Lecture 11

Fundamental Sampling Distributions and Data Distributions I

Lecture Information

Ceng272 *Statistical Computations* at May 10, 2010

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Contents

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- For instance, the distribution of a sample mean $\bar{X}$, which is a random variable because the different samples may result in different values of sample mean $\bar{x}$.
- The use of high speed computer enhances the use of formal statistical inference with graphical techniques.

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  - **Infinite size**: measuring the atmospheric pressure; some finite populations are so large.

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- Sometimes, it is impossible or impractical to observe the entire set of observations that make up the population.

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- If we assume the population of battery lives to be normal, the possible values of any $X_i,\ i = 1, 2, \ldots, 8$, will be precisely the same as those in the original population, and hence X_i has the same identical normal distribution as X.

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- Again this is very related to the standard deviation of a random variable but is not the same thing.

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- **Theorem 8.1**:

If S^2 is the variance of a random sample of size n, we may write

$$S^2 = \frac{1}{n(n-1)}\left[n\sum_{i=1}^{n} X_i^2 - \left(\sum_{i=1}^{n} X_i\right)^2\right]$$

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$$\sum_{i=1}^{6} x_i^2 = 171 \qquad \sum_{i=1}^{6} x_i = 31 \qquad \sigma^2 = \frac{6 * 171 - 31^2}{6 * 5} = \frac{13}{6}$$

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- **Box plot** provides the viewer information about **outliers** which represent rare event.

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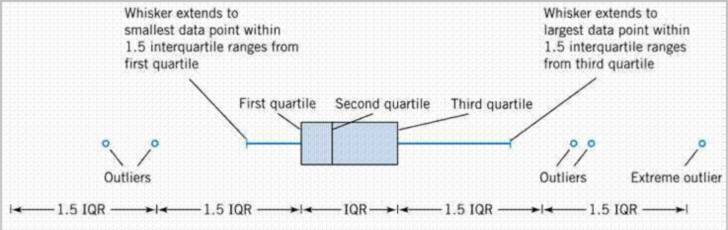

Figure: Box-and-Whisker plot.

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1.79	1.64	2.31	1.58	1.68	2.46	2.09	1.79
2.03	1.51	1.88	1.75	2.28	1.70	1.64	2.08
1.63	1.74	2.17	**0.72**	1.67	2.37	1.47	**2.55**
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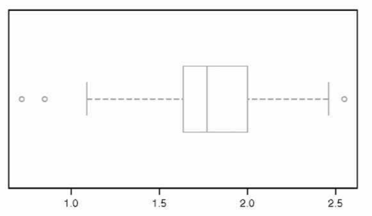

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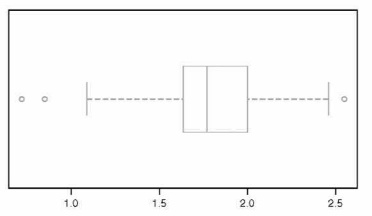

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1.63	1.74	2.17	**0.72**	1.67	2.37	1.47	**2.55**
1.69	1.37	*1.75*	*1.97*	2.11	1.85	1.93	1.69

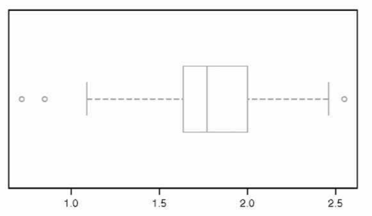

Figure: Box-and-Whisker plot for nicotine data.

- Sample size $n = 40$.

Data Display and Graphical Methods III

- Nicotine content was measured in a random sample of 40 cigarettes. The data is displayed in the table.
- Mild outliers: 0.72, 0.85, and 2.55

Table: Nicotine Data for Example 8.3.

1.09	**0.85**	1.86	1.82	1.40	1.92	1.24	1.90
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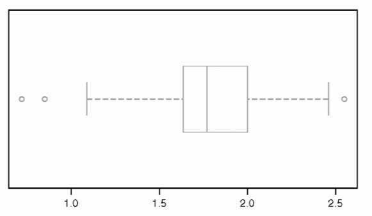

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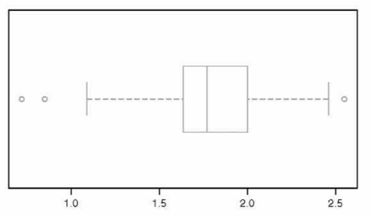

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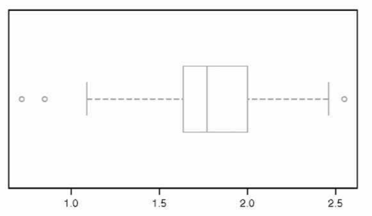

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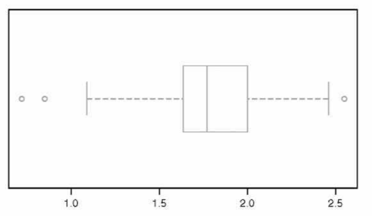

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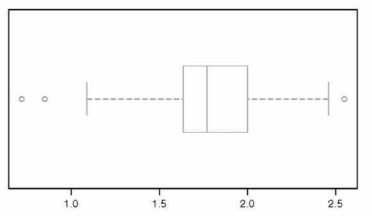

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  2. Make a table: List the stem values as rows. Add each leaf value with a specific stem value to that row.
- Gives an idea about what stem values occur more frequently.

