Lecture 9

Some Continuous Probability Distributions I

Lecture Information

Ceng272 *Statistical Computations* at April 19, 2010

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Contents

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Uniform distribution (Rectangular distribution): The density function of the continuous uniform random variable X on the interval $[A, B]$ is

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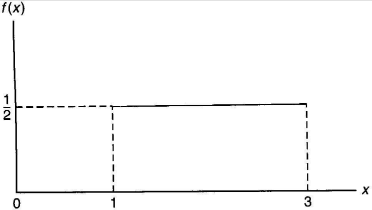

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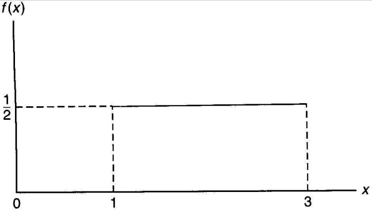

Figure: The density function for a random variable on the interval $[1, 3]$.

Example. Let T be the waiting for a bus when a bus comes every 30 min,

$$f(t) = \frac{1}{30}, \quad 0 \leq t \leq 30$$

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- Mean is at the center of the range as we would expect.

Normal Distribution I

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- The term normal distribution is a historical accident because there is nothing particularly normal about the normal distribution and nor is there anything abnormal about other distribution.

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- Normal Distribution:

The density function of the normal random variable X, with mean μ and variance σ^2 is

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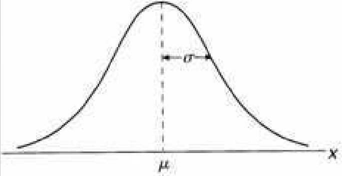

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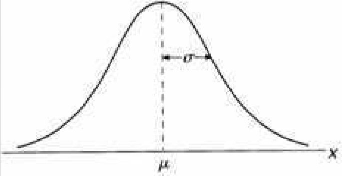

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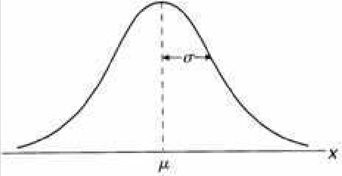

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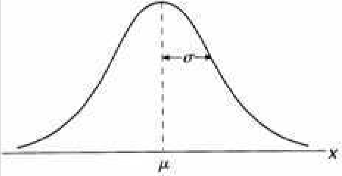

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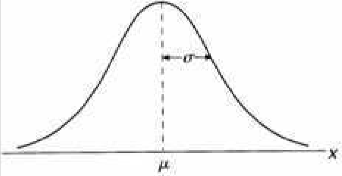

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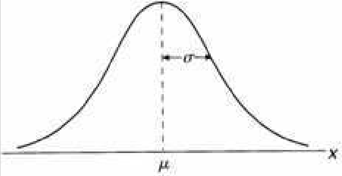

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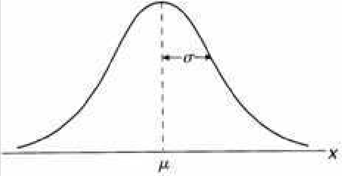

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  - The term $\frac{1}{\sqrt{2\pi}\sigma}$ makes sure $\int_{-\infty}^{\infty} n(x;\mu,\sigma)dx = 1$

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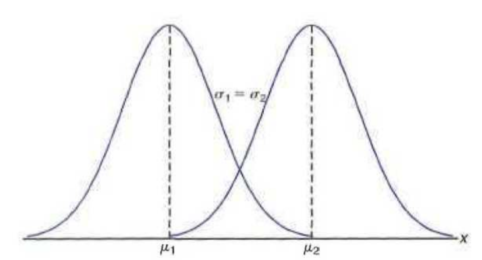

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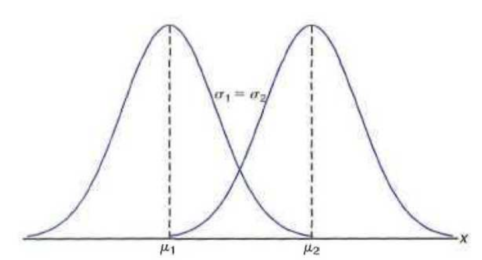

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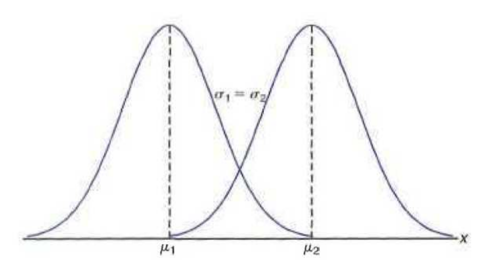

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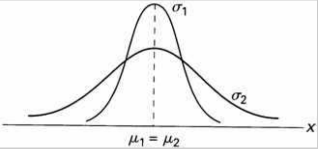

