Chapter 21 - Electric Charge

Physics of electromagnetism → Combination of electric and magnetic phenomena

Begin with electrical phenomena; first step is to discuss the nature of electric charge and electric force.

Every object contains a vast amount of $\underbrace{\text{electric charge}}_{\text{intrinsic characteristic}}$

Two kinds of charge } positive, negative ⇒ electrically neutral } charge balance → ~~equality of~~ amounts of charge

Charged object → charge imbalance, net charge ⇒ charged object inteacts } by exerting forces on one another

Electric Force : Charges with the **same** electrical sign **repel** each other, and chages with **opposite** electrical sign **attract** each other
Coulomb's law of electrostatic force

←⊖ ⊖→
←⊕ ⊕→
⊕→←⊖

Conductors & Insulators

Ability of charge to move through! Conductance

- Conductors : charge can move rather freely ; metals, body, water, ..
- Nonconductors (Insulators) : charge can not move freely ; rubber, plastic, glass
- Semiconductors : intemediate btw conductors and insulators ; (silicon, germanium)
- Superconductors : perfect conductors

We can neutralize (discharge) the object (charge balance) by grounding (creating a pathway btw. $\underbrace{\text{object and Earth's}}$ surface) the object.

Earth is a huge conductor

Seee Lecture notes

Coulomb's Law

Two charged particles are brought near each other → each exert force on the other

The force of repulsion or attraction : $\vec{F}$, electrostatic force

The equation giving the force for charged particles is called Coulomb's law.

$$\boxed{\vec{F} = k \frac{q_1 q_2}{r^2} \hat{r}}$$ Coulomb's law

$$\vec{F} = G \frac{m_1 m_2}{r^2} \hat{r}$$: Newton's law

Both eqns describe inverse square laws

→ always attractive

→ either attractive or repulsive

charge q_1

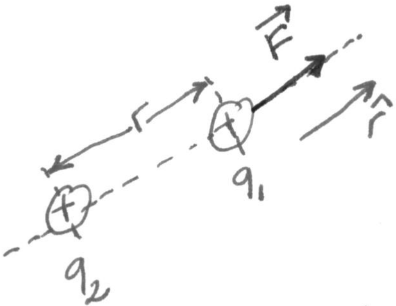
charge q_2

r: distance btw two charged particles

$\hat{r}$: unit vector along an axis

k; constant

See lecture notes



$\vec{F}$, force on particle 1

Unit of charge : Coulomb (C) . Derived from $\underbrace{\text{electric current}}_{}$, $\frac{dq}{dt}$ at which charge moves past a point or through a region

i (Ampere, A)

→ 1 C = (1 A)(1 s)

Electrostatic constant : k

$k = \frac{1}{4\pi\varepsilon_0}$. ε_0: permittivity constant , 8.85×10^{-12} N m^2/C^2 in vacuum

The magnitude of the force in Coulomb's law

$$F = \frac{1}{4\pi\varepsilon_0} \frac{|q_1||q_2|}{r^2}$$

Superposition principle for $\vec{F}$: $\boxed{\vec{F}_{1,net} = \underbrace{\vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \cdots + \vec{F}_{1n}}_{}}$

Here, on particle 1

- n particles
- interacting independently in pairs

} Then, net force on any of them

Shell theorem of electrostatics:

- A shell of uniform charge attracts or repels a charged particle that is outside the shell as if the shell's charge were concentrated at its center.
- If a charged particle is located inside a shell of uniform charge there is no net electrostatic force on the particle from the shell.

Spherical Conductors: if excess charge is placed on a sphericall shell (metal), the excess charge spreads uniformly over the surface.

- place exces electrons
- they repel each other
- spreading over the surface
- uniformly distributed
- Now, first shell theorem works!

Charge is quantized

"Electrical fluid" is made up of multiples of a certain elementary charge.

Charge is one of property of particles like mass.

multiples: $q = ne$ e: elementary charge (electron) 1.602×10^{-19}C

$n: \mp1, \mp2, \mp3, \dots$: quantized, it has discrete values. not continuous

Charge is conserved! In any kind of interaction, charge is transferred. Conservation!

i.e.

Radioactive decay of nuclei

Transformation of ^{238}U to ^{234}Th by emitting apha particle (^4He)

$$\left.\begin{array}{ccc} ^{238}U \rightarrow & ^{234}Th + & ^{4}He \\ 92e & 90e & 2e \\ 238\ p,n & 234\ p,n & 4\ p,n \end{array}\right\} \text{charge is conserved}$$

Example: In figue shown, what are i) horizontal components of the net electrostatic force on the charged particle in the lower left corner of the square. ii) vertical components of that

$q = 1.0 \times 10^{-7}$C

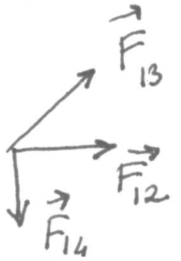
$a = 5.0$ cm

asked

$\vec{F}_{1,net} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}$

$F_{1,net,x} = ?$

$F_{1,net,y} = ?$



$F_{12}: |\vec{F}_{12}| = k\dfrac{|2q||-2q|}{a^2} = k\dfrac{4q^2}{q^2}, \vec{F}_{12} = k\dfrac{4q^2}{a^2}(\hat{i})$

$\longrightarrow \vec{F}_{12}$

$F_{14}: |\vec{F}_{14}| = \dfrac{k|2q||q|}{a^2} = k\dfrac{2q^2}{a^2}, \vec{F}_{14} = \dfrac{k2q^2}{a^2}(-\hat{j})$

$\downarrow$

$F_{13}: |\vec{F}_{13,x}| = \dfrac{k|2q||-q|}{(a\sqrt{2})^2}\overset{\sqrt{2}/2}{\cos 45°}, F_{13,x} = k\dfrac{2q^2}{a^2 2}\dfrac{\sqrt{2}}{2}$

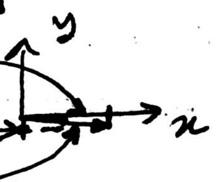
$|\vec{F}_{13,y}| = \dfrac{k|2q||-q|}{(q\sqrt{2})^2}\overset{\sqrt{2}/2}{\sin 45°}, F_{13,y} = k\dfrac{2q^2}{q^2 2}\dfrac{\sqrt{2}}{2}$

i) $F_{1,net,x} = F_{12,x} + F_{13,x} + F_{14,x} = k\frac{q^2}{a^2}\left(4+\frac{\sqrt{2}}{2}\right) = 8.99\times10^{9}\frac{(1.0\times10^{-7})^2}{0.050^2}\left(4+\frac{\sqrt{2}}{2}\right) = \boxed{0.17\,N}$

ii) $F_{1,net,y} = F_{12,y} + F_{13,y} + F_{14,y} = k\frac{q^2}{a^2}\left(-2+\frac{\sqrt{2}}{2}\right) = \boxed{-0.046\,N}$

$$\vec{F}_{1,net} = k\frac{q^2}{a^2}\left[\left(4+\frac{\sqrt{2}}{2}\right)\hat{i} + \left(-2+\frac{\sqrt{2}}{2}\right)\hat{j}\right]$$

$\vec{F}_{1,net} = 0.17\,N\,\hat{i} + 0.046\,(-\hat{j})$



$\tan\theta = \frac{-0.046}{0.17}$

Chapter 22 - Electric Fields

How the chaged particles "know" the presence of othe paticle?
How these interact at a distance althoug they do not touch?
particles

The answer is Electric Field! $\vec{E}$ Particle 2 pushes on particle 1 not by touching it by means of the electric field produced by particle 2.

- Define $\vec{E}$
- How to calculate it for various orragements of chaged particles.

The Electric Field

Temperature, Pressure : Scalar fields

Electric field is a vector field < Distribution of vectors around a chaged object / place a test charge near by

SLN Fig 22-1

Define the electric field $\vec{E}$ at point P due to the charged object as } $\boxed{\vec{E}=\frac{\vec{F}}{q_0}}$ $|\vec{E}|=\frac{|\vec{F}|}{q_0}$ magnitude

Unit N/C

same as $\vec{F}$ ~ direction

$\vec{E}$ exists independently of the test charge.

SLN

Electric Field Lines

To visualize patterns in $\vec{E}$.

- direction of line ⇒ direction of $\vec{E}$
- number of lines per unit area ⇒ magnitude of $\vec{E}$ < lines: close → large $|\vec{E}|$ / lines: away → small $|\vec{E}|$
- extends away from positive charge
- toward negative charge

SLN Fig 22-2 : noncondacting sheet with uniformly distributed positive charge

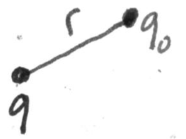
Fig 22-3 : sphere of uniform negative charge

SLN Fig 22-4
Fig 22-5 : Electric Dipole
$\vec{E}$ is tangent to the field line

$\vec{E}$ Due to a Point Charge

To find $\vec{E}$ due to a point charge at any point a distance r } put a test charge, q_0

$$\vec{F}=\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\hat{r}, \quad \boxed{\vec{E}=\frac{\vec{F}}{q_0}=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}}$$



Then, electrostatic force acting on q_0;

magnitude : $E=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

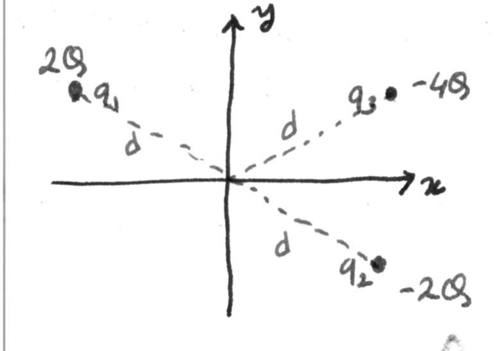
direction : same as the direction of $\vec{F}$

· More than one point charge? Net force, $\vec{F_0}$, acting on test charge, q_0?

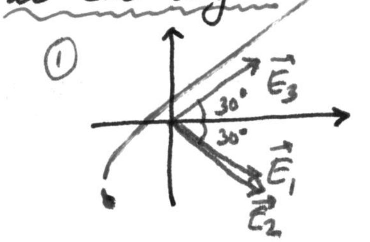
$$\vec{F_0} = \vec{F}_{01} + \vec{F}_{02} + \dots + \vec{F}_{0n} \left\{ \begin{array}{c} \text{net} \\ \vec{E} \end{array} \right| \quad \vec{E} = \frac{\vec{F_0}}{q_0} = \underbrace{\vec{E_1} + \vec{E_2} + \dots + \vec{E_n}}_{\text{principle of superposition}}$$

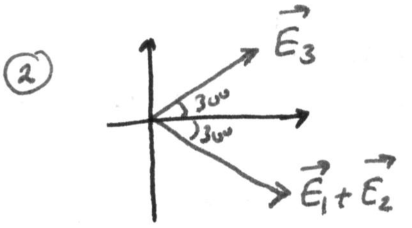
(acting one by one in pairs)

Example



Find the net $\vec{E}$ at the origin





in magnitude $E_1 + E_2 = \frac{1}{4\pi\varepsilon_0}\frac{|2Q|}{d^2} + \frac{1}{4\pi\varepsilon_0}\frac{|-2Q|}{d^2}$; $E_3 = \frac{1}{4\pi\varepsilon_0}\frac{|-4Q|}{d^2}$

in vector $\vec{E}_{net} = \vec{E_1} + \vec{E_2} + \vec{E_3}$ $\Big|$ $\vec{E}_{net,x} = \vec{E}_{1,x} + \vec{E}_{2,x} + \vec{E}_{3,x}$

$\vec{E}_{net,y} = \vec{E}_{1,y} + \vec{E}_{2,y} + \vec{E}_{3,y}$

$\rightarrow \vec{E}_{net,x} = E_1 \cos 30\, \hat{i} + E_2 \cos 30\, \hat{i} + E_3 \cos 30\, \hat{i}$

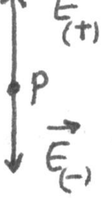
$\vec{E}_{net,y} = E_1 \sin 30(-\hat{j}) + E_2 \sin 30(-\hat{j}) + E_3 \sin 30(\hat{j})$

$$\vec{E}_{net,x} = \frac{1}{4\pi\varepsilon_0}\frac{8Q}{d^2}\frac{\sqrt{3}}{2}\,\hat{i} = \boxed{\frac{6.93Q}{4\pi\varepsilon_0 d^2}\,\hat{i}}$$

$$\vec{E}_{net,y} = \frac{1}{4\pi\varepsilon_0 d^2}(-2Q - 2Q + 4Q)\sin 30\,\hat{j} = \boxed{0\,\hat{j}}$$

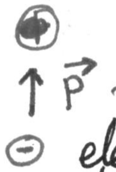
$\vec{E}$ Due to an Electric Dipole

Two charged particles of magnitude q but opposite sign seperated by a distance d. SLN Fig 22-8 $\vec{E}$ due to dipole at a point P, a distance z from the midpoint of the dipole and on the axis through the particles (dipole axis)



$$\underbrace{E = E_{(+)} - E_{(-)}}_{\text{at point P (magnitude)}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(+)}^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(-)}^2} \quad \left\{ \begin{array}{l} r_{(+)} = z - \frac{d}{2} \\ r_{(-)} = z + \frac{d}{2} \end{array} \right.$$



electric
p : dipole
moment

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{(z - d/2)^2} - \frac{1}{(z + d/2)^2}\right)$$

$$\boxed{E = \frac{q}{4\pi\varepsilon_0 z^3}\left(\frac{d}{(1 - (\frac{d}{2z})^2)^2}\right)}$$

at large distances $\frac{d}{2z} << 1$ $(z >> d)$ $\Rightarrow$ $\boxed{E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3} = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}}$

Direction of $\vec{p}$: - → +
specify orientation of a dipole

$\vec{E}$ Due to a Line of Charge

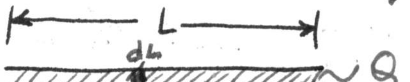
A single point charge ! not realistic. Usually we have charge distributions
(discrete)

Many closely spaced point charges spread
- along a line
- over a surface
- within a volume

} continuous.

Now, we need charge density rather than total charge

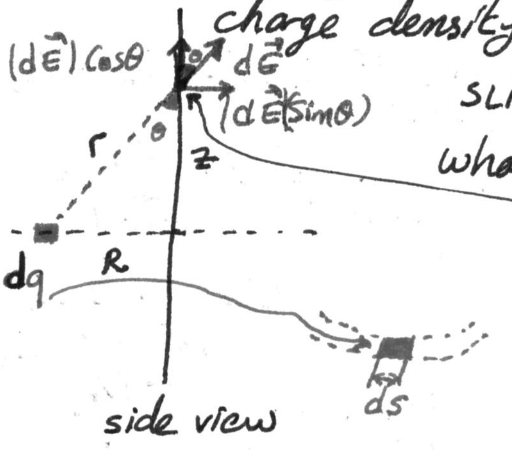
λ, σ, ρ (linear, surface, volume) charge density. SLN Table 22-



$$\lambda = \frac{Q}{L}$$

$$\lambda = \frac{dq}{dL}$$

Example: A ring of radius R with a uniform positive linear charge density. (Plastic or insulator so that charges are fixed, not moving)



SLN Fig 22-10

what is $\vec{E}$ at point P

Previously $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$

$\frac{dq}{ds} = \lambda$ charge per unit length

now, we have dq and we should find $d\vec{E}$.

dq: charge of the differential element

ds: length of any differential element

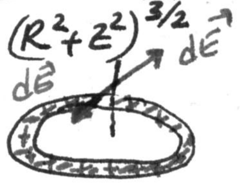
$\Rightarrow$ $d\vec{E}$ at point P. Now, dq can be treated as point charge.

