

**Ceng 272 Statistical Computations**  
**Final**  
**June 02, 2009 13:00 – 15:00**  
**Good Luck!**

Answer all the questions.

Write the solutions explicitly and use the statistical terminology.

1. (10 pts) Consider the density function

$$f(x) = \begin{cases} kx^2, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

- i Evaluate  $k$ .
- ii Find  $F(x)$  and use it to evaluate  $P(0.3 < X < 0.6)$

2. (10 pts) Suppose that  $X$  and  $Y$  have the following joint probability function:

$f(x, y)$		x	
		2	4
y	1	0.10	0.15
	3	0.20	0.30
	5	0.10	0.15

- i Find the expected value of  $g(X, Y) = XY^2$ .
- ii Find  $\mu_x$  and  $\mu_y$ .

3. (20 pts) One hundred new jobs are opening up at an automobile manufacturing plant, but 1500 applicants show up for the 100 positions. To select the best 100 from among the applicants, the company gives a test that covers mechanical skill, manual dexterity, and mathematical ability. The mean grade on this test turns out to be 70, and the scores have a standard deviation 6. Can a person who has an 94 score count on getting one of the jobs? [Hints: Use Chebyshev's theorem. Assume that the distribution is symmetric about the mean.]

4. (20 pts) A foreign student club lists as its members 2 Canadians, 3 Japanese, 5 Italians, and 2 Germans. If a committee of 4 is selected at random, find the probability that
- i all nationalities are represented;
  - ii all nationalities except the Italians are represented.
- .

5. (15 pts) The average life of a certain type of small motor is 15 years with a standard deviation of 3 years. The manufacturer replaces free all motors that fail while under guarantee. If he is willing to replace only 10% of the motors that fail, how long a guarantee should he offer? Assume that the lifetime of a motor follows a normal distribution.

6. (25 pts) The heights of 1000 students are approximately normally distributed with a mean of 174.5 centimeters and a standard deviation of 6.9 centimeters. If 200 random samples of size 25 are drawn from this population and the means recorded to the nearest tenth of a centimeter, determine
- i the mean and standard deviation of the sampling distribution of  $\bar{X}$ ;
  - ii the number of sample means that fall between 172.5 and 175.8 centimeters inclusive;
  - iii the number of sample means falling below 172.0 centimeters.