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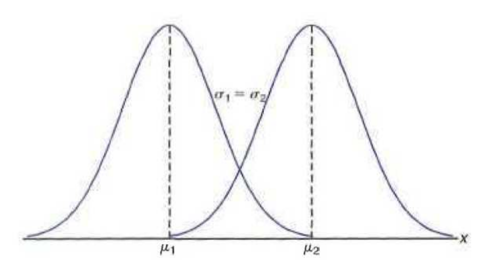

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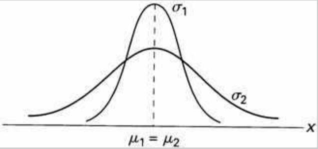

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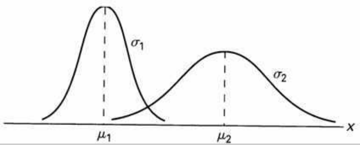

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6. Both tails become dramatically thin beyond $\pm 3\sigma$ from the mean μ.

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- Then, tabulation of normal curve areas is necessary.

Areas Under the Normal Curve I

- The probability of the random variable X assuming a value between x_1 and x_2.

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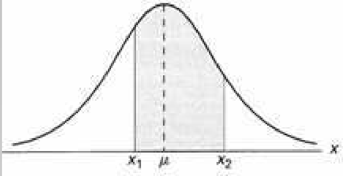

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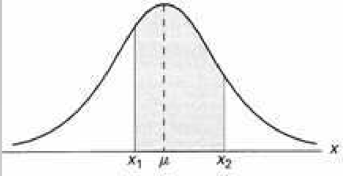

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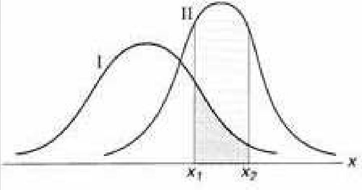

Figure: $P(x_1 < X < x_2)$ for different normal curves.

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  The distribution of a normal random variable with mean 0 and variance 1 is called a **standard normal distribution**.

- Transformation: $Z = \frac{X-\mu}{\sigma}$, $z_1 = \frac{x_1-\mu}{\sigma}$, $z_2 = \frac{x_2-\mu}{\sigma}$
- Then; $E(Z) = \mu = 0$ and $\sigma_Z^2 = 1$

$$P(x_1 < X < x_2) = \frac{1}{\sqrt{2\pi\sigma}} \int_{x_1}^{x_2} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx \Rightarrow \frac{1}{\sqrt{2\pi}} \int_{z_1}^{z_2} e^{\frac{1}{2}(z)^2} dz$$

$$= \int_{z_1}^{z_2} n(z; 0, 1) = P(z_1 < Z < z_2)$$

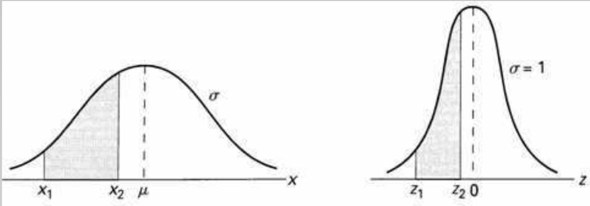

Figure: The original and transformed normal distributions.

Areas Under the Normal Curve III

Example 6.2: Given a standard normal distribution, find the area under the curve that lies

i to the right of $z = 1.84$

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$$1 - 0.9671 = 0.0329$$

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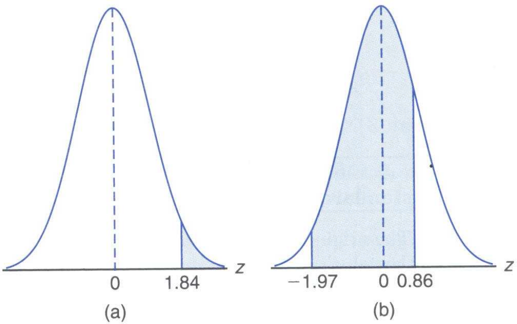
The area to the left of $z = 0.86$ minus the left of $z = -1.97$

$$0.8051 - 0.0244 = 0.7807$$



Areas Under the Normal Curve IV

TABLE A.3 (continued) Areas Under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Areas Under the Normal Curve V

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  - and the first row indicates the second digit under the decimal of the corresponding values of z according to the column values.
  - Suppose we want to find the area between 0 and 1.23, then all we need to do is to read the entry where the row of 1.2 and the column of 0.03 come across.