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda ds}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda ds}{(R^2+z^2)}$$: First step is completed

Second step is to calculate total electric field.

$$|\vec{E}| = \int dE\cos\theta \quad \left\{ \cos\theta = \frac{z}{r} = \frac{z}{(R^2+z^2)^{1/2}} \right\} \quad E = \int_0^{\overset{\sim\text{length of ring}}{2\pi R}} \frac{1}{4\pi\varepsilon_0}\frac{z\lambda ds}{(R^2+z^2)^{3/2}}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\lambda z}{(R^2+z^2)^{3/2}}\int_0^{2\pi R} ds = \frac{z\underbrace{\lambda\, 2\pi R}_{q}}{4\pi\varepsilon_0 (R^2+z^2)^{3/2}}$$

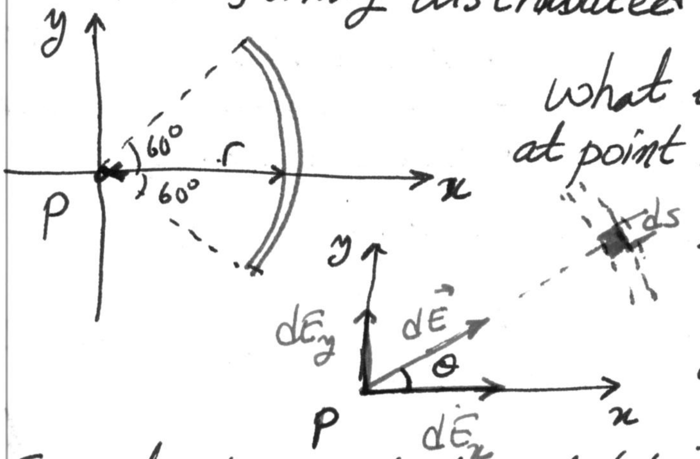


$$\Rightarrow \boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{qz}{(R^2+z^2)^{3/2}}}$$ charged ring

when $z >> R$ $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2}$

At very large distances, ring looks like a point charge.

Example A 120° circular arc rod of radius r with having a uniformly distributed charge −Q. SLN Fig. 22-11



What is $\vec{E}$ at point P? in terms of Q and r

Firs step: Find dE with knowns!

$d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\hat{r}$, $|d\vec{E}| = dE$ magnitude

$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}$, $\lambda = \frac{dq}{ds}$, $dq = \lambda ds$

$\Rightarrow dE = \frac{1}{4\pi\epsilon_0}\frac{\lambda ds}{r^2}$: First Step is comple

Second Step: Calculate total $\vec{E}$

↕ y-components of $d\vec{E}$ are cancelled out.

$\vec{E} = E_x\hat{i} + \overset{0}{E_y}\hat{j}$, $|\vec{E}| = E_x = E$

$E = \int dE_x = \int dE\cos\theta = \int \frac{1}{4\pi\epsilon_0}\frac{\lambda ds}{r^2}\cos\theta$ }

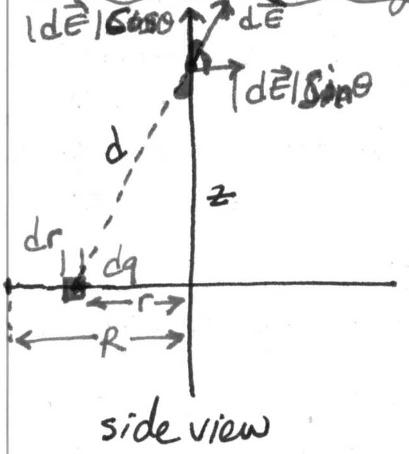
$ds = r d\theta$

$= \frac{1}{4\pi\epsilon_0}\frac{\lambda}{r}\int_{60°}^{60°}\cos\theta\, d\theta \Rightarrow \frac{\lambda}{4\pi\epsilon_0}\left[\sin\theta\right]_{60°}^{60°} = \frac{\lambda}{4\pi\epsilon_0 r}\left[\sin 60° - \sin(-60°)\right]$

$\Rightarrow \boxed{E = \frac{1.73\lambda}{4\pi\epsilon_0 r}}$: Total $\vec{E}$. Not finished. λ should be removed { $\lambda = \frac{Q}{\frac{2\pi}{3}r} \Rightarrow \boxed{E = \frac{0.83Q}{4\pi\epsilon_0 r^2}}$

$\boxed{\vec{E} = \frac{0.83Q}{4\pi\epsilon_0 r^2}\hat{i}}$

$\vec{E}$ Due to a Charged Disk



side view

What is $\vec{E}$ at point P?

$\frac{dq}{dA} = \sigma = \frac{dq}{2\pi r dr}$

$d = \sqrt{r^2+z^2}$

$\cos\theta = \frac{z}{d}$

$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{d^2} = \frac{1}{4\pi\epsilon_0}\frac{\sigma dA}{d^2}$

$dE = \frac{1}{4\pi\epsilon_0}\frac{\sigma 2\pi r dr}{(r^2+z^2)}$: 1st step

$\Rightarrow$ 2nd step: ~~E=~~ $\int dE\cos\theta = dE_z$

~~dE~~ $= \frac{1}{4\pi\epsilon_0}\sigma 2\pi\int\frac{r\,dr}{(r^2+z^2)}\frac{z}{(r^2+z^2)^{1/2}}$

$dE_z = \frac{\sigma z}{4\epsilon_0}\int\frac{2r\,dr}{(r^2+z^2)^{3/2}}$

3rd step: Calculate total $\vec{E}$

$E = \int dE_z = \frac{\sigma z}{4\epsilon_0}\int_0^R (z^2+r^2)^{-3/2}(2r)dr$ { $\int X^M dX = \frac{X^{M+1}}{M+1}$ } $X = (z^2+r^2)$, $M = -3/2$, $dX = (2r)dr$

$E = \frac{\sigma z}{4\epsilon_0}\left[\frac{(z^2+r^2)^{-1/2}}{-1/2}\right]_0^R$

$\boxed{E = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)}$ charged disk { if $R \to \infty$ $\Rightarrow \boxed{E = \frac{\sigma}{2\epsilon_0}}$ infinite sheet

A Point Charge in an $\vec{E}$

A charged particle $\xrightarrow[\text{to}]{\text{located}}$ $\vec{E}$. what happens? An elettrostatic force acts on the particle: $\boxed{\vec{F}=q\vec{E}}$

$\vec{E}$: External electric field

q : charge of the particle

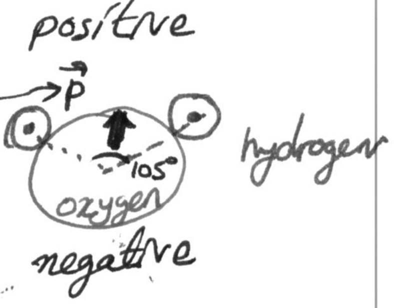
* The direction of $\vec{F}$ and $\vec{E}$ are the same if charge, q, is positive
" " " opposite " " " negative

SLN. Example

A dipole in an $\vec{E}$

The behavior of a dipole in an $\vec{E}$ can be described completely in terms of $\vec{E}$ and $\vec{p}$.

No need any information about dipole's structure



A dipole, $\vec{p}$, in a uniform extenal $\vec{E}$.

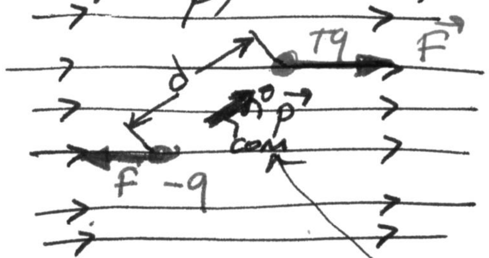
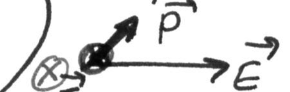


Fig. 22-19

- Forces act in opposite directions.
- Net force on the dipole is zero.
- But, forces produce a net torque, $\vec{\tau}$ on the dipole ~~axis~~ about its com.

$\tau = Fd\sin\theta$

$\tau = pE\sin\theta$



$p=qd$ & $F=qE$

$$\boxed{\begin{array}{l}\vec{\tau}=\vec{p}\times\vec{E}\\ U=-\vec{p}\cdot\vec{E}\end{array}}$$

potential energy of a dipole.

SLN

Chapter 23 - Gauss' Law

Simple way of solving complex problem $\Leftarrow$ due to symmetry

For certain charge distributions → use Gauss' law instead of considering the fields $d\vec{E}$.

Hypothetical closed surface enclosing charge distribution.

$\underbrace{\text{Hypothetical closed surface}}_{\text{Gaussian Surface}}$
- can have any shape
- minimizes our calculations of $\vec{E}$
- mimics the symmetry of the charge distribution

Gauss' law relates the electric fields at points on a (closed) Gaussian surface to the net charge enclosed by that surface. SLN → Fig. 23-1 (OR reverse)

Flux

How much charge is enclosed? need a way → Flux (intercepted field)

Flux: product of an area and the field accross that area. SLN

Flux of an $\vec{E}$: Consider an arbitrary Gaussian surface immersed in a nonuniform electric field.

Divide the surface into small flat squares each having area of ΔA. Since squares are very small electric field can be taken as constant.

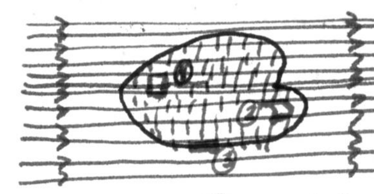
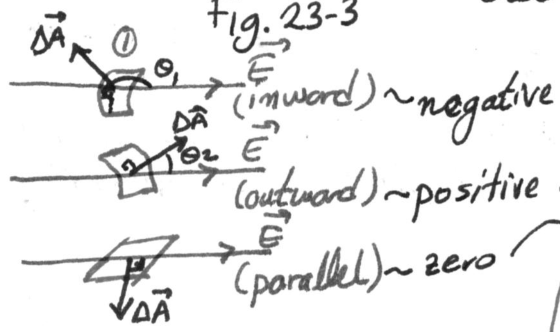


Fig. 23-3



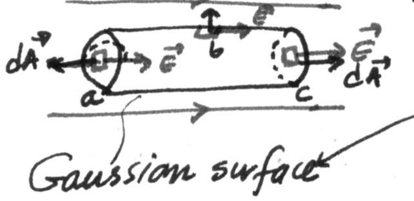
$$\Phi = \sum \vec{E}\cdot\Delta\vec{A} \quad \left\{\begin{matrix}\Delta A \text{ is}\\ \text{smaller}\\ \text{and}\\ \text{smaller}\end{matrix}\right\} \quad \boxed{\Phi = \oint \vec{E}\cdot d\vec{A}}$$

Electric flux through a Gaussian surface

closed surface

> The electric flux Φ through a Gaussian surface is proportional to the net number of electric field lines passing through that surface.

Example Gaussian surface in the form of a cylinder of radius R immersed in a uniform $\vec{E}$. What is Φ?



Gaussian surface

$$\Phi = \oint \vec{E}\cdot d\vec{A} = \int_a \vec{E}\cdot d\vec{A} + \int_b \vec{E}\cdot d\vec{A} + \int_c \vec{E}\cdot d\vec{A}$$

$$= \int E(\cos 180)\,dA + \overset{0}{\cancel{\int E(\cos 90^\circ)\,dA}} + \int E(\cos 0)\,dA$$

$(A = \pi R^2)$

$$= -EA + 0 + EA$$

$\Phi = 0$: net flux zero $\left\{\begin{matrix}\text{input}\\ \text{output}\end{matrix}\right.$ are same in magnitude, but opposite in direction.

SLN

Gauss' law

Gaussian suface

Relates the net flux Φ of an $\vec{E}$ through a closed surface to the net charge q_{enc} → enclosed charge

$\varepsilon_0 \Phi = q_{enc}$ $\xrightarrow[\text{flux}]{\text{subs.}}$ $\boxed{\oint \vec{E}.d\vec{A} = \frac{q}{\varepsilon_0}}$

Gauss' law

- charge is placed in vacuum or air
- net charge $<^{+}_{-}{}_{0}$
- q_{enc} is (+) ⇒ Φ is outward
- q_{enc} is (-) ⇒ Φ is inword

SLN Fig. 23-6

S_1 : $\vec{E}$ is outward → Φ is positive → net charge is (+) as Gauss' law

S_2 : $\vec{E}$ is inward → Φ is negative → net charge is (-)

S_3 : no charge is enclosed, $q_{enc}=0$ → $\Phi=0$: Field enter and also leave the surface

S_4 : net charge is zero → $\Phi=0$ (net flux is zero)

What would happen if a large q is placed neer the suface S_4?

→ The pattern of $\vec{E}$ lines would change, but flux would not change.

SLN Fig. 23-7 & Fig. 23-5

Gauss' Law and Coulomb's Law

Gauss' law } Coulombs law } are diffrent ways of describing the relation btw $\vec{E}$ and q. SLN Fig. 23-8

⇒ So that, derive each from the other is possible.

Consider a positive charge $\xrightarrow[\text{surface}]{\text{a Gaussian}}$ $\varepsilon_0 \oint \vec{E} d\vec{A} = q_{enc}$

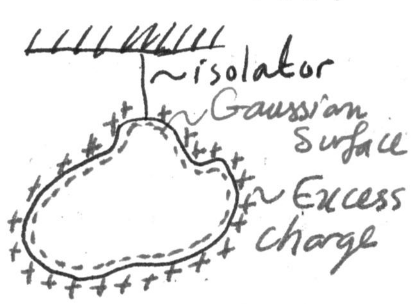
Gauss' law ⟷ Coulomb's law

a differential area from Gaussion surface (spherical).

$$\left.\begin{array}{l}\vec{E} \parallel d\vec{A} \Rightarrow \theta = 0^\circ \\ q_{enc} = q\end{array}\right\} \varepsilon_0 \overset{\text{Gauss' law}}{\oint E dA} = q \Rightarrow \left.\begin{array}{l}\varepsilon_0 E \oint dA = q \\ \varepsilon_0 E(4\pi r^2) = q\end{array}\right\} \boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}}$$

A charged Isolated Conductor

Gauss' law permits us to provide } If an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor. None of these ~~exchange~~ excess charge will be found within the body of the conductor.



GS : Just below the surface

$\vec{E} \to 0$ since net charge inside is 0

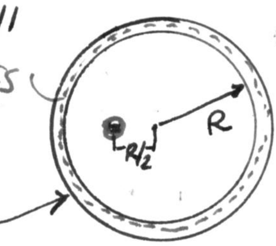
$\vec{E} \to 0$ again

Example Spherical metal shell, $\vec{E}$ and q_{enc}

A negatively charged particle is placed inside the neutral metal shell. Draw $\vec{E}$ lines and charge distribution.

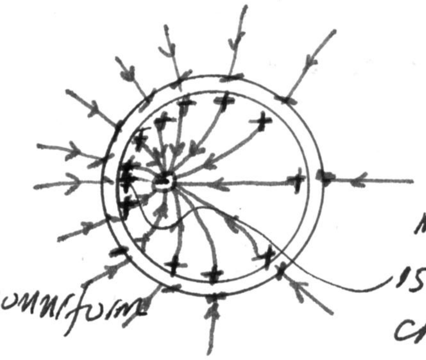
SLN Fig.23-11

neutral metal shell



net charge inside GS is zero

$q = -5\mu C$

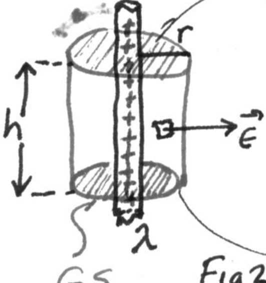


Total charge in shell is conserved

more positive charge is near negative charge.