Data Display and Graphical Methods V

```
Stem Leaf                     #          Boxplot
  25 5                        1             0
  24 6                        1             |
  23 17                       2             |
  22 8                        1             |
  21 17                       2             |
  20 389                      3          +-----+
  19 0237                     4          |     |
  18 2568                     4          |     |
  17 045599                   6          *--+--*
  16 3447899                  7          +-----+
  15 18                       2             |
  14 07                       2             |
  13 7                        1             |
  12 4                        1             |
  11                                        |
  10 9                        1             |
   9
   8 5                        1             0
   7 2                        1             0
     ----+----+----+----+
Multiply Stem.Leaf by 10**-1
```

Figure: Stem-and-leaf plot for the nicotine data.

Data Display and Graphical Methods VI

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Sample	Measurements					Sample	Measurements				
1	29	36	39	34	34	16	35	30	35	29	37
2	29	29	28	32	31	17	40	31	38	35	31
3	34	34	39	38	37	18	35	36	30	33	32
4	35	37	33	38	41	19	35	34	35	30	36
5	30	29	31	38	29	20	35	35	31	38	36
6	34	31	37	39	36	21	32	36	36	32	36
7	30	35	33	40	36	22	36	37	32	34	34
8	28	28	31	34	30	23	29	34	33	37	35
9	32	36	38	38	35	24	36	36	35	37	37
10	35	30	37	35	31	25	36	30	35	33	31
11	35	30	35	38	35	26	35	30	29	38	35
12	38	34	35	35	31	27	35	36	30	34	36
13	34	35	33	30	34	28	35	30	36	29	35
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5	30	29	31	38	29	20	35	35	31	38	36
6	34	31	37	39	36	21	32	36	36	32	36
7	30	35	33	40	36	22	36	37	32	34	34
8	28	28	31	34	30	23	29	34	33	37	35
9	32	36	38	38	35	24	36	36	35	37	37
10	35	30	37	35	31	25	36	30	35	33	31
11	35	30	35	38	35	26	35	30	29	38	35
12	38	34	35	35	31	27	35	36	30	34	36
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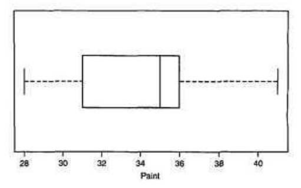

Figure: Box-and-whisker plot for thickness of paint can "ears".

Data Display and Graphical Methods VII

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- Sample median: q(0.5); 75th percentile: q(0.75); 25th percentile: q(0.25).
- A quantile plot simply plots the data values on the vertical axis against an empirical assessment of the fraction of observations exceeded by the data value.

Data Display and Graphical Methods VIII

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- where i is the order of the observations when they are ranked from low to high.
- In other words, if we denote the ranked observations as

$$y_{(1)} \leq y_{(2)} \leq \cdots \leq y_{(n-1)} \leq y_{(n)}$$

then the quantile plot depicts a plot of $y_{(i)}$ against f_i.

Data Display and Graphical Methods IX

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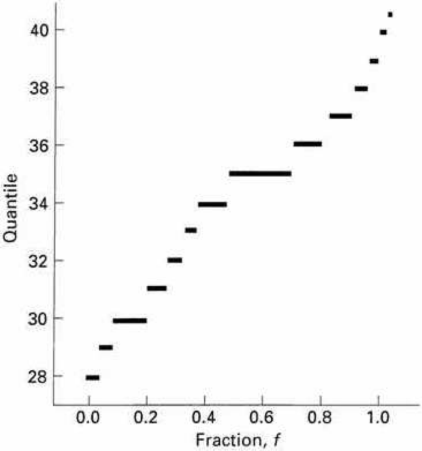
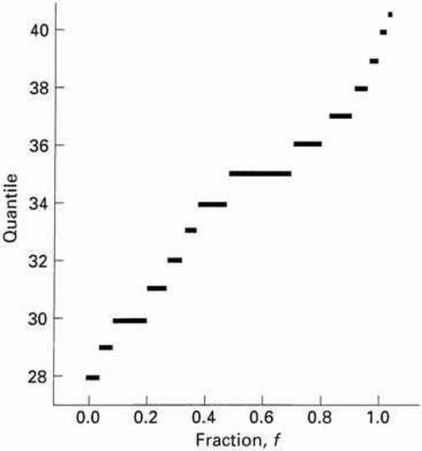


Figure: Quantile plot for paint data.