Areas Under the Normal Curve VI

Example 6.3: Given a standard normal distribution, find the value of k such that

- i $P(Z > k) = 0.3015$

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ii $P(k < Z < -0.18) = 0.4197$

$$P(Z < -0.18) - P(Z < k) = 0.4286 - P(Z < k) = 0.4197 \Rightarrow k = -2.37$$

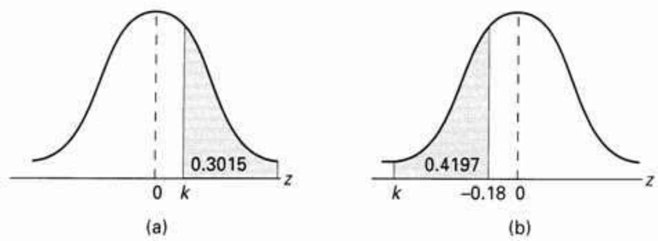

Figure: Areas for Example 6.3.

Areas Under the Normal Curve VII

- **Example 6.4**: Given a random variable X having a normal distribution with $\mu = 50$ and $\sigma = 10$,

Areas Under the Normal Curve VII

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Areas Under the Normal Curve VII

- **Example 6.4**: Given a random variable X having a normal distribution with $\mu = 50$ and $\sigma = 10$,
- Find the probability that X assumes a value between 45 and 62.
- Solution:

$$x_1 = 45 \; and \; x = 62 \xrightarrow{transformation} z_1 = \frac{45 - 50}{10} = -0.5, \; z_2 = \frac{62 - 50}{10} = 1.2$$

$$P(45 < X < 62) = P(-0.5 < Z < 1.2)$$
$$= P(Z < 1.2) - P(Z < -0.5) = 0.8849 - 0.3085 = 0.5764$$

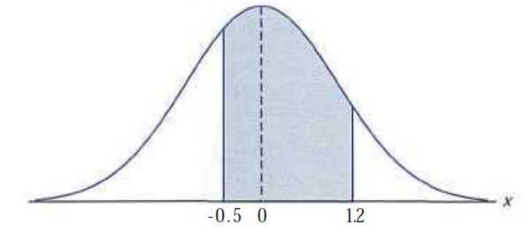

Figure: Area for Example 6.4.

Areas Under the Normal Curve VIII

- **Example 6.5** Given that a normal distribution with $\mu = 300$ and $\sigma = 50$, find the probability that X assumes a value greater than 362.

Areas Under the Normal Curve VIII

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- Solution:

Areas Under the Normal Curve VIII

- **Example 6.5** Given that a normal distribution with $\mu = 300$ and $\sigma = 50$, find the probability that X assumes a value greater than 362.
- Solution:

$$z = \frac{362 - 300}{50} 1.24$$

$$-P(X > 362) = P(Z > 1.24) = 1 - P(Z < 1.24)$$

$$= 1 - 0.8925 = 0.1075$$

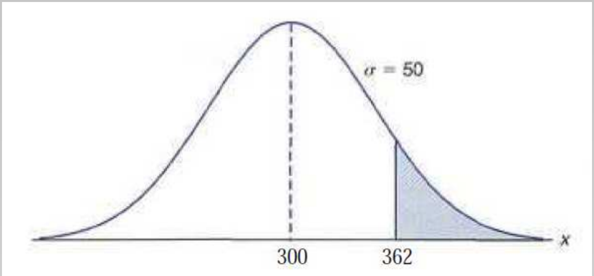

Figure: Area for Example 6.5.

Areas Under the Normal Curve IX

- **Using the Normal Curve in Reverse**

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Areas Under the Normal Curve IX

- **Using the Normal Curve in Reverse**
- We might want to find the value of z corresponding to a specified probability.
- The steps:
  1. Begin with a known area or probability.
  2. Find the z values corresponding to the tabular probability that comes closest to the specified probability.
  3. Determine x by rearranging the formula

$$z = \frac{x - \mu}{\sigma} \text{ to give } x = \sigma z + \mu$$

Areas Under the Normal Curve X

Example 6.6: Given a normal distribution with $\mu = 40$ and $\sigma = 6$, find the value of *x* that has

- 45% of the area to the left

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From Table A.3 we find $P(Z < -0.13) = 0.45$. Hence

$$x = 6 * (-0.13) + 40 = 39.22.$$

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From Table A.3, we find $P(Z < 1.08) = 086$. Hence

$$x = 6 * (1.08) + 40 = 46.48.$$

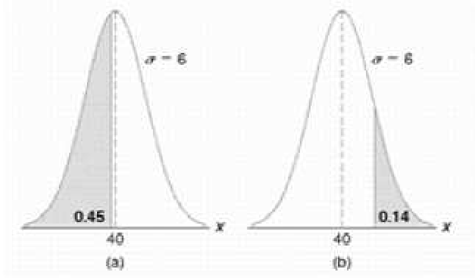

Figure: Areas for Example 6.6.

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- Some of the many problems for which the normal distribution is applicable are treated in the following examples.