Applying Gauss' law

1) Cylindrical Symmetry



GS Fig 23-12

- An infinetely long cylindrical plastic rod, ~~with a~~
- a uniform positive linear charge density, λ
- What is the magnitude of $\vec{E}$ at a distance r?
  - Directed radially outward. (+ charge)
  - Flux through end caps is zero since $\vec{E}$ is parallel to these caps. ($\vec{E} \perp d\vec{A}$)
  - Flux through cylindrical surface,

$$\Phi = EA\cos\theta = E(2\pi rh)\cos 0 = E(2\pi rh)$$

Gauss's law

$$\varepsilon_0 \Phi = q_{enc}$$

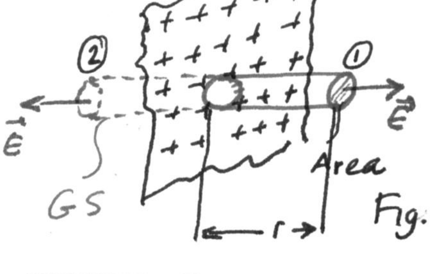
$$\varepsilon_0 E 2\pi rh = \boxed{\lambda h}$$

charge enclosed by GS

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$ line of charge

2) Planar Symmetry

2a Nonconducting Sheet



- Infinite nonconducting sheet
- a uniform positive surface charge density, σ
- What is the magnitude of $\vec{E}$ at a distance r?
  - Again cylindrical GS

Fig. 23-15
  - Directed away from the sheet. (+ charge)
  - Flux through cylindrical surface is zero since $\vec{E}$ ∥ surface ($\vec{E} \perp d\vec{A}$)
  - Flux through end caps $\varepsilon_0(\overset{①}{EA} + \overset{②}{EA}) = \boxed{\sigma A}$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$ sheet of charge

2b Two Conducting Plates

Thin infinite conducting plate

(+) charged plate

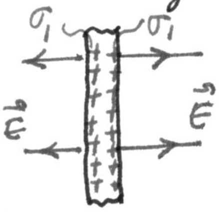
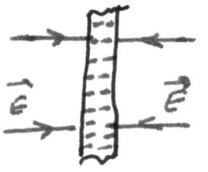
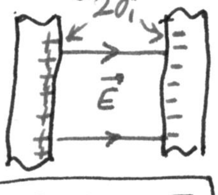


Fig.23-16

(-) charged plate



They come close



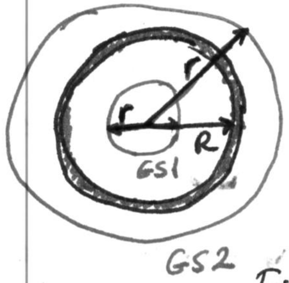
- The excess charge on one plate will attract the excess charge on the other plate, and all the excess charge will move to the inner surfaces.
- charge density on each surface: $2\sigma_1$

$$\varepsilon_0(\overset{E_{q+}}{EA\cos 0} + \overset{E_{q-}}{EA\cos\theta}) = 2\sigma_1 A \Longrightarrow \boxed{E = \frac{2\sigma_1}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}}$$

3) Spherical Symmetry

shell theorems 3LN - Charged spherical shell, q
Radius, R

- Two Gaussian Surfaces GS1 & GS2



- GS2 $r \geq R$ (Remember shell theorem1)

Apply Gauss' law

Fig.23-18 $\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}}$

- GS1 $r < R$

$\boxed{E = 0}$

- Since GS1 encloses no charge
- Remember shell theorem2

$\vec{E}$ in Spherically Symmetric Charge Distribution

charge density, ρ. which is a function of only distance from center

- Entire charge lies within GS

enclosed charge is q. (ρ)

Fig. 23-19a

- $\vec{E}$: as if all the charge were a point charge at the center

- Entire charge does'nt lie within GS

Fig. 23-19b

q' : the charge enclosed by GS

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q'}{r^2}$$

To find q': $\frac{q'}{\frac{4}{3}\pi r^3} = \frac{q}{\frac{4}{3}\pi R^3}$

$$\Rightarrow q' = q\frac{r^3}{R^3}$$

$$\Rightarrow \boxed{E = \frac{q}{4\pi\varepsilon_0 R^3} r}$$

Chapter 24 - Electric Potential

A force is conservative?
- thus, has an associated electrical potential energy
- Path independence (i.e. Gravitational force)
- Apply the principle of the conservation of mechanical energy.

height ↔ equipotential sufaces

Electric Potential Energy

Charged particles → Electrostatic force acting btw them → Assign an electric potential energy, U

$W = \vec{F}\cdot\vec{d}$; $\vec{F} = q\vec{E}$

system has a configuration → initial state, $i \Rightarrow U_i$

a change ↓

final state, $f \Rightarrow U_f$

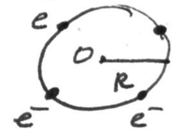
$\Delta U = U_f - U_i = -W$ Change in potential energy

Since electrostatic force is conservative ⇒ path independence whic means that work done on the particle is the same for all paths.

Electric Potential

- The potential energy per unit charge $\left(\frac{U}{q}\right)$ is independent of the charge, q.
- Characteristic only of the electric field.

Example



V at O?

$\vec{E}$ at O?

scalar

$V = -4 \times \frac{1}{4\pi\varepsilon_0}\frac{e}{R}$:)

$\vec{E} = 0$: vector quantity symmetry cancelled

$V = \frac{U}{q}$ ÷ Electric Potential. A scalar not a vector (OR Potential)

↳ Potential energy per unit charge

- Electric Potential Difference btw two points, $\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \boxed{\frac{\Delta U}{q}} = \boxed{\frac{-W}{q}}$ SLN

Take $U_i = 0$ at infinity then V at infinity is equal to zero.

$V = -\frac{W_\infty}{q}$ work done by $\vec{E}$ on a charged particle to move from infinity to point f.

Unit of potential

$\frac{\text{Joule}}{\text{Coulomb}} \equiv 1$ Volt

- Redefine the unit of $\vec{E}\left(\frac{N}{C}\right)$: $1\,\frac{N}{C} = 1\,\frac{N}{J/V} = 1\,\frac{NV}{Nm} = 1\,V/m$
- Work to move an electron through 1V is called one electron-volt (1eV)

Magnitude of the work: $q\Delta V \Rightarrow 1\,eV = e(1V) = (1.6\times10^{-19}C)(1\,J/c)$

$\Rightarrow \boxed{1\,eV = 1.6\times10^{-19}J}$

Work Done by an Applied Force

a charged particle, q } in an $\vec{E}$ → experiences force { move from point i to f } Change in KE

$\Delta K = K_f - K_i = W_{app} + W$

force: Applied one / From $\vec{E}$ itself } W_{app}: work done by the applied force; W: work done by $\vec{E}$

if $K_f = K_i \Rightarrow W_{app} = -W$

~~stationary~~ particle is stationary before and after move. } $\Delta U = -W$, $\Delta U = W_{app}$ } $\Delta U = U_f - U_i = \boxed{W_{app} = q\Delta V}$

(+), (-) or zero

Equipotential Surfaces

- Adjacent points that have the same electric potential form an equipotential surface.
- Work done on a particle on a given equipotential surface is zero.

SLN Fig. 24-2, Fig. 24-3 $\quad -W = q\Delta V = 0 \rightsquigarrow \Delta V = 0$

① Calculating the Potential from the Field

What is the potential difference btw two points ① and ② in an $\vec{E}$?

SLN Fig. 24-4 · Consider a positive charge q_0 moving btw points $\underset{i}{①}$ and $\underset{f}{②}$ along the path shown

- $dW = \vec{F}\cdot d\vec{s}$, $dW = q\vec{E}\cdot d\vec{s}$ } Differential work } $\underbrace{W = q_0 \int_1^2 \vec{E}\cdot d\vec{s}}_{\text{Total work}} \rightarrow \frac{W}{q_0} = \int_1^2 \vec{E}\cdot d\vec{s} \Rightarrow \boxed{V_f - V_i = -\int_1^2 \vec{E}\cdot d\vec{s}}$ $\quad \Delta V = \frac{-W}{q}$
- Since $\vec{F}$ is consevative, all paths btw 1-2 yield the same work.

② Potential due to a Point Charge

Consider a positive charged particle, q and a positive test charge, q_0 at point P at a distance R from the fixed charged particle. What is the potential at point P? SLN Fig. 24-6

i: q_0 at R; f: q_0 at ∞ } $V_i = V$, $V_f = V_\infty = 0$ } $V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s}$ } $0 - V_i = -\int_R^\infty E\,\underbrace{ds}_{dr}\,\overset{1 \text{ since } \theta = 0°}{\cos\theta}$

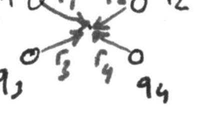
$\rightarrow V_i = V(\text{at } R) = +\int_R^\infty \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,dr \rightarrow \boxed{V = +\frac{1}{4\pi\varepsilon_0}\frac{q}{R}}$ ↗ + charge produces +V ↘ − charge produces −V

③ Potential due to a Group of Point Charges

Net Potential. Superposition principle. Potential is equal to sum of potential resulting from each charge at the given point

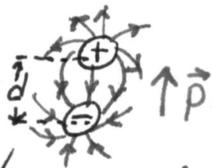
$\boxed{V = \sum_{i=1}^{\overset{\sim \#\text{charge}}{n}} V_i = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}}$

Example square center



$V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \frac{q_4}{r_4}\right)$

④ Potential due to an Electric Dipole



$\uparrow \vec{p} \sim qd$

SLN Fig. 24-10 Potential at an arbitrary point P, $V = \sum_{i=1}^{2} V_i$

- At point P: + charge sets a +V; − charge sets a −V $\Big\}$ $\overbrace{V = V_{(+)} + V_{(-)}}$: Net potentia

$$\Rightarrow V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_{(+)}} + \frac{-q}{r_{(-)}}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{r_{(-)} - r_{(+)}}{r_{(-)}\,r_{(+)}}\right)$$

Since naturally occuring dipoles are small, $r >> d$
$\Rightarrow r_{(-)} - r_{(+)} \sim d\cos\theta$
$r_{(-)}\,r_{(+)} \sim r^2$

$$\Rightarrow V = \frac{q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2} \Rightarrow \boxed{V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}}$$

; p: magnitude of electric dipole moment

SLN Fig. 24-11

⑤ Potential due to a Continuous Charge Distribution

Not point charges anymore. But Continuous charge distribution.

$q \longrightarrow V$ $\qquad$ $\Downarrow$ $dq \rightarrow dV$ $\quad dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$

Then, integrate over the entire charge distribution to find the potential, V

$$V = \int dV = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$$

Line of Charge:

non-conducting thin rod.

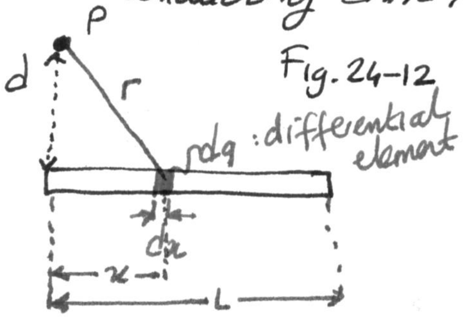


Fig. 24-12

What is V at point P? positive charge linear density, λ length, L

Start by

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} \rightarrow V = \int dV \quad \Big\{ \; dq = \lambda dx, \quad r = \sqrt{x^2+d^2}$$

$$\Rightarrow V = \int_0^L \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{(x^2+d^2)^{1/2}} \checkmark \Rightarrow \boxed{V = \frac{1}{4\pi\varepsilon_0}\ln\left[\frac{L+(L^2+d^2)^{1/2}}{d}\right]}$$

Charged Disk:

non-conducting (plastic) disc

SLN Fig. 24-13

What is V at point P? surface charge density, σ

Start by

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} \rightarrow V = \int dV \quad \Big\} \; dq = \sigma dA = \sigma\, 2\pi R' dR', \quad r = \sqrt{R'^2 + z^2}$$

$$\Rightarrow V = \frac{1}{4\pi\varepsilon_0}\int_0^R \frac{\sigma\, 2\pi R' dR'}{\sqrt{R'^2+z^2}} \Rightarrow V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R'^2} - z\right)$$

Calculating the Field from the Potential

SLN Fig. 24-14

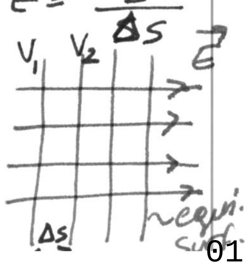
$\frac{\Delta U}{q_0} = \Delta V$, $\Delta U = -W$

$W = \vec{F}\cdot\Delta\vec{s}$, $W = q_0\vec{E}\cdot d\vec{s}$ $\Big\}$ $-q_0 dV$, $q_0\vec{E}\cdot d\vec{s}$ $\Big\}$ $-q_0 dV = q_0 \underbrace{E\cos\theta}_{E_s} ds$

$\Rightarrow \boxed{E_s = \frac{-dV}{ds}}$ OR $\frac{\partial V}{\partial s}$

in general $E_x = -\frac{\partial V}{\partial x}$ (y, z)

$E = \frac{-\overset{(V_2 - V_1)}{\Delta V}}{\Delta s}$



Electric Potential Energy of a System of Point Charges

SLN Fig.24-15

When we bring q_2 from infinity to a point near q_1, we must do a work since q_1 exerts electrostatic force on q_2.

That work should be equal to $q_2 V$, where V is the potential that is created by q_1

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r} \Rightarrow \boxed{U = W = q_2 V = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}}$$

U q_1, q_2

(+) ← same charges

(−) ← opposite charges

Chapter 25 – Capacitance

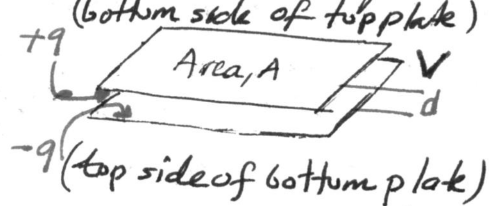
The Capacitor is a device → electrical energy can be stored.
How Much → capacitance.

Capacitance

SLN Fig. 25-2

Basic elements of any capacitor ⇒ Two isolated conductors of any shape
→ Plates (flat or not)

Parallel Plate Capacitor:



SLN Fig. 25-36

$\underbrace{\text{Two parallel conducting plates}}^{\text{Equipotential Surface}}$ of area A, seperated by a distance d.

—| |— ; symbol ; used for capacitors of all geometries

- * No material medium (glass, plastic) is present btw the plates for ~~now~~ now
- * Capacitor is charged ⇒ plates have charges of equal magnitudes but opposite signs ; +q & -q
  → However, charge of capacitor is q
- * Potential difference ⇒ btw two plates → ΔV, use V

↳ The charge q and the potential differene V for a capacitor are proportional to each other. Proportionality Constant : $C\left(=\frac{q}{V}\right)$

$$\Rightarrow \boxed{q = CV}$$

1 farad, 1 F, 1 Coulomb per volt
(usually μF or pF)

Charging a capacitor

Place the capacitor in an $\overbrace{\text{electric circuit}}^{\text{charge can flow through}}$ with a batte maintains a potential difference btw its terminals
- higher potential → + terminal
- lower potential → – terminal

SLN, Fig 25-4 S → open ; incomplete circuit
S → closed ; complete circuit

electrons are driven through the wires by an $\vec{E}$ that the battey sets up in the wire.

- plate h ; becomes + (loosing $\bar{e}$s)
- plate l ; becomes – (gaining $\bar{e}$s)

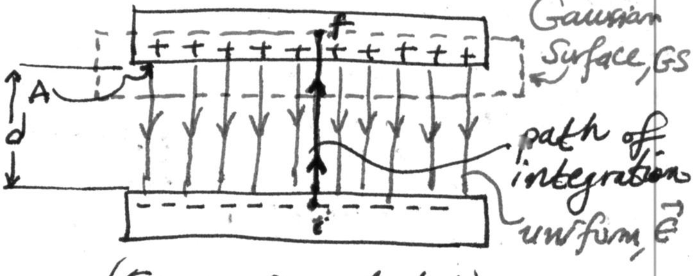
Initially	During Charge	Fully charged
no charge	+q & -q	V equals to battery
V=0	V increases	$\vec{E}=0$
$\vec{E}=0$	$\vec{E}$ increases	charge, q $\boxed{\frac{q}{V}=C}$

Calculating the Capacitance

Known geometry ⇒ capacitance of a capacitor

① assume a charge q on the plates
② calculate $\vec{E}$ using Gauss' law
③ calculate V (knowing $\vec{E}$)
④ calculate C (knowing q & V)

Fig. 25-5



(Fringing is neglected)

② Calculating $\vec{E}$, using Gauss' law. $\boxed{\varepsilon_0 \oint \underbrace{\vec{E}\cdot d\vec{A}} = q}$ → net charge enclosed by a GS

$\underbrace{\vec{E}\cdot d\vec{A}}$: net electric flux through the surface

knowing $\vec{E}$ ↓

$\vec{E} \parallel d\vec{A}$ $\Rightarrow \boxed{q = \varepsilon_0 E A}$

③ Calculating V

$$\boxed{\underbrace{V_f - V_i}_{} = -\int_i^f \vec{E}\cdot d\vec{s}}$$

- choose a path that follows an $\vec{E}$ line
- from negative to positive plate

} $\vec{E}$ & $d\vec{s}$ opposite directions

$\Rightarrow -E\,ds$

$$\boxed{V = \int_-^+ E\,ds} \qquad V = E\int_0^d ds = Ed$$

$$q = CV \quad \} \quad \varepsilon_0 E A = C E d$$

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

Parallel Plate Capacitor

ε_0: permittivity constant of air. $8.85 \times 10^{-12} \frac{F}{m}$

Capacitance does only depend on geometrical factors; A, d

$C\uparrow \Leftarrow A\uparrow$
$C\downarrow \Leftarrow d\uparrow$

A Cylindrical Capacitor

SLN Fig. 25-6

- A cylindrical capacitor of length L
- Two coaxial cylinders of radii a & b
- Charge, q

} $L \gg b$ → neglect fringing of $\vec{E}$

radius of GS

② $q = \varepsilon_0 E A = \varepsilon_0 E (2\pi r L) \Rightarrow E = \dfrac{q}{2\pi\varepsilon_0 L r}$

③ $V = \int_-^+ E\,ds = -\dfrac{q}{2\pi\varepsilon_0 L}\int_b^a \dfrac{dr}{r} = \dfrac{q}{2\pi\varepsilon_0 L}\ln\left(\dfrac{b}{a}\right)$

$(ds = -dr)$

④ $q = CV \rightarrow \boxed{C = \dfrac{2\pi\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)}}$

Cylindrical Capacitor

A Spherical Capacitor

SLN Fig. 25-6 Same figure but two concentric spherical shells of radii a & b.

