ }

reference point

$$w = U_f = k\frac{q_1 Q}{2d} + k\frac{\left(-\frac{q_1}{2}\right)Q}{d}$$

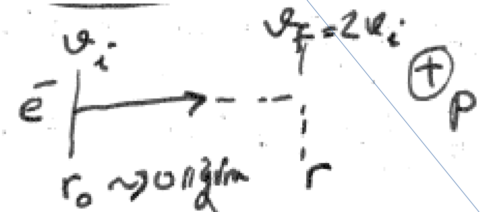
$= 0$

$$\frac{kq_1}{2d} + \frac{kq_2}{d}$$

$$U_{Q1} = Q V_1 + U_{Q2} = Q V_2$$

7. An electron is projected with an initial speed of 3.2×10^5 m/s directly toward a proton that is fixed in place. If the electron is initially a great distance from the proton, at what distance from the proton is the speed of the electron instantaneously equal to twice the initial value

H (55)



$v_i = 3.2 \times 10^5$ m/s

p: fixed

r at distance when $v = 2v_0$ = ?

$\Delta K = W$
$-\Delta U = W$
$\Delta K + \Delta U = 0$
$(K_f - K_i) + (U_f - U_i) = 0$

change in potential energy?

$V_f - V_i = \Delta U$

which is equal to change in kinetic energy ΔK

$-(K_2 - K_1) = U_2 - U_1 = \frac{-e^2}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r_0}\right)$ ($\frac{1}{r_0} \to 0$)

$-\left(\frac{1}{2} m_e v_f^2 - \frac{1}{2} m_e v_i^2\right) \Rightarrow \frac{1}{2} m_e 4 v_i^2 - \frac{1}{2} m_e v_i^2 = \frac{e^2}{4\pi\varepsilon_0} \frac{1}{r}$ ($v_f^2 = (2v_i)^2$)

$r = \frac{2e^2}{4\pi\varepsilon_0 v_i^2 3 m_e}$

$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$

$r = \frac{2 \times (1.6 \times 10^{-19} C)^2}{4\pi\, 8.85 \times 10^{-12} \frac{C^2}{Nm^2} (3.2 \times 10^5 m/s)^2\, 3 \times 9.11 \times 10^{-31} kg}$

$r = 1.65 \times 10^{-9}$ m

Electric Potential

- The electric potential V at point P in the electric field of a charged object:

$$V = \frac{-W_\infty}{q_0} = \frac{U}{q_0}, \quad \textbf{Eq. 24-2}$$

Electric Potential Energy

- Electric potential energy U of the particle-object system:

$$U = qV. \quad \textbf{Eq. 24-3}$$

- If the particle moves through potential ΔV:

$$\Delta U = q\,\Delta V = q(V_f - V_i). \quad \textbf{Eq. 24-4}$$

Mechanical Energy

- Applying the conservation of mechanical energy gives the change in kinetic energy:

$$\Delta K = -q\,\Delta V. \quad \textbf{Eq. 24-9}$$

- In case of an applied force in a particle

$$\Delta K = -q\,\Delta V + W_{app}. \quad \textbf{Eq. 24-11}$$

- In a special case when $\Delta K=0$:

$$W_{app} = q\,\Delta V \quad (\text{for } K_i = K_f). \quad \textbf{Eq. 24-12}$$

Finding V from *E*

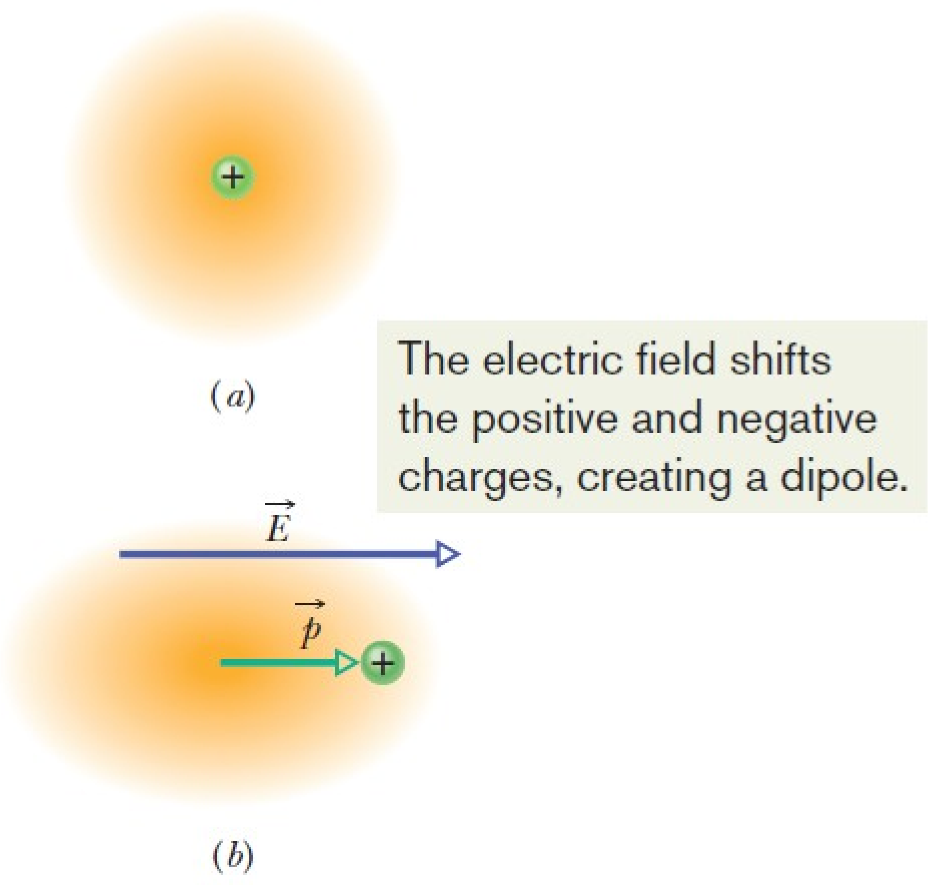
- The electric potential difference between two point I and f is:

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}; \quad \textbf{Eq. 24-18}$$

Additional Materials

Induced Dipole Moment

Fig. 24-11 (a) An atom, showing the positively charged nucleus (green) and the negatively charged electrons (gold shading).The centers of positive and negative charge coincide.
(b) If the atom is placed in an external electric field ***E***, the electron orbits are distorted so that the centers of positive and negative charge no longer coincide. An induced dipole moment ***p*** appears. The distortion is greatly exaggerated here.

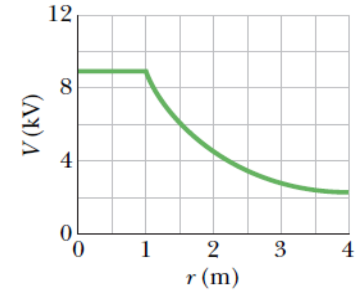


An excess charge placed on an isolated conductor will distribute itself on the surface of that conductor so that all points of the conductor—whether on the surface or inside—come to the same potential. This is true even if the conductor has an internal cavity and even if that cavity contains a net charge.

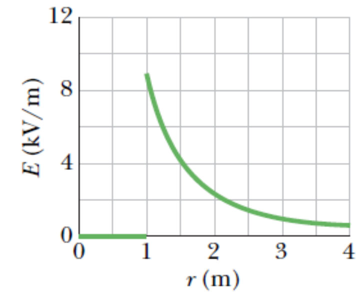
We know that

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

Since for all points ***E*** = *0* within a conductor, it follows directly that $V_f = V_i$ for all possible pairs of points *i* and *f* in the conductor.



(*a*)



(*b*)

Fig. 24-18 (*a*) A plot of $V(r)$ both inside and outside a charged spherical shell of radius 1.0 m. (*b*) A plot of $E(r)$ for the same shell.

Spark Discharge from a Charged Conductor

Fig. 24-19 A large spark jumps to a car's body and then exits by moving across the insulating left front tire (note the flash there), leaving the person inside unharmed. *(Courtesy Westinghouse Electric Corporation)*

On nonspherical conductors, a surface charge does not distribute itself uniformly over the surface of the conductor. At sharp points or edges, the surface charge density-and thus the external electric field, -may reach very high values. The air around such sharp points or edges may become ionized, producing the corona discharge that golfers and mountaineers see on the tips of bushes, golf clubs, and rock hammers when thunderstorms threaten. Such corona discharges are often the precursors of **lightning strikes. In such circumstances, it is wise to enclose yourself in a cavity inside a conducting shell, where the electric field is guaranteed to be zero.** A car (unless it is a convertible or made with a plastic body) is almost ideal.

Isolated Conductor in an Isolated Electric Field

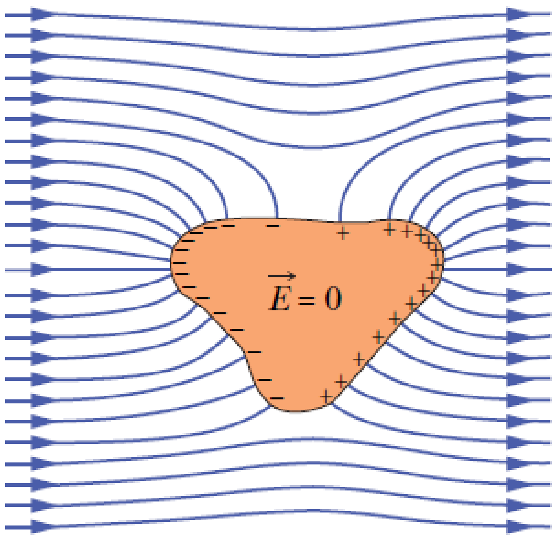


Fig. 24-20 An uncharged conductor is suspended in an external electric field. The free electrons in the conductor distribute themselves on the surface as shown, so as to reduce the net electric field inside the conductor to zero and make the net field at the surface perpendicular to the surface.

- If an isolated conductor is placed in an external electric field, all points of the conductor still come to a single potential regardless of whether the conductor has an excess charge.
- The free conduction electrons distribute themselves on the surface in such a way that the electric field they produce at interior points cancels the external electric field that would otherwise be there.
- Furthermore, the electron distribution causes the net electric field at all points on the surface to be perpendicular to the surface.
- If the conductor in Fig. 24-20 could be somehow removed, leaving the surface charges frozen in place, the internal and external electric field would remain absolutely unchanged.