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$$z = \frac{2.3 - 3}{0.5} = -1.4 \Rightarrow P(X < 2.3) = P(Z < -1.4) = 0.0808$$

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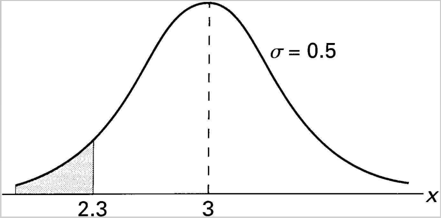

Figure: Area for Example 6.7.

Applications of the Normal Distribution II

- **Example 6.8**: An electrical firm manufactures light bulbs that have a life, before burn-out, that is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours.

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- Solution:

$$z_1 = \frac{778 - 800}{40} = -0.55 \textit{ and } z_2 = \frac{834 - 800}{40} = 0.85$$

$$P(778 < X < 834) = P(-0.55 < Z < 0.85)$$

$$= P(Z < 0.85) - P(Z < -0.55) = 0.8023 - 0.2912 = 0.5111$$

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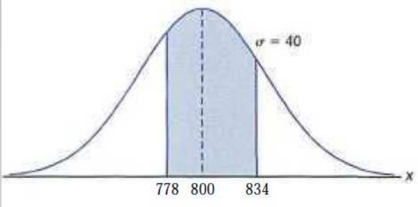

Figure: Area for Example 6.8.

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- On the average, how many manufactured ball bearings will be scrapped?.
- Solution:

$$z_1 = \frac{2.99 - 3.0}{0.005} = -2.0$$

$$z_2 = \frac{3.01 - 3.0}{0.005} = 2.0$$

$$\Rightarrow P(2.99 < X < 3.01)$$

$$= P(-2.0 < Z < 2.0)$$

$$= 1 - 2 * P(Z < -2.0)$$

$$= 1 - 2 * 0.0228 = 0.9544$$

Applications of the Normal Distribution III

- **Example 6.9**: The buyer sets specifications on the diameter to be 3.0 ± 0.01 cm.
- It is known that in the process the diameter of a ball bearing has a normal distribution with mean $\mu = 3.0$ and standard deviation $\sigma = 0.005$.
- On the average, how many manufactured ball bearings will be scrapped?.
- Solution:

$$z_1 = \frac{2.99 - 3.0}{0.005} = -2.0$$

$$z_2 = \frac{3.01 - 3.0}{0.005} = 2.0$$

$$\Rightarrow P(2.99 < X < 3.01)$$

$$= P(-2.0 < Z < 2.0)$$

$$= 1 - 2 * P(Z < -2.0)$$

$$= 1 - 2 * 0.0228 = 0.9544$$

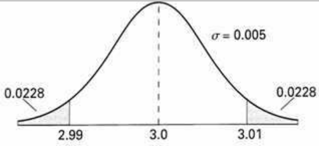

Figure: Area for Example 6.9.

Applications of the Normal Distribution IV

- **Example 6.10**: Gauges are used to reject all components where a certain dimension is not within the specification $1.50 \pm d$.

Applications of the Normal Distribution IV

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- Solution:

From Table A.3 we know that

$$P(-1.96 < Z < 1.96) = 0.95$$

$$1.96 = \frac{(1.50 + d) - 1.50}{0.2}$$

$$\Rightarrow d = 0.2 * 1.96 = 0.392$$

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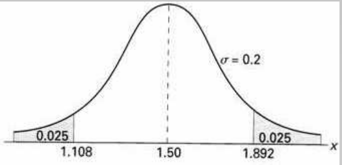

Figure: Specifications for Example 6.10.

Applications of the Normal Distribution V

- **Example 6.11**: A certain machine makes electrical resistors having a mean resistance of 40 ohms and a standard deviation of 2 ohms.

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From Table A.3 we know that

$$z = \frac{43 - 40}{2} = 1.5$$

$$P(X > 43) = P(Z > 1.5)$$

$$= 1 - P(Z < 1.5) = 1 - 0.9332$$

$$= 0.0668$$

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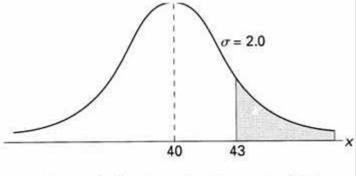

Figure: Area for Example 6.11.

Applications of the Normal Distribution VI

- **Example 6.13**: The average grade for an exam is 74, and the standard deviation is 7.

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- Solution:

$$1 - 0.12 = 0.88 = P(Z < 1.175)$$

$$1.175 = \frac{x - 74}{7} \Rightarrow$$
$$x = 7 * 1.175 + 74 = 82.225$$

The lowest *A* is 83 and the highest *B* is 82.

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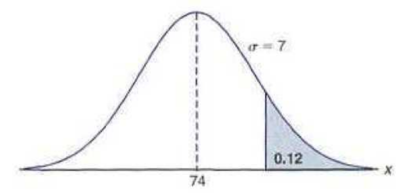

Figure: Area for Example 6.13.