② $q = \varepsilon_0 E (4\pi r^2) \Longrightarrow E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}$

③ $V = -\dfrac{q}{4\pi\varepsilon_0}\int_b^a \dfrac{dr}{r^2} = \dfrac{q}{4\pi\varepsilon_0}\left(\dfrac{b-a}{ba}\right)$

④ $\boxed{C = 4\pi\varepsilon_0 \dfrac{ab}{b-a}}$ Spherical Capacitor

An Isolated Sphere

- Single isolated spherical conductor of radius, R
- Assuming missing plate is a conducting sphere of infinite radius

$(b \to \infty)$

$$C = 4\pi\varepsilon_0 \frac{a}{1 - a/b} \boxed{= 4\pi\varepsilon_0 R}$$

$a/b \to 0$ & $a \to R$

Example

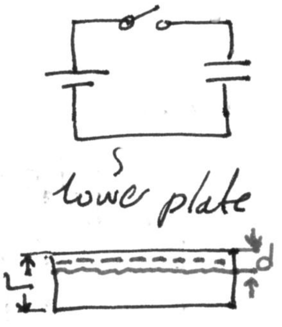


Fig. 25-7

$C = 0.25\,\mu F$

$V = 12\,V$

$L = 0.5\,cm$

$A = 2.0 \times 10^{-4}\,m^2$

$n = 8.49 \times 10^{18}\,\frac{e^-s}{m^3}$ (in copper)

$q = CV = 3.0 \times 10^{-6}\,C$

Number of e^-s } $N = \dfrac{q}{e}$

$N = \dfrac{3.0 \times 10^{-6} C}{1.602 \times 10^{-19} C}$

$n = \dfrac{N}{\text{Volume}} = \dfrac{N}{Ad} \Rightarrow d = \dfrac{N}{An}$

$\Rightarrow d = 1.1 \times 10^{-12}\,m = 1.1\,pm$

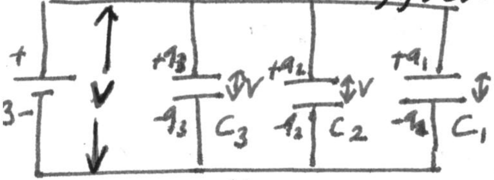
- Electrons very close to the plate face.

Capacitors in Parallel and in Series

An equivalent capacitor : a single capacitor that has the same capacitance as the actual combination of capacitors

- Capacitors in Parallel

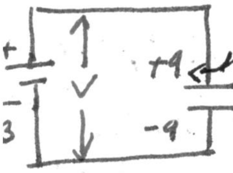
Potential difference, V : applied accross several capacitors in parallel



- Same V is applied accross each capacitor } $C_1 V = q_1$, $C_2 V = q_2$, $C_3 V = q_3$
- total charge, q, is the sum of the charges stored on all the capacitors } $q = q_1 + q_2 + q_3$

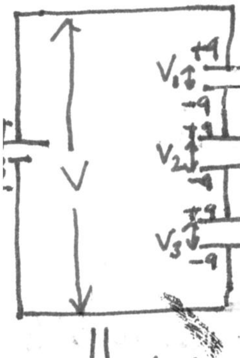
$$= \underbrace{(C_1 + C_2 + C_3)}_{C_{eq}} V$$

||
Equivalent
⇓



$$\Rightarrow C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3 \boxed{= \sum_{j=1}^{n} C_j}$$ n capacitors in parallel

- Capacitors in Series



A chain reaction ⇒ charging of each capacitor causes the charging of the next capacitor

From C_3 to C_1

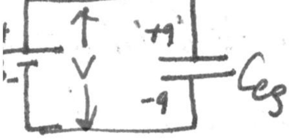
B - produces charge −q on the bottom plate of C_3 ⇒ +q on the top plate of C_3 (due to repelling)

C_3 - " " " " " " " " C_2 ⇒ " C_2

C_2 - " " " " " " " " C_1 ⇒ " C_1

C_1 - supplies +q to the battery ⇒ V is sustained

||
Equivalent
⇓



- Same charge q accross each capacitor
- same total potential difference, V

$$V_1 = \frac{q}{C_1}, \quad V_2 = \frac{q}{C_2}, \quad V_3 = \frac{q}{C_3}$$

$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) = q\, C_{eq} \Rightarrow \boxed{\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}}$$

n capacitors in series

Energy Stored in an Electric Field

To charge a capacitor ⇐ Work must be done by an external agent

- e^-s are removed from one plate → transfered to the other plate
- $\vec{E}$ (btw the plates) tends to oppose further transfer
- Thus, as charge accumalates on the capacitor plates ⇒ more work is required to transfer e^-s
- → Required Chemical energy stored in battery $\xRightarrow{\text{becomes}}$ Electric Potential Energy

Incremental work, dw } $dw = V' dq' = \frac{q'}{C} dq'$ } $w = \int dw = \frac{1}{C}\int_0^q q' dq' = \boxed{\frac{q^2}{2C} = U}$

potential difference, q' btw plates, $\frac{q'}{C}$

~ Extra increment of charge dq' (to be transfered)

$$\boxed{U = \frac{q^2}{2C} = \frac{1}{2} C V^2}$$ Stored potential energy, U in the capacit

Capacitor with a Dielectric

An insulating material (dielectric; plastic...) → btw the plates of a capacitor } what happens? } Capacitance increased by a numerical factor, κ

κ: Dielectric constant of the insulating material

SLN Table 25-1

⇓

Limits the potential difference btw the plates to V_{max} ⤳ breakdown potential

when $V >> V_{max}$ → dielectric material will breakdown and form a conducting path between the plates

Dielectric Strength: $\vec{E}_{max}$ that material can tolerate without breakdown

$C = \varepsilon_0 \mathcal{L}$ $\left(\mathcal{L} = \frac{A}{d}\right) \Rightarrow C = \kappa\varepsilon_0 \mathcal{L} = \kappa C_{air}$ } $\varepsilon_0 \Leftrightarrow \kappa\varepsilon_0$ } $E = \frac{1}{4\pi\kappa\varepsilon_0}\frac{q}{r^2}$: point charge

$E = \frac{\sigma}{\kappa\varepsilon_0}$: isolated conductor

- For a fixed distribution of charges, the effect of a dielectric is to weaken the electric field.

Dielectric and Gauss' Law

SLN Fig. 25-16 q': Induced charges , q: free charge on plate

Without dielectric material

$\varepsilon_0 \oint \vec{E}\cdot d\vec{A} = \varepsilon_0 E A = q$

$\rightarrow E_0 = \frac{q}{\varepsilon_0 A}$

$\Rightarrow \boxed{\varepsilon_0 \oint \kappa \vec{E}\cdot d\vec{A} = q}$

Gauss' law with dielectric

with dielectric material

$\varepsilon_0 \oint \vec{E}\cdot d\vec{A} = \varepsilon_0 E A = q - q'$

$\rightarrow E = \frac{q-q'}{\varepsilon_0 A}$ } we know $E = \frac{E_0}{\kappa}$; weaken original field

$\frac{q-q'}{\varepsilon_0 A} = \frac{q}{\kappa\varepsilon_0 A} \Rightarrow q - q' = \frac{q}{\kappa}$

Induced surface charge { $* \; q' = 0 \rightarrow \kappa = 1$ no dielectric present

{ $* \; q' < q \rightarrow \kappa > 1 \Rightarrow$ weakens $\vec{E}$

Example

SLN Fig 25-17

$A = 115\text{ cm}^2 = 115\times10^{-4}\text{ m}^2$

$d = 1.24\text{ cm} = 1.24\times10^{-2}\text{ m}$

$V_0 = 85.5\text{ V}$

$b = 0.78\text{ cm} = 0.78\times10^{-2}\text{ m}$

$\kappa = 2.61$

$d > b$

without slab

i) $C_0 = ?$

$C_0 = \frac{\varepsilon_0 A}{d}$

$= \frac{(8.85\times10^{-12}\frac{F}{m})(115\times10^{-4}m^2)}{1.24\times10^{-2}m}$

$\boxed{C_0 = 8.21\text{ pF}}$ 10^{-12}

ii) $q = ?$

$q = C_0 V_0 = (8.21\times10^{-12}F)\, 85.5\text{ V}$

$\boxed{q = 702\text{ pC}}$

iii) $E_0 = ?$ btw the plates and slab

$\varepsilon_0 \oint \vec{E_0}\cdot d\vec{A} = q \rightarrow \varepsilon_0 \int E_0 dA \cos 0 = q \rightarrow \varepsilon_0 E \kappa A = q$

$\overset{\uparrow}{1}$ in air

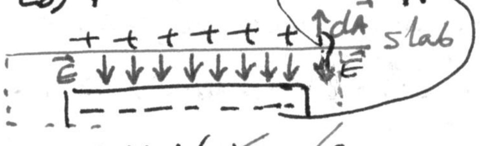
$E = E_0 = \frac{q}{\varepsilon_0 A} = \frac{702\times10^{-12}C}{(8.85\times10^{-12}F/m)(115\times10^{-4}m^2)} \boxed{= 6.90\text{ kV/m}}$

with slab

iv) $E_1 = ?$ in slab

$\varepsilon_0 \oint \vec{E_1}\cdot d\vec{A} = q_{enc}$

$\varepsilon_0 \oint E_1 \kappa\, dA \cos 180° = -q$



$\varepsilon_0 E_1 \kappa A \cancel{(-1)} = \cancel{-}q$

$E_1 = \frac{q}{\kappa\varepsilon_0 A} \boxed{= 2.64\text{ kV/m}}$

v) $V = \int_{-}^{+} E ds = E_0(d-b) + E_1 b$

$\boxed{V = 52.3\text{ V}}$

iv) $C = \frac{q}{V} = 13.4$ pF

slab → $\boxed{13.4\text{ pF} > 8.21\text{ pF}!}$

Chapter 26 - Current and Resistance

Up to now: Electrostatics → physics of stationary charges.
From now on: physics of electric currents ← charges in motion
streams of moving charges

Electric Current

Not all moving charges constitute an electric current → Net flow of charge is needed.
→ e^-s are in random motion with 10^6 m/s !! ← no net transport of charge (cancel each other!) SLN Fig. in Slide 2
→ Connect a battery → Create a potential difference ⇒ Now, we have a net transport of charge. SLN Fig in Slide 3

Steady Currents of conduction e^-s moving through metallic conductors. SLN Fig. in Slide 4

Fig. 26-1 Consider a battery in the loop
- $\vec{E}$ act inside the material. + − $\vec{E}$ loop
- $\vec{E}$ creates a force which acts on conduction e^-. SLN Fig in Slide 4

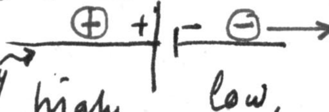
→ This force causes e^-s to move and electric current is established.
⇒ After a very short time, e^- flow reaches a constant value and the current is in its steady state (does not vary with time)

$\boxed{i = \frac{dq}{dt}}$ SI Unit 1 ampere = 1 A = 1 C/s | Total charge from 0 to t $q = \int dq = \int_0^t i\,dt$ SLN

Fig. 26-2 SLN $i(aa') = i(bb') = i(cc')$ ↔ charge is conserved ↔ SLN Fig. 26-3 ($i_0 = i_1 + i_2$)
The current has same value at each plane

Direction of Current:

i ← direction of current (conventionally accepted!!)



→ Actual charge carriers are negative and move in opposite direction (electrons)
Battery
high voltage
low voltage
SLN

Current Density:

Flow of charge through a cross section → current density $\vec{J}$

$|\vec{J}| = \boxed{J = \frac{i}{A}} = \frac{\Delta i}{\Delta A} \rightarrow \frac{di}{dA} = J$ $\left\{ i = \int di = \int \vec{J} \cdot d\vec{A} = JA \right\}$

total area
Magnitude & direction
Unit A/m^2

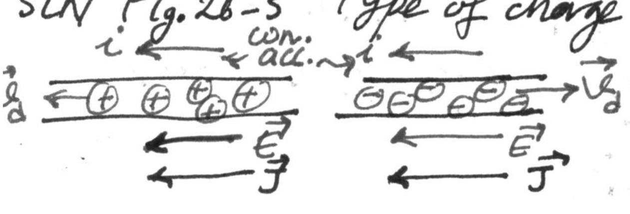
charge carrier (cc)	Direction of $\vec{J}$
+	same with (cc)
−	opposite to (cc)

SLN Fig. 26-4
Charge is conserved. However, current density ($\vec{J}$) changes.
(same i) ——— (different area)

Drift Speed:

Random motion with 10^6 m/s + applied $\vec{E}$ (or V) ⇒ Drift motion with v_d drift speed ($\sim 10^{-5}$ m/s!!)

SLN Fig. 26-5 Type of charge carriers!?



- A conductor of length L
  → a current in which
  - n: # of cc / volume (number density)
  - A: cross sectional area
  - (1.602×10^{-19}C)

$q = (nAL)e$: Total cc in length L

$t = \frac{L}{v_d}$: Time interval that charges move through L

$$i = \frac{q}{t} = \frac{nALe}{L/v_d} = nAe\,\overset{?}{v_d} \rightarrow v_d = \frac{\overset{J}{i}}{nAe}$$

$$\Rightarrow v_d = \frac{J}{ne} \Rightarrow \boxed{\vec{J} = ne\,\vec{v_d}} \quad \frac{A}{m^2} \equiv \frac{C}{m^3}\,m/s$$

ne: charge carrier (cc) density

Example: Current density: uniform & nonuniform

A cylindrical wire with radius $R = 2.0\,mm$

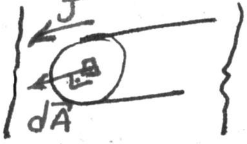
i) J : uniform $2.0\times10^5\,A/m^2$. What is the current btw $\frac{R}{2}$ & R?

SLN Fig. 26-6a $J = \frac{i}{A}$ | $A = ?$ | not total cross-section area but effective one $A' = \pi R^2 - \pi(\frac{R}{2})^2 = \frac{3\pi R^2}{4}$

$$\Rightarrow 2.0\times10^5\,A/m^2 = \frac{i}{\frac{3\pi}{4}(2.0\times10^{-3}m)^2} \Rightarrow \boxed{i = 1.9A}$$

ii) $\vec{J} = \underset{3\times10^{11}\,A/m^2}{a}r^2$: non uniform $(J \rightarrow J(r))$. What is the current btw $\frac{R}{2}$ & R?

SLN Fig-26-6 b-e $J = \frac{di}{dA} \rightarrow i = \int di = \int \vec{J}\cdot d\vec{A}$



$= \int J\,dA\,\cos 0$

$\Rightarrow$


$dA = 2\pi r dr$

$$i = \int_{R/2}^{R} J\,2\pi r dr = \int_{R/2}^{R} ar^2 2\pi r dr = 2\pi \underset{3\times10^{11}\,A/m^2}{a} \frac{r^4}{4}\Big|_{R/2}^{R \rightarrow 2.0\times10^{-3}m} \boxed{= 7.1A}$$

Sample Problem SLN In a current, the conductions $\bar{e}$s move very slowly ($v_d = 4.9\times10^{-7}m$ for sample problem

Resistance and Resistivity

Copper & Glass (at similar shapes) } same $\vec{E}$ (or V) applied $\Rightarrow$ different currents are obtained } why? The answer is resistance of the material! SLN Characteristics of the material

$\boxed{R = \frac{V}{i}}$ SI unit $\Omega = \frac{Volts}{Ampere}$ $(1\Omega = 1\frac{V}{A})$ The conductor which provide resistance is called resistor.