Data Display and Graphical Methods IX



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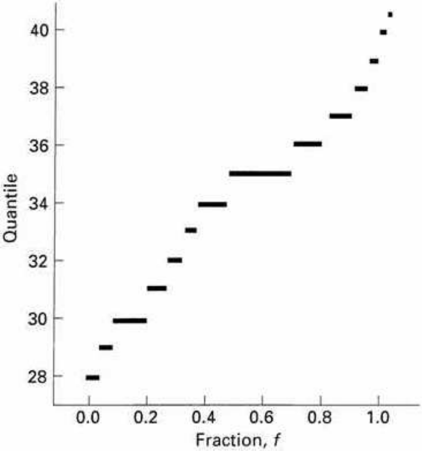

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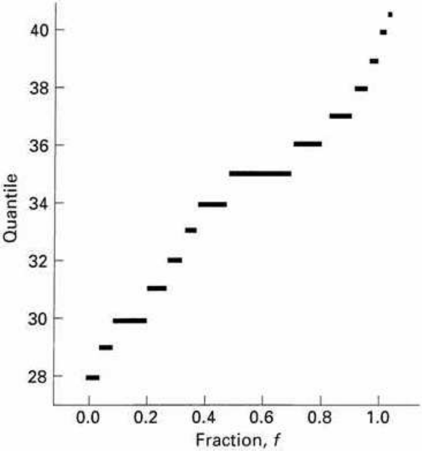
Data Display and Graphical Methods IX



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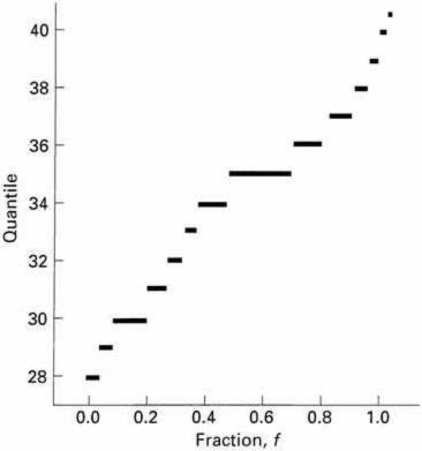

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- Sparse data: steeper slopes. e.g.: 28-30

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- Question: Did this sample come from a population with a normal distribution?
- Tool: We can take advantage of what is known about the quantiles of the normal distribution to answer this question.
- The diagnostic plot can often nicely augment a formal **goodness-of-fit test** on the data.

Data Display and Graphical Methods XI

- Approximation of quantile of normal distribution

$$q_{\mu,\sigma}(f) = \mu + \sigma \left\{ 4.91 \left[f^{0.14} - (1-f)^{0.14} \right] \right\}$$

$\mu = 0$ and $\sigma = 1$ for standard normal distribution

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The **normal quantile-quantile plot** is a plot of $y_{(i)}$ ordered observations against $q_{0,1}(f_i)$, where

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- A nearly straight line relationship suggests that the data came from a normal distribution.

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The **normal quantile-quantile plot** is a plot of $y_{(i)}$ ordered observations against $q_{0,1}(f_i)$, where

$$f_i = \frac{i - \frac{3}{8}}{n + \frac{1}{4}}$$

- A nearly straight line relationship suggests that the data came from a normal distribution.
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Data Display and Graphical Methods XI

- Approximation of quantile of normal distribution

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Data Display and Graphical Methods XII

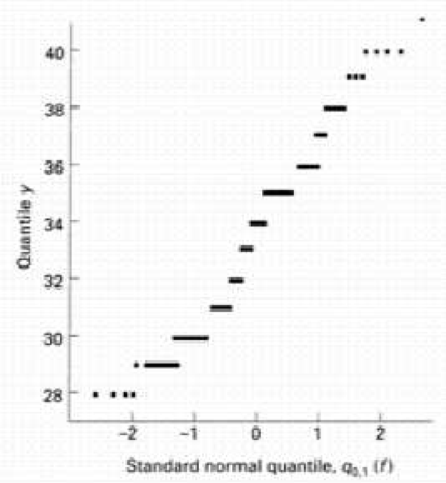

Figure: Normal quantile-quantile plot for paint data.

Data Display and Graphical Methods XIII

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- If the normal distribution adequately describes the data, the plotted points will fall approximately along a straight line.
- Construct a normal quantile-quantile plot and draw conclusions regarding whether or not it is reasonable to assume that the two samples are from the same $N(\mu, \sigma)$ distribution.