Resistance

In case of having V $\Rightarrow$ deal with i & R. Resistance is a property of an object. (Shape dependency)

In case of having $\vec{E} \rightarrow$ deal with J & ρ : Resistivity is a property of a material. (Shape does not matter)

$R = \frac{V}{i} \longrightarrow \boxed{\rho = \frac{E}{J}}$ SI unit $\Omega\cdot m$ Resistivity $\left(\frac{V/m}{A/m^2}\right)$

$\boxed{\sigma = \frac{1}{\rho}}$ SI unit $\frac{1}{\Omega m}$ Conductivity

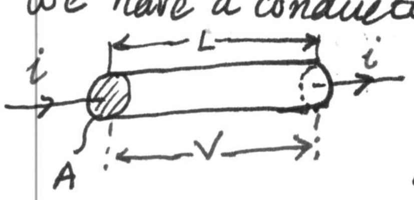
In vector forms

$$\boxed{\begin{aligned}\vec{E} &= \rho\vec{J}\\ \sigma\vec{E} &= \vec{J}\end{aligned}}$$

SLN Table 26-1

Calculating Resistance from Resistivity

we have a conductor as below



Assumption: J is uniform $\Downarrow$ E is constant all over the wire

$$\rho = \frac{E}{J} = \frac{V/L}{i/A}$$

$$\rho = R\frac{A}{L}$$

$$\boxed{R = \rho\frac{L}{A}}$$

Resistivity vary with temperature SLN Fig. 26-10

$$\rho - \rho_0 = \rho_0\alpha(T - T_0)$$

Ohm's Law

A resistor is a $\boxed{\text{conductor}}$ ~ conducting device with a specified resistance — not change with V. / applied potential difference — changes

SLN Fig. 26-11. Not Change ⇒ Ohm's Law (OR not law)

$\Rightarrow \boxed{V = iR}$ Ohm's Law! (not only)

$\Downarrow$ i vs V : i directly proportional to V

$\frac{i}{V} = \frac{1}{R}$ independent of magnitude and polarity

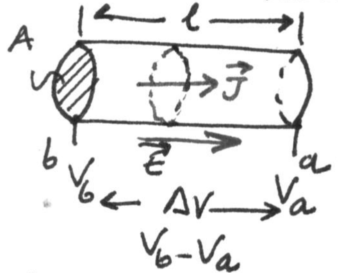
↓ Applies to all conducting devices, whether they obey Ohm's Law or not. (Fig. 26-11c)

Ohm's Law: i vs V is a linear plot ⇒ R is independent of V.

More general way: $\boxed{\vec{E} = \rho \vec{J}}$ ~ conducting materials (microscopic version)

A conducting material obeys Ohm's Law when the resistivity of the material is independent of the magnitude and direction of the applied field.

* Consider a section of a wire with length of l and cross-sectional area A:



ΔV is applied → $\vec{E}$ created & a current I

$\Delta V = V_b - V_a = \int_a^b \vec{E} \cdot d\vec{s} = El \Rightarrow E = \frac{\Delta V}{l}$

$J = \sigma E = \sigma \frac{\Delta V}{l} \Leftrightarrow \frac{I}{A} = \underbrace{\sigma}_{1/\rho} \frac{\Delta V}{l}$

$\Rightarrow \underbrace{\rho \frac{l}{A}}_{R} I = \Delta V \Rightarrow \boxed{\Delta V = I \Delta R}$

Macroscopic version of Ohm's Law

Microscopic View of Ohm's Law:

e^- & m_e & $\vec{E}$ → Force experienced → acceleration of e^-

$a = \frac{F}{m} = \frac{eE}{m}$ } Random Motion & Drift Velocity } Mean free path, Mean free time (τ) } without collision!

Random Motion ⇓ collisions; Drift Velocity ⇓ $v_d = a\tau$

$\Rightarrow v_d = a\tau = \frac{eE\tau}{m}$ & $\boxed{J = ne}\, v_d$ } $v_d \equiv \frac{J}{ne} = \frac{eE\tau}{m}$ } $E = \underbrace{\left(\frac{m}{e^2 n \tau}\right)}_{\rho} J$ } $\vec{E} = \rho \vec{J}$

SLN Sample Problem

Power In Electric Circuits

Fig. 26-13 SLN

Battery → potential difference $V_a > V_b$ set up → a steady state current $i = \frac{dq}{dt}$ maintained } from a to b in Fig.

- dq moves btw.
- Decrease in V ($V_a > V_b$)

$\Rightarrow \frac{U}{q} = V$ } $\frac{\Delta U}{\Delta q} = V$ } $\frac{dU}{dq} = V$ } $dU = i\,dt\,V$ } $\frac{dU}{dt} \boxed{= iV = P}$ ⇒ Decrease in U

Rate of electrical energy transfer!! (of all kinds)

SI unit

$1\,VA = 1\left(\frac{J}{C}\right)\left(\frac{C}{s}\right) = 1\frac{J}{s} = 1\,W$

$\boxed{P = i^2 R \quad P = \frac{V^2}{R}}$ Rate of electrical (potential) energy transfer to thermal energy for a resistor. SLN

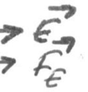
Rate of Electrical Energy Dissipation due to a Resistance.

Chapter 28 - Magnetic Fields

- How an $\vec{E}$ can produce an $\vec{F}_E$ on a charged object?
- How an $\vec{B}$ can produce a $\vec{F}_B$ on a (moving) charged particle or on a magnet object
- $\vec{E}$ is produced by an electric charge.
- $\vec{B}$ is produced
  - by moving electrically charged particles → current → electromagne
  - by means of e^-s (intrinsic $\vec{B}$ around them): spinning motion → permanent magnet

SLN

- The Definition of $\vec{B}$
- Put a charged particle in an $\vec{E}$ and measure the $\vec{F}_E$ on it.



- Put a charged moving particle in an $\vec{B}$ and measure $\vec{F}_B$

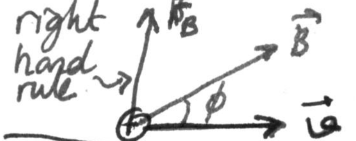
Experiment

① $\vec{F}_B$ is proportional to particles velocity v, and q

② Magnitude and direction of $\vec{F}_B$ depends on $\vec{v}$ and $\vec{B}$

③ when $\vec{v} /\!/ \vec{B}$, $\vec{F}_B \to 0$
when $\vec{v}$ makes an angle ϕ with $\vec{B}$, then $\vec{F}_B \perp \vec{v}, \vec{B}$
Magnitude of $\vec{F}_B \propto \sin\phi$

All these experimental results can be summarized as



$$\boxed{\vec{F}_B = q\,\vec{v} \times \vec{B}}$$

Definition of $\vec{B}$

$$\boxed{|\vec{F}_B| = F_B = |q|\,vB\sin\phi}$$

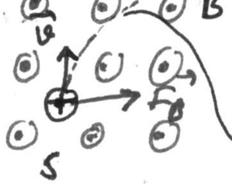
Magnitude

- * if $q=0$ or $v=0 \Rightarrow F_B=0$
- * if $\vec{v} /\!/ \vec{B}$ $\begin{pmatrix}\phi=0\\ \phi=180\end{pmatrix} \Rightarrow F_B=0$
- * if $\vec{v} \perp \vec{B} \to F_B$ is maximum
- - Direction is found by Right Hand Rule SLN (1,2,3/5)
- - Direction of $\vec{F}_B$ is always perpendicular to plane of v and B SLN (4,5/5)
- - SI Unit. 1 Tesla $= 1\,\frac{N}{C\,m/s} = 1\,\frac{N}{A\,m}$

- $\vec{F}_B$ never has a component parallel to $\vec{v}$
  $\Rightarrow \vec{F}_B$ can not charge the particle's speed, ~~it~~ can only change its direction.

Example

Consider an uniform $\vec{B}$ of 1.2 mT directed towards out of the paper. What will be the magnetic force and direction of motion of proton if the proton has a KE of 5.3 MeV and moves initially from S to N?



$$K = \tfrac{1}{2}mv^2 \to v = \sqrt{\frac{2(5.3\times10^{6}\,eV)(1.6\times10^{-19}\,J/eV)}{(1.67\times10^{-27}\,kg)}} = \boxed{3.2\times10^{7}\,m/s}$$

$$F_B = |q|\,vB\sin\phi = (1.6\times10^{-19}\,C)(3.2\times10^{7}\,m/s)(1.2\times10^{-3}\,T)\sin 90 \boxed{= 6.1\times10^{-15}}$$

Proton motion direction is changed and goes to right.

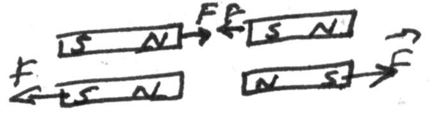
- Magnetic Field Lines.
- Similar to electric field lines.
- North Pole & South pole (emerge from) (enter into)

SLN Fig. 28-4 Fig-28-5

North Geographic Pole
South Magnetic Pole

South geographic Pole
North Magnetic Pole

- Magnet has two poles ⇒ magnetic dipole
- Like poles repel, opposite poles attract

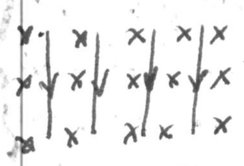


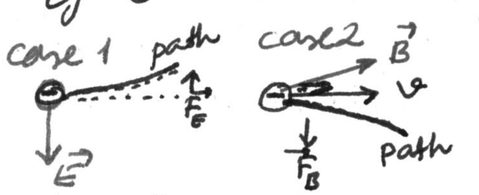
SLN Sample Problem for large acceleration

Crossed Fields: Discovery of the Electron

$\vec{E}$ $\xrightarrow{\text{produce}}$ $\vec{F}_E$ on a charged particle. When $\vec{E} \perp \vec{B}$: crossed fields
$\vec{B}$ $\quad$ $\vec{F}_B$

what happens if e^- move in cross-fields? SCN Fig. 28-7



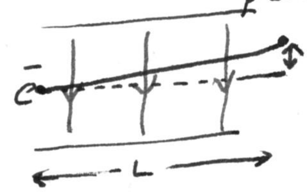


When $\vec{B}=0$

$y=\frac{1}{2}a_y t^2$

$x=L=v_{0x}t \Rightarrow t=\frac{L}{v_x}$

$$\boxed{y=\frac{1}{2}\frac{|q|E}{m_e}\frac{L^2}{v^2}}$$



Deflecting

$a_y=\frac{F}{m}$

When $F_e > F_B$

When $F_E < F_B$

when $\vec{B} \neq 0$ and $|\vec{F}_B| = |\vec{F}_e|$

$qE = |q|vB \sin 90° \rightarrow \boxed{v=\frac{E}{B}}$

- velocity of the charged particle can be measured.

if we substitute $v=\frac{E}{B}$ into deflection equation $\boxed{\frac{m}{|q|}=\frac{B^2L^2}{2yE}}$

→ all the quantities at the right hand side can be measured

→ Allowed us to measure $\frac{m}{|q|}$ of moving particle : Discovery of electron

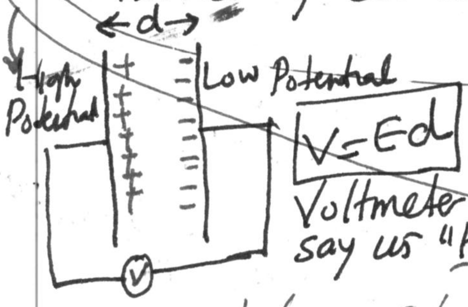
Crossed Fields: The Hall Effect

Previously; $\bar{e}$ was in vacuum and deflected by $\vec{B}$ (and/or $\vec{E}$)

Now, $\bar{e}$ are in a copper wire! < Drifting / Deflected by $\vec{B}$ } Hall effect

- Hall effect allows us to determine
  - type of charge carriers
  - number of carriers per unit volume

SCN Fig. 28-8



$\boxed{V=Ed}$

Voltmeter will say us "Higher Potential"

When $F_e=F_B \rightarrow eE=ev_dB \rightarrow v_d=\frac{E}{B}$

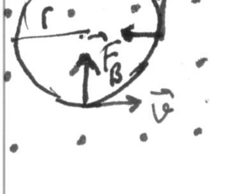
we know $v_d=\frac{J}{ne}=\frac{i}{neA} \Rightarrow n=\frac{i}{v_d eA}=\frac{i}{\frac{E}{B}eA}$

density of charge carrier } $n=\frac{iB}{EeA}=\frac{iB}{\frac{V}{d}edl}=\boxed{\frac{iB}{VeL}}$ all measurable quantities

A circulating Charged Particle

(Uniform Circular Motion)

Consider a beam of $\bar{e}$s projected into a chamber < speed v / uniform B - directed out of page



- Magnetic force
- A circular path
- Continuous Deflection

$F_B=|q|vB\sin 90°$
$F=m\frac{v^2}{r}$

$m\frac{v^2}{r}=|q|vB$

$\boxed{r=\frac{mv}{|q|B}}$ $v\uparrow r\uparrow$ $v\downarrow r\downarrow$

Radius of Circular Path

$T=\frac{2\pi r}{v}=\frac{2\pi m}{|q|B}$

$f=\frac{1}{T}=\frac{|q|B}{2\pi m}$

$\omega=2\pi f=\boxed{\frac{|q|B}{m}}$

No Velocity Dependence

$\frac{|q|}{m}$!!

Example SLN Fig. 28-12 Uniform Circular Motion of a charged particle in a $\vec{B}$

$B = 80.000 \text{ mT}$
$V = 1000.0 \text{ V}$
$q = 1.6022 \times 10^{-19} C$
$x = 1.6254$: strikes
$m = ?$ in atomic mass unit

$r = \frac{\cancel{m}\,\cancel{v}}{|q|\,B}$

We need $\vec{v}$ when enters the chamber.

$\cancel{K_2} + U_2 = \cancel{K_1} + \cancel{U_1}$ 0

$\frac{1}{2} m v^2 - qV = 0$

$\Rightarrow v = \sqrt{\frac{2qV}{m}}$

$r = \frac{m}{|q|B}\sqrt{\frac{2qV}{m}} = \frac{1}{B}\sqrt{\frac{2mV}{q}} = \frac{x}{2}$

$\Rightarrow m = \frac{B^2 q x^2}{8V}$: all known values

$\boxed{m = 3.3863 \times 10^{-25} \text{ kg}} = 203.93 \text{ u}$

divided by 1.6605×10^{-27} kg (atomic mass unit)

Magnetic Force on a Current-Carrying Wire

Current Carrying wire, Magnetic Field } Force on electrons → This force is transmitted to the wire } So that, a current carrying wire will also experience a $\vec{F}_B$

SLN Fig. 28-14

Inside the wire

All conduction e's move through planes $(xx')_1$ & $(xx')_2$

Time required, $t = \frac{L}{v_d}$ & Charge $q = it$

$\Rightarrow \boxed{q = i\frac{L}{v_d}}$ $F_B = |q| v_d B \sin\phi = i\frac{L}{\cancel{v_d}}\cancel{v_d} B \sin 90°$

$\Rightarrow \boxed{F_B = iLB.}$ $\boxed{\vec{F} = i\vec{L}\times\vec{B}}$ Generalization

- Magnetic force acting wire of length L in $\vec{B}$
- $\vec{L}$: length vector $L \parallel i$

- wire of arbitrary shape.
  - Divide the wire into small segments having a length of $d\vec{L}$

$\Rightarrow \boxed{d\vec{F}_B = i\,d\vec{L}\times\vec{B}}$ magnetic force acting on this segment, $d\vec{F}_B$

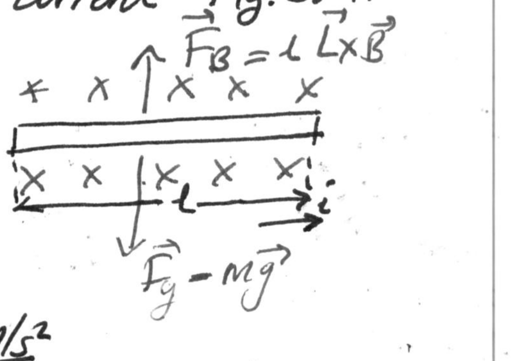
Total Magnetic Force

$F_B = i\int_a^b d\vec{L}\times\vec{B} = i\left(\int_a^b d\vec{L}\right)\times\vec{B}$

$F_B = i\,\vec{L}\times\vec{B}$

If wire form a closed loop : $\vec{F}_B = 0$ $\left(\oint_a d\vec{L} = 0\right)$

Example Magnetic Force on a wire carrying current Fig. 28-17 SLN



straight, horizontal copper wire
$i = 28$ A
$B_{min} = ?$
to suspend the wire to balance $\vec{F}_g$
$\frac{m}{L} = 46.6$ g/m

$\vec{F}_B = i\,\vec{l}\times\vec{B} \Rightarrow ilB\sin\phi$

$\vec{F}_g = m\vec{g} \Rightarrow mg$

$ilB\sin\phi = mg$

for minimum B; $\phi = 90°$

$B = \frac{mg}{iL} = \frac{46.6\times10^{-3} \text{ kg/m } 9.8 \text{ m/s}^2}{28 \text{A}}$

$\boxed{B = 1.6\times10^{-2} T}$

~160 times the strength of Earth's magnetic field

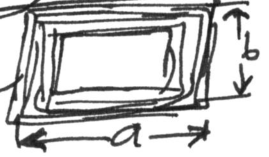
Torque on a Current Loop

Rotation in electric motors is created by torque produced by $\vec{F}_B$

SLN Fig. 28-18 A rectangular loop in $\vec{B}$.

· Fig. 28-19 $\tau' = \underbrace{(iaB\sin\theta)\frac{b}{2}}_{\vec{r}\times\vec{F}_1} + \underbrace{(iaB\sin\theta)\frac{b}{2}}_{\vec{r}\times\vec{F}_3} \Big\} |\vec{r}| = \frac{b}{2}$

$\tau' = iabB\sin\theta \rightarrow \boxed{\tau = N\tau' = (Ni\overset{ab}{A})B\sin\theta}$



· For a coil containing N loop (turns)

$\tau = N\tau' = (NiA)B\sin\theta \Big\} A = ab$

· For a coil containing N loop (circular turns)

$\tau = N\tau' = (NiA)B\sin\theta \Big\} A = \pi r^2$

The Magnetic Dipole Moment

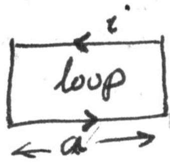
A current-carrying coil placed in a $\vec{B}$ → a torque acts

is said to be a magnetic dipole

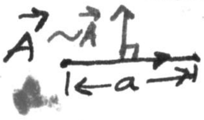
magnetic dipole moment, $\vec{\mu}$

we know that the magnitude of the torque in a N-loop coil

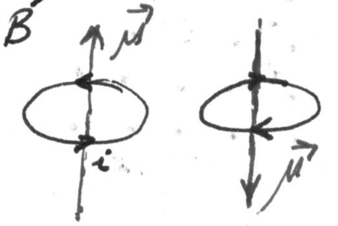
$\tau = NiabB\sin\theta = Ni(\text{Area})B\sin\theta = \mu B\sin\theta = \vec{\mu}\times\vec{B}$



$\vec{\mu} = Ni\vec{A}$



} Unit Am^2 } Direction Right Hand Rule



· Magnetic moment vector tries to align with $\vec{B}$ → minumum enegy SLN Fig. 28-20

· Torque on the coil, τ : $\vec{\tau} = \mu\times B$

· Enegy of magnetic dipole in an external $\vec{B}$: $U(\theta) = -\vec{\mu}\cdot\vec{B}$ — U : max θ=180° ; U : min θ=0 } dipole's orientation

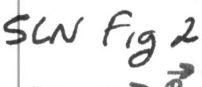
· Work done on dipole : There is torque → Rotates the dipole $\theta_i \Rightarrow \theta_f$

$\rightarrow W_a = U_f - U_i$

Example Rotating a magnetic dipole in a B

	Electric	Magnetic
τ	$\vec{p}\times\vec{E}$	$\vec{\mu}\times\vec{B}$
$U(\theta)$	$-\vec{p}\cdot\vec{E}$	$-\vec{\mu}\cdot\vec{B}$

SLN Fig 28.21



circular coil

$N = 250$

$A = 2.52\times10^{-4}\ m^2$

$i = 100\ \mu A$

$B = 0.85T$

at rest $\vec{B} \parallel \vec{\mu}$

i) Direction of the current?

RHR → ↓i

ii) How much work : $\theta = 0 \rightarrow \theta = 90°$ (initial orientation) (final orientation)

$W_a = U(90°) - U(0) = -\mu B\cos 90 - (-\mu B\cos 0))$

$= 0 + \mu B = \mu B = (NiA)B$

$= (250)(100\times10^{-6}A)(2.52\times10^{-4}m)(0.85T)$

$= 5.355\times10^{-6}J\ \boxed{\sim 5.4\ \mu J}$

Chapter 30 Induction and Inductance

A current produces Magnetic Field } More Suprisingly { A magnetic field can produ an electric field that can drive a current.

$\vec{B}$ induces $\vec{E}$ → Faraday's law of induction.

Experiment 1

SCN Fig. 30-1

- Initially no current passing through the wire
- Move magnet → current on wire
  - move towards the loop → in one direction
  - move away from the loop → in another direction

Experiment 2

SCN Fig. 30-2

- Initially, no current
- S is closed → a sudden current in one direction,
- S is opened → a sudden current in another direction
- Steady state current in right hand loop → No current in left hand loop

---

- The current produced in the loop is called induced current.
- The work done per unit charge to produce that current is called induced emf.
- The process of producing the current and emf is called induction.

{ electro motive force

Faraday's law of induction

Faraday realized that an emf and a current can be induced in a loop by changing the amount of magnetic field passing through the loop.

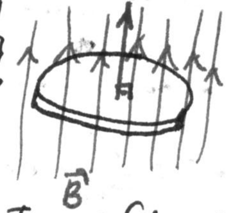
of $\vec{B}$ field lines that pass through the loop $\xrightarrow{\text{changing}}$ An Emf is induced in the loop } Faraday's Law of Induction.

$\Phi_B = \int \vec{B}.d\vec{A}$: Magnetic flux. Magnetic field through area A.

$(\Phi_E = \int \vec{E}.d\vec{A})$

Weber (Wb) 1Wb = 1 Tm²

$\Phi_B = \int B\,dA \cos\theta$



$\Phi_B = B\int dA = BA$

since $\theta = 0$

$$\mathcal{E} = \boxed{-}\frac{d\Phi_B}{dt}$$

minus means that emf, $\mathcal{E}$, tends to oppose flux change

How we can change the Φ_B through a coil?

① Change the magnitude of B,

② Change the area of the coil, OR
" " " " " " by sliding in or out of $\vec{B}$

③ Change the angle btw $\vec{B}$ and $d\vec{A}$

Example Induced $\mathcal{E}$ in a coil due to a solenoid. SCN Fig. 30-3

$N_S = 220$ turn/cm

$i = 1.5A$

$D = 3.2cm$

$N_C = 130$ turn/cm

$d = 2.1cm$

$\Delta t = 25ms$

$B_s = \mu_0 in = \mu_0(1.5A)(22000\text{ turn/m})$

$\Phi_{B_s} = B_s A = B(\pi(\frac{d}{2})^2) = 1.44\times10^{-5}$ Wb

$$\frac{d\Phi_B}{dt} = \frac{\Delta\Phi_B}{\Delta t} = \frac{\overset{0}{\cancel{\Phi_{B,f}}} - \Phi_{B,i}}{\Delta t} = \frac{-1.44\times10^{-5}\,\text{W}}{25\times10^{-3}\,\text{s}}$$

$= -5.76\times10^{-4}$ V

$\mathcal{E} = N\frac{d\Phi}{} = 7.5\times10^{-2}$ V

Lenz's Law

Detemining the direction of induced current in the loop.

An induced current has a direction such that the magnetic field due to current opposes the change in the magnetic flux that induces the current

SLN Lenz law can be evaluated in two difflent ways.

SLN Fig. 30-5

Example Changing Uniform B

A conducting loop

half-circle $r = 0.20m$

three straight sections

$B = 4.0t^2 + 2.0t + 3.0$

$\mathcal{E}_{bat} = 2.0\ V$

$R = 2.0\ \Omega$

Fig. 30-6

CW

i) $\mathcal{E}_{ind} = ?$ at $t = 10s$

$$\mathcal{E}_{ind} = \frac{d\Phi_B}{dt} = A\frac{dB}{dt} = \pi r^2 (8.0t + 2.0)$$

at $t = 10s$

$\mathcal{E}_{ind} = 5.152V$

ii) current $= ?$ at $t = 10s$

$$i = \frac{\mathcal{E}_{net}}{R} = \frac{\mathcal{E}_{ind} - \mathcal{E}_{bat}}{R} = \frac{5.152V - 2.0V}{2.0\Omega} = 1.58A$$

CCW

Example Changing nonuniform B

Fig. 30-7

Rectangular loop

$B = 4t^2x^2$

$W = 3.0m$

$H = 2.0m$

$\mathcal{E}_{ind} = ?$ at $t = 0.1s$

B changing $\Rightarrow \Phi_B$ changing

Φ_B changing $\Rightarrow$ induced $\mathcal{E}$

$$\Phi_B = \int B\,dA = \int BH\,dx \Big|_0^3 = \int 4t^2x^2H\,dx = 4t^2H\frac{x^3}{3}\Big|_0^3 = 72t^2$$

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d(72t^2)}{dt} = 144t \Big\} \underset{t=0.1s}{\cong} 14V$$

$\otimes B \quad \odot_{B\,ind} \Rightarrow i \to ccw \leftarrow \mathcal{E}$

Induction and Energy Transfes

A rectangular loop
- one end in a uniform external magnetic field
- has a resistance of R
- pull this loop to the right at a constant velocity

SLN Fig. 30-8

- Flux change with time
- current will be induced in CW.

Applied force → energy transfered to loop + magnet system → thermal energy

↓

Mechanical work done during pulling } Magnetic force ← Apply a constant force → } The rate of work done: Power $\boxed{P_F = Fv}$ speed of the loop

Apply Faraday's law: $\Phi_B = BA = BLx$ } $\mathcal{E} = \frac{d\Phi}{dt} = BL\frac{dx}{dt} \boxed{= BLv}$

Representation of the loop as a circuit: SLN Fig. 30-9 $\quad i = \frac{\mathcal{E}}{R} = \frac{BLv}{R}$

Deflection Forces in Fig. 30-8: $\vec{F}_2$ & $\vec{F}_3$ cancels each other. $\vec{F}_1$ is opposing force

$$\vec{F} = i\vec{l} \times \vec{B} \rightarrow F_1 = i l B \sin 90° = \left(\frac{Blv}{R}\right) Bl \boxed{= \frac{B^2l^2v}{R}}$$

$$P = \frac{B^2l^2v^2}{R} = i^2R \ : \text{Power}$$

Thermal energy rate

⇒ Work done on the loop transformed to thermal energy in the loop

SLN Fig. 30-17. Physical meaning of τ_L: when $t = \tau_L \Rightarrow i = 0.63\ \mathcal{E}/R$ } since $e^{-1} = 0.63$

$\Rightarrow \tau_L$ is the time it takes current in the circuit to reach about 63% of its final equilibrium value ($\mathcal{E}/R$)

- When S is on "b" (Fig. 30-15, Fig. 30-16)

$$L\frac{di}{dt} + iR = 0 \rightarrow \boxed{i = \frac{\mathcal{E}}{R} e^{-t/\tau_L} = i_0 e^{-t/\tau_L}}$$ Decay of current

Example RL circuit immediately after switching and after a long time

SLN Fig. 30-18a
$R = 9\ \Omega$
$L = 2.0\ mH$
$\mathcal{E} = 18\ V$
(ideal battery)

i) $i = ?$ Just after the switch is closed

- Initially, an inductor acts like broken wire Fig. 30-18b
- Long later, it acts like ordinary wire Fig. 30-18c

$\mathcal{E} - iR = 0 \rightarrow i = \mathcal{E}/R = \frac{18V}{9.0\Omega} = \underline{2.0A}$

ii) $i = ?$ long after the switch is closed

$i = \frac{\mathcal{E}}{R_{eq}} = \frac{18V}{3.0\Omega} = \underline{6.0A}$

$U_B = \frac{1}{2} L i^2$: Energy Stored in Magnetic Fields (SLN. Fig. 30-16) by an inductor L carrying a current i.

$U_E = \frac{q^2}{2C}$: Energy Stored in Electric Fields by a capacitor with capacitance C and charge q.

Example. Energy Stored in Magnetic Fields

$L = 53\ mH$
$R = 0.35\ \Omega$

i) $\mathcal{E} = 12\ V$ $U_B = ?$ $i_\infty = \frac{\mathcal{E}}{R} = \frac{12V}{0.35\Omega} = 34.3A$ } $U_B = \frac{(53\times10^{-3}H)}{2}(34.3A)^2$ $\boxed{= 31J}$

equilibrium current (steady state)

ii) when $U_B \rightarrow \frac{1}{2} U_{B\infty}$ } $i = \frac{1}{\sqrt{2}} i_\infty$ — final value, initial is zero — $\mathcal{E}/R$ } $\frac{\mathcal{E}}{R}(1 - e^{-t/\tau_L}) = \mathcal{E}/R \frac{1}{\sqrt{2}}$ } $\frac{t}{\tau_L} = -\ln 0.293$

$\frac{1}{2}Li^2 \rightarrow \frac{1}{2}\left(\frac{1}{2}Li_\infty\right)$; $\frac{\mathcal{E}}{R}(1 - e^{-t/\tau_L})$; $e^{-t/\tau_L} = 1 - \frac{1}{\sqrt{2}} = 0.293$; $\Rightarrow t = 1.23\ \tau_L$

In Solenoid; Area A, length l } volume is Al } $u_B = \frac{U_B}{Al} = \left(\frac{L}{l}\right)\frac{i^2}{2A} = (\mu_0 n^2 A)\frac{i^2}{2A} = \boxed{\frac{B^2}{2\mu_0}}$

(long, near the middle) $B = \mu_0 i n$

Magnetic Energy Density

$u_E = \frac{1}{2}\varepsilon_0 E^2$: Electric Energy Density

Mutual Induction

Not self-induction. Mutual Interaction of two coils : Mutual Induction

(only one coil is involved) SLN Fig. 30-2

Fig. 30-19a
- Two circular close-packed coils
- Near each other
- A common central axis.

Mutual Inductance of coil 2 wrt coil 1

$M_{21} = \frac{N_2 \Phi_{21}}{i_1}$ { $M_{21} = M_{12}$

$\boxed{\mathcal{E}_{②} = -M\frac{di_{①}}{dt}}$ $\boxed{\mathcal{E}_{①} = -M\frac{di_{②}}{dt}}$

an $\mathcal{E}$, emf appears in coil ② due to changing current in coil ①.

M: proportionality constant

minus sign - indicate the direction

Induced Electric Fields

A changing magnetic field produces an electric field.

SLN. Fig. 30-11
- There is an extenal magnetic field in cylindrical form
- A copper ring of radius r is placed in that field
- Change magnetic field at a steady rate → Flux will change →
  - Faraday's law
  - induced emf
  - induced current (ccw)

⇒ If there is a current in the ring (induced) → Fig. 30-11a an electric field must be present along the ring (induced)

> * Electric field due to changing magnetic field is induced even if there is no copper ring. Fig. 30-11b

SLN

- A Refurmulation of Faraday's Law

Fig. 30-11b.
- A circular path ($2\pi r$)
- Work done in one revolution. $W = q_0 \mathcal{E}$ ~ test charge ~ induced emf

} Remember $W = U = q_0 V$ But with caution!

$\rightarrow W = \int \vec{F} \cdot d\vec{s} = (q_0 E)(2\pi r) \Rightarrow q_0 E 2\pi r = q_0 \mathcal{E} \Rightarrow \boxed{\mathcal{E} = 2\pi r E}$

$W = \oint \vec{F} \cdot d\vec{s} = q_0 \oint \vec{E} \cdot d\vec{s} = q_0 \mathcal{E} \Rightarrow \boxed{\mathcal{E} = \oint \vec{E} \cdot d\vec{s}}\ \boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}} \Rightarrow \boxed{\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}}$

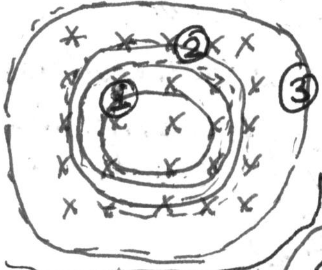
closed path

Faraday's law

Reformulation

$\underbrace{-\frac{d\Phi_B}{dt}}_{\text{changing magnetic field}} = \underbrace{\oint \vec{E} \cdot d\vec{s}}_{\text{produces electric field}}$

} This can be applied any closed loop in changing magnetic field.