Data Display and Graphical Methods XIV

Table: Data for Example 8.5.

Number of Organisms per Square Meter			
Station 1		Station 2	
5,030	4,980	2,800	2,810
13,700	11,910	4,670	1,330
10,730	8,130	6,890	3,320
11,400	26,850	7,720	1,230
860	17,660	7,030	2,130
2,200	22,800	7,330	2,190
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- Unlikely!

Data Display and Graphical Methods XV

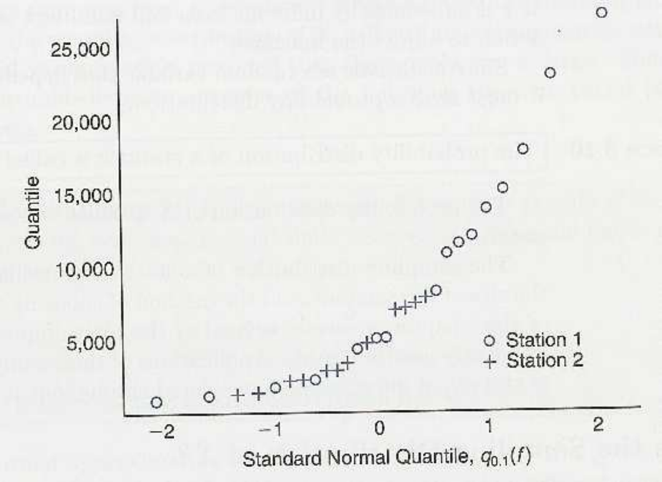

Figure: Standard Normal Quantile, $q_{0,1}(f)$.

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- We call $\left(\frac{N-n}{N-1}\right)$ the finite population correction and it approaches 1 as $N \to \infty$.

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- There will be $\binom{5}{4} = 5$ ways of making combinations.

Sampling Distribution of Means III

- The following table shows the list all the possible samples (without replacement) that can be selected from this population.

Sample No	Sample	Sample Mean $\bar{x}$
1	(A,B,C,D) = 7,8,12,7	8.5
2	(A,B,C,E) = 7,8,12,20	11.75
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5	(B,C,D,E) = 8,12,7,20	11.75

- Calculate the sample mean for each of these samples. Then, the sampling distribution of $\bar{X}$ is

$\bar{X}$	8.5	10.5	11.5	11.75	$\sum p(\bar{x})$
$p(\bar{x})$	1/5	1/5	1/5	2/5	1.0

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- If a random sample of *n* observations is selected from a population (any population), then when *n* is sufficiently large, the sampling distribution of will be approximately a normal distribution.

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- **Theorem 8.2**:

Central Limit Theorem. If $\bar{X}$ is the mean of a random sample of size n taken from a population with mean μ and finite variance σ^2, then the limiting form of the distribution of

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- **Theorem 8.2**:

  Central Limit Theorem. If $\bar{X}$ is the mean of a random sample of size n taken from a population with mean μ and finite variance σ^2, then the limiting form of the distribution of

  $$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

  as $n \to \infty$, is the standard normal distribution $n(z; 0, 1)$.

- The normal approximation for $\bar{X}$ will generally be good if $n \geq 30$.
- If $n < 30$, the approximation is good only if the population is not too different from a normal distribution.
- This is true no matter what the population distribution may be as long as the population has a finite variance σ^2.
- This marvellous and famous fact in probability theory is called the Central Limit Theorem.
- This is remarkable and an universal probability law.
- If the population is known to be normal, the sampling distribution of $\bar{X}$ will follow a normal distribution exactly, no matter how small the size of the samples.

Sampling Distribution of Means VI

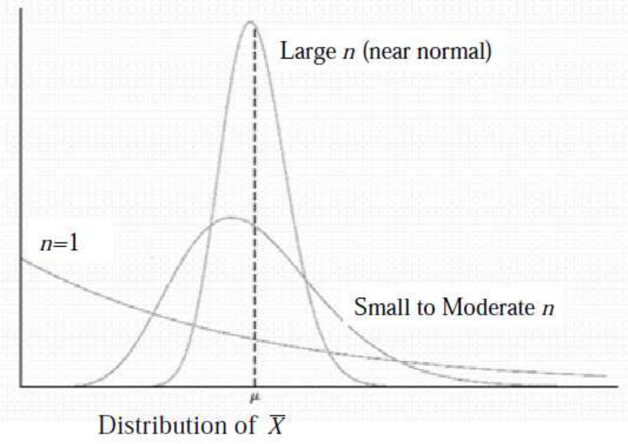

Figure: Illustration of the central limit theorem (distribution of $\bar{X}$ for $n = 1$, moderate n, and large n).