$E_1 > E_2$

$E_3 = ?$ outside of the changing magnetic field

① $\vec{E}$ can be produced by static charges. (previous knowledge)

② $\vec{E}$ can be produced by changing magnetic flux.

① These lines never produce closed loops From (+) charge to (−) charge

② These lines form closed forms

Electric potential has meaning only for electric fields that are produced by static charges. It has no meaning for electric fields that are produced by induction.

$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s} = 0$ Since $V_f = V_i$ in closed loop.

Example: Induced electric field due to changing magnetic field inside and outside

Fig. 30-11b $R = 8.5\,\text{cm}$ & $\frac{dB}{dt} = 0.13\ T/s$

i) inside $r < R$ $\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$

$r = 5.2\,\text{cm}$

$\rightarrow E(2\pi r) = (\pi r^2)\frac{dB}{dt}$ (drop minus sign)

$\rightarrow E = \frac{r}{2}\frac{dB}{dt} = \frac{(5.2 \times 10^{-2} m)}{2}(0.13\ T/s)$

$E = 0.0034\ V/m \boxed{= 3.4\ mV/m}$

ii) $r = 12.5\,\text{cm}$ Outside $r > R$

$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$ } $E 2\pi r = \underbrace{(\pi R^2)}_{\text{flux is only in this area}}\frac{dB}{dt}$

$\Rightarrow E = \frac{R^2}{2r}\frac{dB}{dt}$

$= \frac{(8.5 \times 10^{-2} m)^2}{2(12.5 \times 10^{-2} m)}(0.13\ T/s) \boxed{= 3.8\ mV/m}$ see Fig. 30-12

Inductors and Inductance

Capacitor → to produce electric field
Inductor → to produce magnetic field


(symbol)

A long solenoid → basic type of inductor

$L = \frac{N\Phi_B}{i}$ ~ magnetic flux linkage; i ~ current in the windings (N turns) } Inductance

1 henry = $1\,H = 1\,\frac{Tm^2}{A}$

- Inductance of a Solenoid

Consider a long solenoid of cross-sectional area A. $L = ? \rightarrow N\Phi_B = ?$

$N\Phi_B = (\underbrace{n}_{\text{\# of turns per unit length}} l) BA = (nl)(\mu_0 i n)A \rightarrow L = \frac{(nl)(\mu_0 i n)A}{i} = \mu_0 n^2 l A \Rightarrow \boxed{\frac{L}{l} = \mu_0 n^2 A}$ (solenoid)

⇓

Inductance depends only on the geometry of the device! (like capacitance) $C = \varepsilon_0 \frac{A}{d}$

- For long solenoid, $L \gg r$ → uniform magnetic field at the middle.

Self Induction

Two inductors are near each other (① ② coils) } current i ① produces ⇒ magnetic flux Φ_B ② } changing i ① ⇒ induced $\mathcal{E}$ ② } An induced emf appears in the first coil as well!

⇒ An induced emf $\mathcal{E}_L$ appears in any coil in which the current is changing. ⇒ Self-Induction → Self-induced emf } $\mathcal{E}_L = \ominus \frac{d(\overbrace{N\Phi_B}^{Li})}{dt} \boxed{= -L\frac{di}{dt}}$ magnitude

Faraday's law

SLN Fig. 30-13 : current changes with time. } Direction of induced emf (From Lenz's Law) : Opposes the change in current i

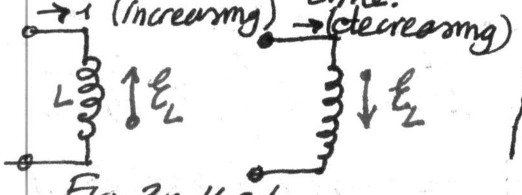


Fig. 30-14a,b

- We can define potential difference V_L btw the terminals of inductor
- We can not define potential within the inductor itself
- For an ideal inductor ; $V_L \simeq \mathcal{E}_L$
- If has some resistance, they deffer

RL Circuits

RC Circuit charge ~
- The charge on the capacitor does not build up immediately to its final equilibrium value (steady state).
- But approaches it an exponential fashion. $q = C\mathcal{E}(1 - e^{-t/\tau_C})$

discharge ~
- Remove $\mathcal{E}$ from circuit. Does not immediately fall to zero.
- But approaches zero in an exponential fashion. $q = q_0 e^{-t/\tau_C}$

} τ_C : capacitive time constant, RC

RL Circuit SLN Fig 30-15
- If there were no inductor, current will rise suddenly to $\mathcal{E}/R$
- When there is an inductor
  - Self-induced emf $\mathcal{E}_L$ appears in the circuit (Lenz's law)
  - This opposeses the rise of current
  - The current will be less than $\mathcal{E}/R$. By time the current increases less rapidly and self-induced emf becomes smaller
  - Thus the current in the circuit approaches $\mathcal{E}/R$ asymptotically.

* An inductor acts to oppose changes in the current through it. A long time later, it acts like ordinary connecting wires

- When S is on "a" (Fig. 30-15) - current → cw

$-iR - \underbrace{L\frac{di}{dt}}_{\mathcal{E}_L} + \mathcal{E} = 0$

$L\frac{di}{dt} + Ri = \mathcal{E}$

} Rise of current $i = \frac{\mathcal{E}}{R}(1 - e^{-Rt/L}) \boxed{= \frac{\mathcal{E}}{R}(1 - e^{-t/\tau_L})}$

} τ_L : inductive time constant L/R

$t = 0 \rightarrow e^0 = 1 \rightarrow i = 0$

$t = \infty \rightarrow e^{-\infty} = 0 \rightarrow i = \frac{\mathcal{E}}{R}$ ste sta

Chapter 31 - Electromagnetic Oscillations and Alternating Current

Energy is stored (either electric or magnetic energy) $\xrightarrow{\text{to be transfered}}$ Then, usage.

↓ Mostly, as a sinusoidally oscillating current, ac

Now, we will study electrically oscillating systems. LC systems

RC $\tau_C=RC$	Charging a Capacitor $q=C\mathcal{E}(1-e^{-t/\tau_C})$ $i=\frac{\mathcal{E}}{R}e^{-t/\tau_C}$	Discharging a capacitor $q=q_0e^{-t/\tau_C}$ $i=-\frac{q_0}{RC}e^{-t/\tau_C}$
RL $\tau_L=\frac{L}{R}$	Rice of current $i=\frac{\mathcal{E}}{R}(1-e^{-t/\tau_L})$	Decay of current $i=i_0e^{-t/\tau_L}$

} Charge, current potential difference ⇒ grow & decay exponentially

τ_C & τ_L are capacitive and inductive time constants

LC Systems: Circuits containing inductance and capacitance } vary sinusoidally (with period T and angular frequency ω)

↓

Oscillations ← Capacitor's electric field, $u_E=\frac{q^2}{2C}$

Oscillations ← Inductor's magnetic field. $u_B=\frac{1}{2}Li^2$

⇓

Electromagnetic Oscillations SLN Fig. 31-1: Eight stages in a single cycle of oscillation of LC circuits

As circuit oscillates, energy shift back and forth from one type of stored energy to the other but total amount of energy is conserved.

V_C: time-varying potential difference across the capacitor, C

V_R: time-varying potential difference across the resistor, R

} Oscillating: $V_C=\left(\frac{1}{C}\right)\textcircled{q}$, $V_R=R\textcircled{i}$ SLN Fig. 31-1 } Oscillating means that the quantity is varying with time such as sinusoidally.

- The Electrical Mechanical Anology

Block-spring system $\frac{d^2x}{dt^2}+\frac{k}{m}x=0 \rightarrow x(t)=x_0\cos(\overset{\sim\sqrt{k/m}}{\omega}t+\phi)$: Oscillation Sinusoidal!

F=ma F=-kx

$m\frac{d^2x}{dt^2}=-kx$

$-L\frac{d^2q}{dt^2}-\frac{q}{C}+\underset{0}{\overset{\sim\text{battery.}}{\cancel{\mathcal{E}}}}=0 \rightarrow q(t)=Q_0\cos(\underset{\sim 1/\sqrt{LC}}{\omega}t+\phi$: LC Oscillator SLN Table 31-

SLN The LC Oscillator, Quantitatively

$L\frac{d^2q}{dt^2}+\frac{1}{C}q=0$

charge, $q=Q\cos(\omega t+\phi)$

current, $i=-\omega Q\sin(\omega t+\phi)$

$i=-I\sin(\omega t+\phi$

} Q, I: Amplitudes

ω: angular frequency of EM oscillations

ϕ: phase constant

angular frequency $-L\omega^2Q\cos(\omega t+\phi)+\frac{1}{C}Q\cos(\omega t+\phi)=0 \Rightarrow \boxed{\omega=\frac{1}{\sqrt{LC}}}$

- Electrical & Magnetic Energy Oscillations

$u_E=\frac{q^2}{2C}=\frac{Q^2}{2C}\cos^2(\omega t+\phi)$

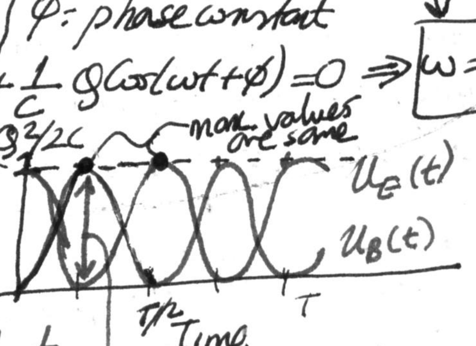
$u_B=\frac{1}{2}Li^2=\frac{1}{2}L\textcircled{\omega}^2Q^2\sin^2(\omega t+\phi)$

$=\frac{Q^2}{2C}\sin^2(\omega t+\phi)$

} $\textcircled{\omega}=\frac{1}{\sqrt{LC}}$

Energy

$\underbrace{u_E+u_B=\frac{Q^2}{2C}}$



- total energy, constant
- when $u_E\rightarrow$ max, then $u_B\rightarrow 0$, and conversly

Example LC Oscillator, potential charge, rate of current charge

C=15 μF
V=57 V
L=12 mH
LC circuit

i) $V_L(t)=?$ At time t=0 → $V_L(t)=V_C(t)$ Maximum (also; charge & $\phi=0$) ↓ $q=Q\cos\omega t$, t=0 → $q=Q$ (maximum charge)

$\Rightarrow \frac{q}{C}=\frac{Q}{C}\cos\omega t$ } $V_C=V_C\cos\omega t=V_L$ (57V ??) } $\omega=\frac{1}{\sqrt{LC}}\cong 7500$ rad/s

$\Rightarrow V_L=(57V)\cos(7500 \text{ rad/s})t$ $\frac{Q}{C}=V_C$

ii) $\left(\frac{di}{dt}\right)_{max}=?$ $i=\frac{dq}{dt}=-Q\omega\sin\omega t$ | $\frac{di}{dt}=-Q\omega^2\cos\omega t=-CV_C\omega^2\cos\omega t=-\frac{V_C}{L}\underbrace{\cos\omega t}_{1 \text{ for maximum}}$ ($\omega^2 = \frac{1}{LC}$)

$\Rightarrow\left(\frac{di}{dt}\right)_{max}=-\frac{V_C}{L}=\frac{57V}{0.012H}\sim 4800$ A/s

Alternating Current

Energy should be supplied to the circuit against "damping" (heat dissipation etc). It is mostly by AC

Alternatic Current, AC: Oscillating currents (and emfs) < vary sinusoidally with time / reversing its direction

Direct Current, DC: Nonoscillating current (i.e from a battery)

* As the current alternates, so does the magnetic field that surrounds the conductor.
↳ Possible Use of Faraday's Law: Generators, motors, etc. SLN Fig. 31-6

In the loop of Fig. 31-6:

$\mathcal{E}=\mathcal{E}_m\sin\omega_d t$ < induced
- ω_d: Angular frequency of emf (ω_d) ≡ angular speed of rotation (ω)
  - ↳ $\omega_d t$: phase of emf
- $\mathcal{E}_m$: amplitude of the emf

⇓

Produces sinusoidally (alternating) current with ω_d → Driving angular frequency ($2\pi f_d$) $\Rightarrow i=I\sin(\omega_d t-\phi)$ → needed since current and emf may not be in phase → depends on the generator

Three Simple Circuits

Before RLC circuit, simpler "pure" circuits

① Resistive Load: Fig. 31-8 R & AC generator. Loop Rule $\mathcal{E}-v_R=0$ ($\to i_R R$)

$\to v_R=\mathcal{E}_m\sin\omega_d t\boxed{=V_R\sin\omega_d t}$ (equal) } $i_R=\frac{V_R\sin\omega_d t}{R}\equiv I_R\sin(\omega_d t-\phi)$ } v_R, i_R } Time varying

means $\phi=0$ & $\frac{V_R}{R}=I_R$

Fig. 31-9a → Do not decay due to "damping" ↳ Oscillating

$\Rightarrow i_R$ & v_R are in phase (SLN Fig. 31-9a) $\boxed{i_R=I_R\sin\omega_d t}$

Fig. 31-9b "Phasor" representation of v_R & i_R

Phasor's

Angular Speed: ω_d, Length: Amplitude (V_R & I_R), Projection: At vertical axis ($\sin(\omega_d t)$), Rotation Angle: $\omega_d t$

Voltage & Current are in phase for Resistive Load. SLN Sample Problem

② Capacitive Load: Fig. 31-10 C & AC generator. Loop Rule $\mathcal{E}-v_C=0$ (q_C/C)

$\to\boxed{v_C=V_C\sin\omega_d t}$ } $Cv_C=CV_C\sin\omega_d t=q_C$ } $i_C=\frac{dq_C}{dt}=\omega_d CV_C\cos\omega_d t$ | Define Capacitive Reactance $X_C=\frac{1}{\omega_d C}$ ohm

$\Rightarrow i_C=\frac{V_C}{X_C}\cos\omega_d t=\frac{V_C}{X_C}\sin(\omega_d t+90°)\boxed{=I_C\sin(\omega_d t-\phi)}$ $\phi=-90°$ pure capacitive load; phase shifted sine } v_C, i_C } · Time varying · out of phase

$\boxed{V_C=I_C X_C}$ SLN Fig-31-11a & b Current "leads" the potential difference by 90°. ← SLN Sample Problem

③ Inductive Load: Fig. 31-12 L & AC generator. Loop Rule $\mathcal{E} - \underbrace{v_L}_{} = 0$ $\sim L\frac{di_L}{dt}$

$\rightarrow \boxed{v_L = V_L \sin \omega_d t}$ $\Big\}$ $L\frac{di_L}{dt} = V_L \sin \omega_d t$ $\Big\}$ $\frac{di_L}{dt} = \frac{V_L}{L} \sin \omega_d t$ $\Big\}$ $i_L = \int di_L = \frac{V_L}{L} \int \sin \omega_d t\, dt$

$\rightarrow i_L = -\left(\frac{V_L}{\omega_d L}\right) \cos \omega_d t$ $\Big\}$ Define Inductive Reactance $X_L = \omega_d L$ $\Big\}$ $i_L = \frac{V_L}{X_L} \underbrace{\sin(\omega_d t - 90°)}_{-\cos \omega_d t} \Rightarrow \boxed{i_L = \frac{V_L}{X_L} \sin(\omega_d t - \phi)}$

$\boxed{I_L = V_L / X_L}$

$\phi = 90°$ pure inductive load

SLN Fig. 31-13 a & b Current "lags" the potential difference by 90°

SLN Sample Problem Table 31-2

The Series RLC Circuit

SLN Fig. 31-7

Now, full RLC circuit
R, L and C are in series ← Applied Alternating Emf $\mathcal{E} = \mathcal{E}_m \sin \omega_d t$ → in series: ~~same current~~ $i = I \sin(\omega_d t - \phi)$

$I = ?$ $\phi = ?$
① ②
⇓
by phasor diagrams!

① Current Amplitude, I

Phasor diagrams of current and voltage. SLN Fig 31-14

Fig. 31-14a

$\omega_d t - \phi$ {
- Angle of rotation of phasor
- Phase of the current at time t

length of phasor: the current amplitude I

Fig. 31-14c

length of phasor: the emf amplitude $\mathcal{E}_m$

Loop Rule ↦ $\mathcal{E} = \underbrace{v_R + v_C + v_L}_{}$ instantaneous voltages

⇓

$\mathcal{E}_m$ is the vector sum: $\vec{\mathcal{E}}_m = \vec{V}_R + \vec{V}_L + \vec{V}_C$

- ↳ First combine V_L & $V_C \Rightarrow \vec{V}_L - \vec{V}_C$
- ↳ Then combine with $V_R \Rightarrow \vec{V}_R + (\vec{V}_L - \vec{V}_C)$

Fig 31-14b

Resistor: current and voltage in phase
Capacitor: voltage is behind current by 90
Inductor: voltage is ahead of current by 90

Lengths of phasors: Instantaneous voltages v_R, v_L, v_C at time t.

Fig. 31-14d

$$\mathcal{E}_m^2 = V_R^2 + (V_L - V_C)^2$$

: Now, see Table 31-2

$$= (IR)^2 + (IX_L - IX_C)^2$$

$$\Rightarrow I = \frac{\mathcal{E}_m}{\underbrace{\sqrt{R^2 + (X_L - X_C)^2}}_{}} = \boxed{\frac{\mathcal{E}_m}{Z} = I}$$

↳ impedance

Also we know that $X_L = \omega_d L$, $X_C = \frac{1}{\omega_d C}$

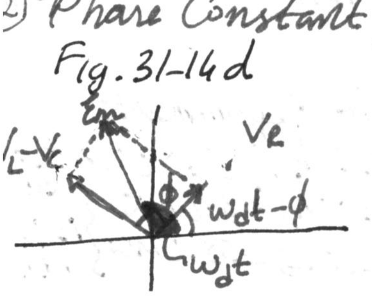
$$\Rightarrow \boxed{I = \frac{\mathcal{E}_m}{\sqrt{R^2 + (\omega_d L - \frac{1}{\omega_d C})^2}}}$$

← Current Amplitude

The steady state current that occurs after alternating emf has been applied for some time.

② Phase Constant, ϕ

Fig. 31-14d



$$\tan \phi = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR}$$

$$\boxed{\tan \phi = \frac{X_L - X_C}{R}}$$ phase constant

$X_L > X_C$: Positive ϕ. Phasor I rotates behind phasor $\mathcal{E}_m$
Circuit is more inductive than capacitive
Fig. 31-15 a & b

$X_L < X_C$: Negative ϕ. Phasor I rotates ahead of phasor $\mathcal{E}_m$
Circuit is more capacitive than inductive
Fig. 31-15 c & d

$X_L = X_C$: Zero ϕ. Phasors I and $\mathcal{E}_m$ rotate together
The circuit is said to be in resonance.
Fig. 31-15 e & f

Power in Alternating-Current Circuits

RLC Circuit ← AC generator provides energy to RLC } stored in the electric field in the capacitor, C
\+ stored in the magnetic field of the inductor, L
\+ dissipated as thermal energy, R } In steady-state average stored energy remains const

The "instantaneous" dissipated energy in the resistor: $P = i^2R = [I\sin(\omega_d t - \phi)]^2 R$
$= I^2 R \sin^2(\omega_d t - \phi)$

What about "average" dissipated energy in the resistor? SCN Fig. 31-17 a & b

Average over time. One complete cycle $\begin{cases} \sin\theta \rightarrow \textcircled{0} \text{ Fig. 31-17a} \\ \sin^2\theta \rightarrow \textcircled{1/2} \text{ Fig. 31-17b} \end{cases}$ ⇒ Average Values

$\Rightarrow P_{avg} = I^2 R \left(\frac{1}{2}\right) = \left(\frac{I}{\sqrt{2}}\right)^2 R$

$\frac{I}{\sqrt{2}}$ is called root-mean-square, or rms value of current i. $\boxed{I_{rms} = \frac{I}{\sqrt{2}}}$

~ $1/\sqrt{2}$ average

$\Rightarrow \boxed{P_{avg} = I_{rms}^2 R}$ If we switch to the rms current, we can compute the average rate of energy dissipation for AC circuits just as DC circuits.

Average Power

$I_{rms} = \frac{I}{\sqrt{2}}$, Similarly $V_{rms} = \frac{V}{\sqrt{2}}$, $\mathcal{E}_{rms} = \frac{\mathcal{E}_m}{\sqrt{2}}$
rms voltage rms emf

$I_{rms} = \frac{\mathcal{E}_{rms}}{Z}$ & $P_{avg} = I_{rms}^2 R$

$\Rightarrow P_{avg} = \left(\frac{\mathcal{E}_{ms}}{Z}\right) I_{rms} R = \mathcal{E}_{rms} I_{rms} \textcircled{\frac{R}{Z}}$

$\cos\phi = \frac{V_R}{\mathcal{E}_m} = \frac{IR}{IZ} = \frac{R}{Z}$

$\Rightarrow \boxed{P_{avg} = \mathcal{E}_{rms} I_{rms} \cos\phi}$ Average Power

$\cos\phi$: Power Factor

if $\cos\phi \rightarrow 1 \Rightarrow$ maximum P_{avg}

220 V ~> rms voltage ⇒ maximum voltage

$\rightarrow V = V_{rms}\sqrt{2} \cong 311 V$ $V_{rms} = \frac{V}{\sqrt{2}}$

Example Driven RLC circuit: power factor and average power

RLC Circuit
$\mathcal{E}_{rms} = 120V$
$f_d = 60.0 Hz$
$R = 200 \Omega$
$X_L = 80.0 \Omega$
$X_C = 150 \Omega$

i) Power factor. $\cos\phi = ?$ $\phi = ?$

$\cos\phi = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{200\Omega}{211.90\Omega} \cong 0.944$

$\rightarrow \phi = \pm 19.3°$ which sign is correct?

Since $X_C > X_L \rightarrow$ must be negative!

ii) Average rate $P_{avg} = ?$ in resistance

$P_{avg} = \mathcal{E}_{rms} I_{rms} \cos\phi = \mathcal{E}_{rms}\left(\frac{\mathcal{E}_{rms}}{Z}\right)\cos\phi \cong 61.4 W$ (OR from $P_{avg} = I_{rms}^2 R$)

iii) P_{avg} is maximum $\rightarrow C_{new} = ?$ } $\cos\phi = 1 \xrightarrow{\phi = 0} X_C = X_L = \frac{1}{\textcircled{\omega_d} C_{new}} \Rightarrow 80\Omega = \frac{1}{2\pi 60 Hz\, C_{new}}$

$2\pi f_d$ ~> ω_d

$\rightarrow \underline{C_{new} = 33.2 \mu F} \Rightarrow P_{avg,max} = 72.0 W$

Transformers

At home: low voltages
At transmission: low current } desired.

Energy Transmission Requirements

Example Case: $\mathcal{E} = 735$ kV, $I = 500$ A, $R = 220\Omega$

Supply Rate ⇒ $P_{avg} = \mathcal{E}I = (735 kV)(500A) = 368 MW$
Dissipated ⇒ $P_{avg} = I^2R = (500A)(220\Omega) = 55.0 MW$ } 15% ?!!

⇒ We need transformers
- high-voltage transmission
- safe-voltage at home

SCN Fig. 31-18 Ideal Transformer

$\mathcal{E} = \mathcal{E}_m \sin\omega t$. AC → Induction at both coils
generator

} I_{mag}: magnetization current } $\boxed{V_S = V_P \frac{N_S}{N_P}}$

where
$V_P = \mathcal{E}_{turn} N_P$
$V_S = \mathcal{E}_{turn} N_S$
$\mathcal{E}_{turn} = \frac{d\Phi_B}{dt}$ — primary / secondary same

ELECTROMAGNETIC WAVES
- CHAPTER 33 -

Information age ⟸ physics of electromagnetic waves ⟹ signals

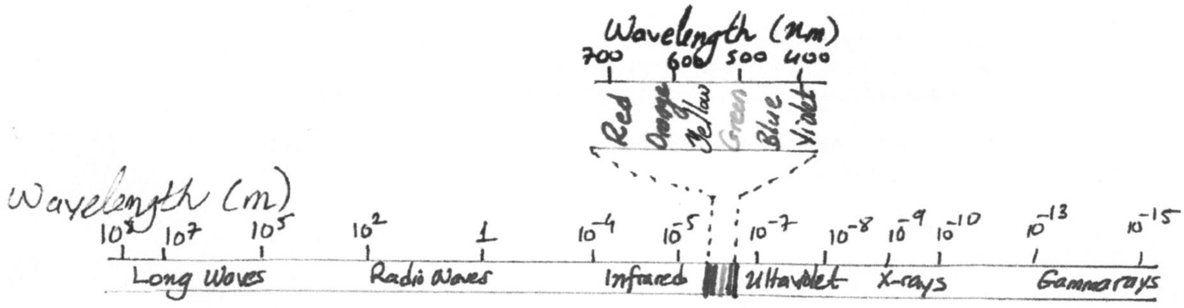
Maxwell's Rainbow

A beam of light is a travelling wave of electric and magnetic fields

an electromagnetic wave (EMW)

Optics, the study of visible light, is a branch of electromagnetism. In Maxwell's time, the visible, infrared and ultraviolet forms of light were the only electromagnetic waves known.

Hertz discovered radio waves → same speed as visible light.



Frequency (Hz)   Maxwell's Rainbow, Spectrum of EMWs

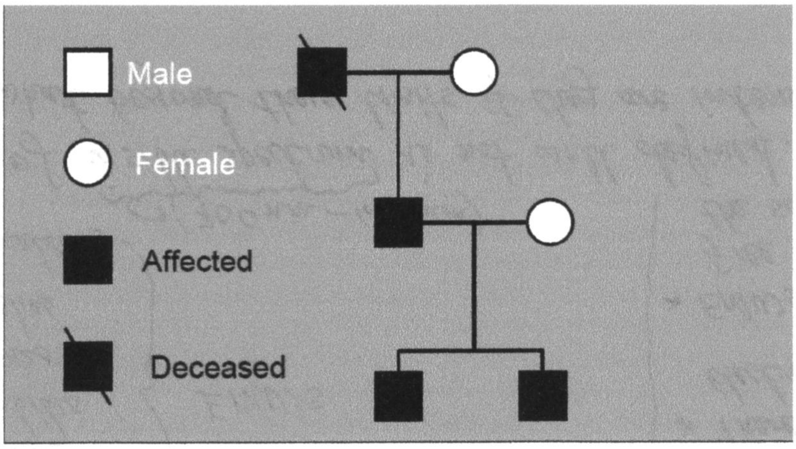
The Sun: dominant source
Radio and Television signals
Microwaves: Radar and Telephone relay systems
Lightbulbs
Heated engine blocks
  of automobiles
x-ray machines
Lighting flashes
Radioactive materials
} EMWs

- scale with a factor of 10
- scale is open-ended. The wavelengths of EMWs have no inherent upper or lower bonds.
- There are no gaps in the electromagnetic spectrum
- EMWs travel through free space (vacuum) with the same speed c.

(700nm – 400nm)

The limits of visible spectrum is not well defined. The eye can detect EMWs somewhat beyond these limits if they are intense enough.

Pedigree Notation



Traveling EMW

How EMWs are generated.

Restriction: $\lambda \cong 1$ mm $\Rightarrow$

$\Rightarrow$ source of the radiation is both macroscopic and manageable dimensions } NOT, (the sources) are of atomic or nucleor size. But, an antenna. An LC oscillator with $\omega = \frac{1}{\sqrt{LC}}$

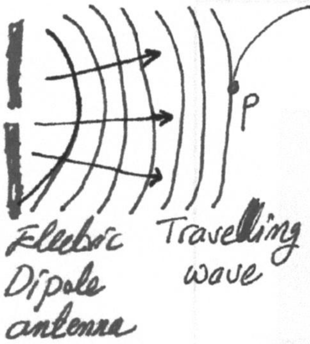
The antenna has the effect of an electric dipole.

- * Electric dipole moment varies sinusoidally in magnitude and direction along the antenna.
  - $\Rightarrow$ produced, $\vec{E}$, varies in magnitude and direction
- * Also, the current varies (in magnitude and direction)
  - $\Rightarrow$ produced, $\vec{B}$, varies

$\to$ The changing fields form an EMW that travels away from the antenna at speed c.

The angular freguency of this wave is ω, the same as that of the LC oscillator.

A distant point P: Plane wave (curvature of the waves is small enough to neglect)



Key Features:

i) $\vec{E}$ & $\vec{B}$ are always pependicular to the direction in which the wave is travelling. Transvese wave

ii) $\vec{E} \perp \vec{B}$

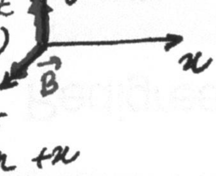
iii) $\vec{E} \times \vec{B}$ gives the direction in which the wave travels

iv) The fields always vary sinusoidally

Assume that, P in +x,

$\vec{E} \parallel y$ (i) (ii)

$\vec{B} \parallel z$



$\vec{E} \times \vec{B}$ (iii) in +x

(iv) $E = E_m \sin(kx - \omega t)$

$B = B_m \sin(kx - \omega t)$

E_m & B_m: amplitudes

ω: angular freguency

k: angular wave number

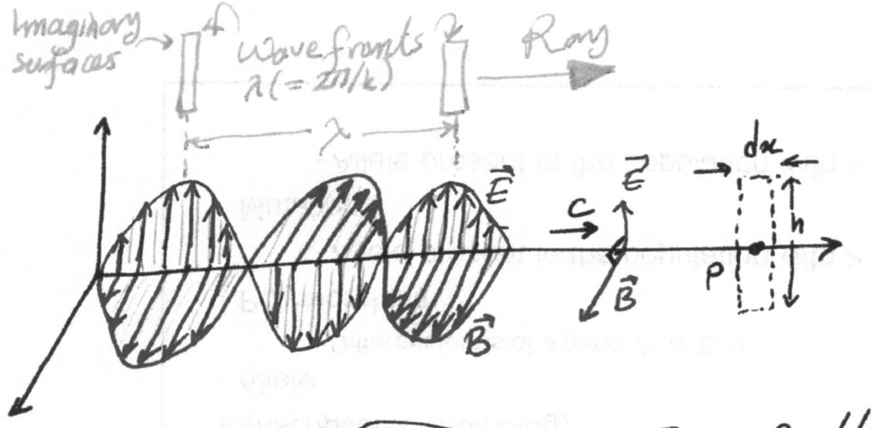
$\downarrow$ Magnetic wave component

Electric wave component $\Rightarrow$ Each also forms its own wave

Two fields form the EMW $\Rightarrow$ Two component can not exist independently

The speed of the wave is $\frac{\omega}{k}$. In EMW, $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3\times10^8$ m/s

All EMWs, including visible light, have the same speed c in vacuum



$\frac{E_m}{B_m} = c$ Amplitude ratio

$\frac{E}{B} = c$ Magnitude ratio

First consider $\vec{B}$;
- varies sinusoidally
- ⇒ induces a perpendicular $\vec{E}$ (Faraday's law of induction)

Induced $\vec{E}$
- perpendicular to $\vec{B}$
- also varies sinusoidally
- ⇒ induces a perpendicular $\vec{B}$

Two fields continuously create each other via induction.

Resulting sinusoidal variations in the fields travel as a wave, EMW.

Energy Transport and the Poynting Vector

An EMW can transport energy.

$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$: The rate of energy transport per unit area in wave.

Poynting vector. Wave's direction and direction of energy transport

$\left(\frac{\text{energy/time}}{\text{area}}\right)_{\text{inst}} = \left(\frac{\text{Power}}{\text{area}}\right)_{\text{inst}}$

$S = \frac{1}{\mu_0} EB$ deal with E: $B = \frac{E}{c}$ $S = \frac{1}{c\mu_0} E^2$

$I = S_{avg} = \left(\frac{\text{power}}{\text{area}}\right)_{avg} = \frac{1}{c\mu_0} [E^2]_{avg} = \frac{1}{c\mu_0} [E_m^2 \sin^2(kx - \omega t)]_{avg} = \frac{1}{c\mu_0} E_{rms}^2$

Over a full cycle $[\sin^2\theta]_{avg} \to \frac{1}{2}$ $\Big\}$ $E_{rms} = \frac{E_m}{\sqrt{2}}$

$u_E \overset{?}{=} u_B$: Energy densities $\left(\equiv \frac{1}{2}\varepsilon_0 E^2\right)$

$u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 (cB)^2$

$= \frac{1}{2}\varepsilon_0 \frac{1}{\mu_0 \varepsilon_0} B^2 = \frac{B^2}{2\mu_0} = u_B$ $\Big\}$ $\boxed{u_E = u_B}$

$\Rightarrow \boxed{I = \frac{1}{c\mu_0} E_{rms}^2}$